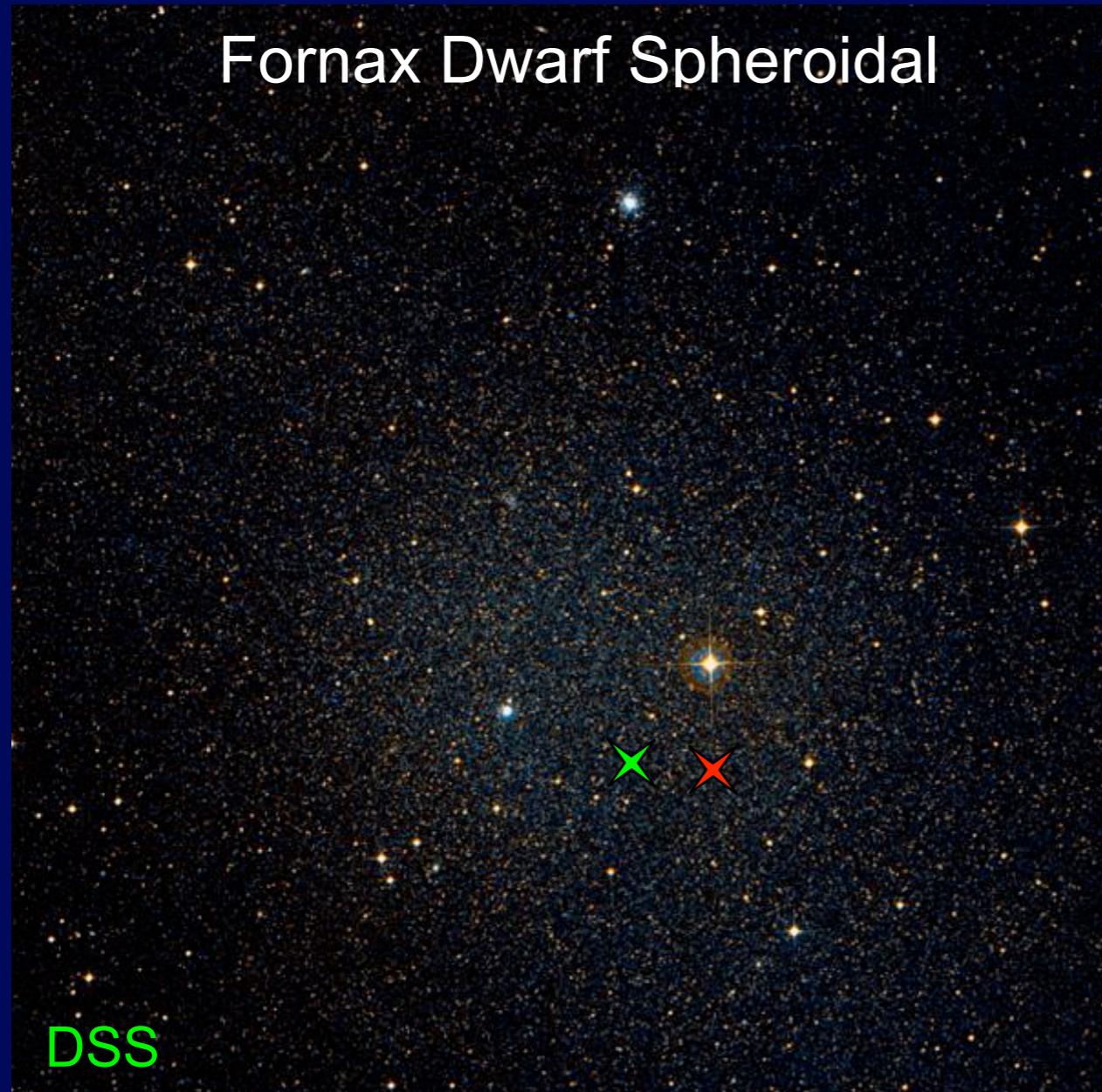


# *Joint mass and anisotropy modeling of the Fornax Dwarf Spheroidal*



with  
Chris GORDON (*Oxford*)  
Andrea BIVIANO (*Trieste*)

