The large quasar structures at high redshift and their compatibility with concordance cosmology

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Clustering Measurements of Active Galactic Nuclei Garching, Germany July 17, 2014



Outline

Quasar large-scale structure

Large Quasar Groups

Mock quasar catalogues

Mock Large Quasar Groups

Compatibility with concordance cosmology

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Ongoing Work and Future work

Summary

Quasars are detectable at very high redshifts thanks to their high luminosities and with large quasar redshift surveys has been possible to trace the large-scale structure of the Universe.

- Their clustering is similar to bright galaxies.
- Quasars live in high mass haloes $(10^{12} 10^{13} M_{\odot})$.
- However quasar number density is about 100 1000 times lower than host haloes.

 \Rightarrow The fraction of active quasars (duty cycle) must be of the order of $10^{-2}-10^{-3}.$ Mean quasar lifetime is $\sim 10-100$ Myrs.

 Low density means that clustering measures are dominated by Poissonian noise.

Structures in the Cosmic Web

- The most widely used measures of clustering are expected values across the entire survey volume, as the correlation functions (power spectrum) or count-in-cells.
- However, the testing environmental dependencies or the presence of outliers requires the identification of local overdensities (peak or cluster search) or underdensities (void search).
- Historically, cluster searches have been very successful in obtaining new information about the large scale distribution of matter (galaxy clusters and groups, superclusters).

 Extreme overdensities might indicate deviations from the standard cosmology.

Large Quasar Groups

- Large Quasar Groups (LQG) are large associations of quasars in the LSS (some other names are used in the literature).
- Detected using Friends-of-Friends method (hierarchical clustering) plus a statistical significance test.
- They comprise the largest structures known with the mean size is of 240 h₇₀⁻¹ Mpc. Mainly filamentary in geometry, similar to superclusters (Einasto et al. 2007).

- They might correspond to large superclusters or walls.
- Large volume surveys are required for their detection.

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- Large volume surveys are required for their detection.
- More about LQG observations in Luis's talk.

- Clowes et al. have constructed a new catalogue of LQGs using the SDSS-DR7QSO redshift.
- Quasar sample: Northern Galactic Cap, $1.0 \le z \le 1.8$, $i \le 19.1$.
- Clowes et al. (2013) shown the result of the largest LQG in the catalogue, named Huge-LQG.
- The properties of this LQG make it a good candidate to be an outlier from the expected quasar large-scale structure.

The Huge-LQG

- It is the largest LQG in our catalogue consisting in 73 quasars.
- ▶ The estimated volume is $1.21 \times 10^8 h_{70}^{-3} \text{ Mpc}^3$, equivalent to a characteristic size $(V)^{\frac{1}{3}} = 495 h_{70}^{-1} \text{ Mpc}$. Peak overdensity of 1.2. Longest axis ~ $1240 h_{70}^{-1}$ Mpc.
- The significance of the structure against random catalogues is 3.81 s.d.
- Detection in MgII absorbers.





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 Scale of transition to homogeneity is approx. 260h⁻¹ Mpc for LCDM (Yadav et al. 2010), but this is the expected value for the underlying matter field.

- BUT quasars populate massive haloes, which are more clustered (higher bias).
- Risk of percolation.

Extreme structures in the LSS

- There have been many claims of extreme outliers in the LSS. These include the cold spot in the CMB (Vielva et al. 2004; Cruz et al. 2005), an underdense region in the galaxy LSS with dimensions of 300 h⁻¹Mpc (Frith et al. (2003)), large overdensities of galaxies (Gott et al. 2005, Baugh et al. 2004; Croton et al. 2004).
- However, the statistical analysis of these claims has shown that these are not significant enough to be consider a problem for the standard model (e.g. Mikelsons et al. 2009, Yaryura et al. 2011, Sheth & Diaferio 2011, Davis et al. 2011, Harrison et al. 2011).

Compatibility with concordance cosmology

- Do observational and mock LQG populations have similar properties?
- Can we find LQG like Huge-LQG in the LCDM cosmology?

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Can we infer properties of the galaxy LSS using LQGs?

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Can we infer properties of the galaxy LSS using LQGs?

A mock LQG catalogue is needed in order to answer these questions.

Mock quasar catalogue constrains

- Very large structures require a good handle of cosmic variance.
 ⇒ Large volume cosmological simulations are needed.
- Large volumes come at expense of mass and spatial resolution.
 Large volume simulations available are only dark matter.

 Very large merger trees and low mass resolution make a semi-analytical simulation impractical

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 \Rightarrow Halo Occupation Distribution is the ideal method to construct the mocks.