# *Motivations*

Large velocity data sets in Dwarf Spheroidals

Walker et al. 09

2267 member velocities in Fornax!

constraints on *Dark Matter normalization*

constraints on *Dark Matter inner slope*

constraints on *velocity anisotropy of stars*

# Mass anisotropy degeneracy

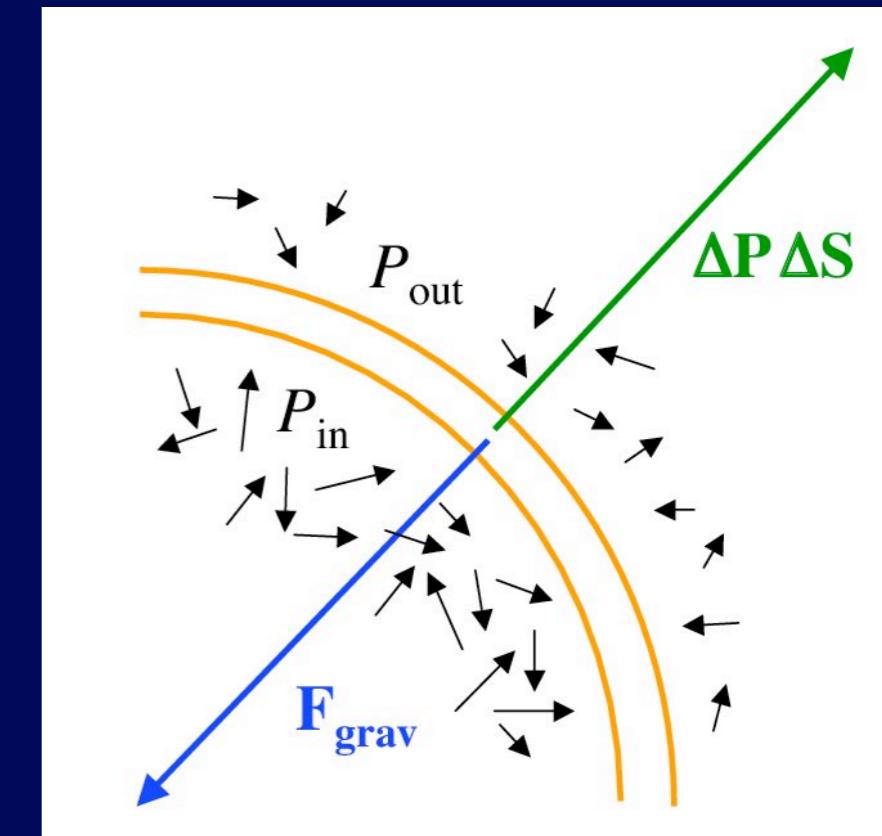
## Spherical stationary Jeans equation

anisotropic dynamical pressure

$$\frac{d(\nu\sigma_r^2)}{dr} + 2\frac{\beta(r)}{r}\nu\sigma_r^2 = -\nu\frac{GM(r)}{r^2}$$

$$\beta(r) = 1 - \frac{\sigma_\theta^2(r)}{\sigma_r^2(r)} = \text{velocity anisotropy}$$

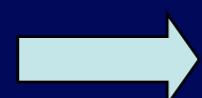
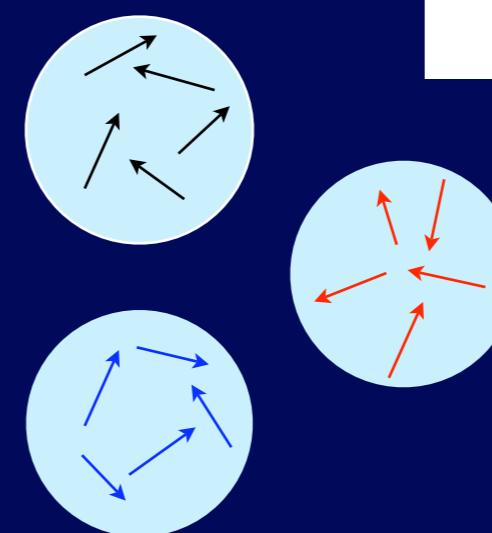
tracer density