Mock quasar catalogue construction

- We used the Horizon Run 2 N-body simulation (Kim et al. 2009,2011).
 ACDM with WMAP 5-year parameters, boxsize of 7200*h*⁻¹
 Mpc and minimum halo mass 3.75*h*⁻¹ × 10¹² M_☉.
- We constructed 11 mock sample volumes in redshift range 1.2 ≤ z ≤ 1.6 (to avoid bias evolution).
- Populating Haloes with quasars using a Halo Occupation Distribution using Monte Carlo simulation. Luminosities assigned using Halo Abundance Matching (Vale & Ostriker 2004)
- 10 independent realizations of the HOD model. A total of 110 mock surveys.

We assumed that quasars follow the same HOD than galaxies but with a constant duty cycle in order to reproduce quasar number density (Padmabadham et al 2009). We used Berlind & Weinberg (2001) galaxy HOD model.

 $N_{cen} \sim Bernoulli(\langle N|M \rangle_{cen}), N_{sat} \sim Poisson(\langle N|M \rangle_{cen})$

$$\langle N|M \rangle_{cen} = f_{on}\Theta(M - M_{min})$$

 $\langle N|M \rangle_{sat} = f_{on}\left(\frac{M}{M_1}\right)^{\alpha}\Theta(M - M_{min})$

Fiducial Halo Occupation Distribution

- We use the Kravtsov et al. (2004) scaling relations to avoid overfitting of the model to observations.

 α = 1 and M₁ = 20M_{min}
- Free parameters, M_{min} and f_{on}, are fitted to quasar number density and two-point correlation function.
 M_{min} = 6.2 × 10¹² M_☉
 f_a = 0.002 ⇒ t_a ≃ 20Myr

Autocorrelation function comparison



Mock Large Quasar Groups

- Distribution of characteristic size (D = V¹/₃) for DR7-LQGs and mock LQGs are consistent with same parent populations. (KS test p-value 0.21%)
- $P(D > 495) = 7 \times 10^{-4}$
- However the distribution of the maximum provides a better assessment of the likelihood of this object in the survey volume.



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The maximum in any sample can be used as statistic and it tends to an asymptotic distribution (as mean tends to a normal dist.), the Generalized Extreme Value distribution (GEV) (Gumbel 1958)

$$G(z) = \exp\left[-\left\{1+\gamma\left(\frac{z-\mu}{\sigma}\right)\right\}_{+}^{-1/\gamma}\right],$$

where $y_+ = max(y, 0)$. When $\gamma \longrightarrow 0$, G(z) tends to a Gumbel distribution

$$G_0(z) = \exp\left[-\exp\left\{-\left(\frac{z-\mu}{\sigma}\right)\right\}\right],$$

Is Huge-LQG compatible with ACDM?

- Empirical distribution of maximum: P(D > 495) = 0.07
- Asymptotic GEV: Extreme value index γ consistent with γ = 0 (i.e. Gumbel type) under a Likelihood-ratio test. Characteristic size: μ = 406.4 ± 3.2, σ = 31.8 ± 2.4
- Probability of Huge-LQG is P_G(D > 495Mpc) = 0.06. Return level (1/p): 17 survey volumes.

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 \Rightarrow Huge-LQG is compatible with the concordance cosmology if there is not a similar or larger structure a survey 17 times larger.





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Statistics for quasar number

 DR7 LQGs and mock LQGs distribution in quasar number are also consistent with same parent population. (KS test p-value 13%)

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$$P(N_q > 73) = 4 \times 10^{-4}$$

- GEV $P_G(N_q > 73) = 0.025$ ($\mu = 45.78 \pm 0.74$, $\sigma = 7.42 \pm 0.56$)
- Return level: 40 volumes (More unlikely)



Statitics for quasar number





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Ongoing Work and Future work

Ongoing Work

Tracing LQG back to galaxy large-scale structure. Sensitivity to shot noise? Potentials? Weaknesses?

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Minkowski funtionals.

Ongoing Work and Future work

Ongoing Work

- Tracing LQG back to galaxy large-scale structure. Sensitivity to shot noise? Potentials? Weaknesses?
- Minkowski funtionals.

Future work

- Is FoF the best cluster finder for quasars?
- Better HOD.
- Testing different cosmologies and non-Gaussianity. Simulation rescaling (Angulo & White 2010) Approximate dark matter halo simulations: Lognormal simulations (Cole et al. 2005), 2LPT (PThalos; Manera et al. 2013), Quick Particle Mesh (White et al. 2013)

Summary

- Quasars can be used as tracers to detect structures at redshifts higher than galaxies. LQGs is the best example of it.
- The largest LQG in observations (Huge-LQG) is compatible with a ACDM as far there is not similar structures in a larger survey.

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 More work is needed in order to link LQGs to matter large-scale structure.