isotropic orbits:  $\beta = 0$

radial orbits:  $\beta = 1$

circular orbits:  $\beta \rightarrow -\infty$



Mass / Anisotropy Degeneracy **MAD**

# *Exploratory Mass-Anisotropy Modeling*

1) Model-fitting of Line-of-Sight velocity dispersion profile

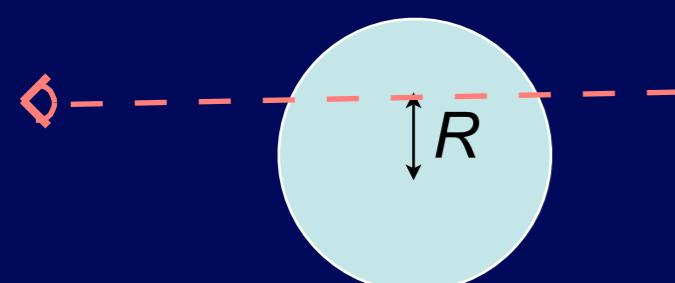
$$M_{\text{tot}}(r) + \beta(r) \longrightarrow \sigma_{\text{LOS}}(R)$$

# 1) assume both $M(r)$ & $\beta(r)$ & fit the LOS velocity dispersions

for  $\beta = 0$

line-of-sight velocity dispersion

Tremaine et al. 94; Prugniel & Simien 97



$$\Sigma(R) \sigma_{\text{los}}^2(R) = 2G \int_R^\infty \frac{\sqrt{r^2 - R^2}}{r^2} v(r) M(r) dr$$

tracer  
surface density

kernels for other simple  $\beta(r)$

Mamon & Łokas 05b

# *Exploratory Mass-Anisotropy Modeling*

1) Model-fitting of Line-of-Sight velocity dispersion profile

$$M_{\text{tot}}(r) + \beta(r) \longrightarrow \sigma_{\text{LOS}}(R)$$

2) Anisotropy inversion

$$\left\{ \begin{array}{l} \Sigma(R) \\ \sigma_{\text{LOS}}(R) \end{array} \right. + M_{\text{tot}}(r) \longrightarrow \beta(r)$$

Binney & Mamon 82

Tonry 83; Bicknell et al. 89

Solanes & Salvador-Solé 90

Dejonghe & Merritt 92

# *Exploratory Mass-Anisotropy Modeling*

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$$\left\{ \begin{array}{l} \Sigma(R) \\ \sigma_{\text{LOS}}(R) \end{array} \right. + M_{\text{tot}}(r) \longrightarrow \beta(r)$$

3) Mass inversion

$$\left\{ \begin{array}{l} \Sigma(R) \\ \sigma_{\text{LOS}}(R) \end{array} \right. + \beta(r) \longrightarrow M_{\text{tot}}(r)$$

Mamon & Boué 10  
Wolf et al. 10

# 3) Mass inversion

Mamon & Boué 10; Wolf et al. 10

*Kinematic deprojection & mass inversion of spherical systems with known anisotropy*

anisotropic kinematic projection

$$P(R) = 2 \int_R^\infty \left(1 - \beta \frac{R^2}{r^2}\right) p \frac{r dr}{\sqrt{r^2 - R^2}}$$

$p = \rho \sigma_r^2$  = dynamical pressure

$P = \Sigma \sigma_{\text{los}}^2$  = “projected pressure”

deprojection

$$(1 - \beta) p = \int_r^\infty K[\beta(s)] \int_s^\infty \frac{dP}{dR} \frac{R dR}{\sqrt{R^2 - r^2}}$$

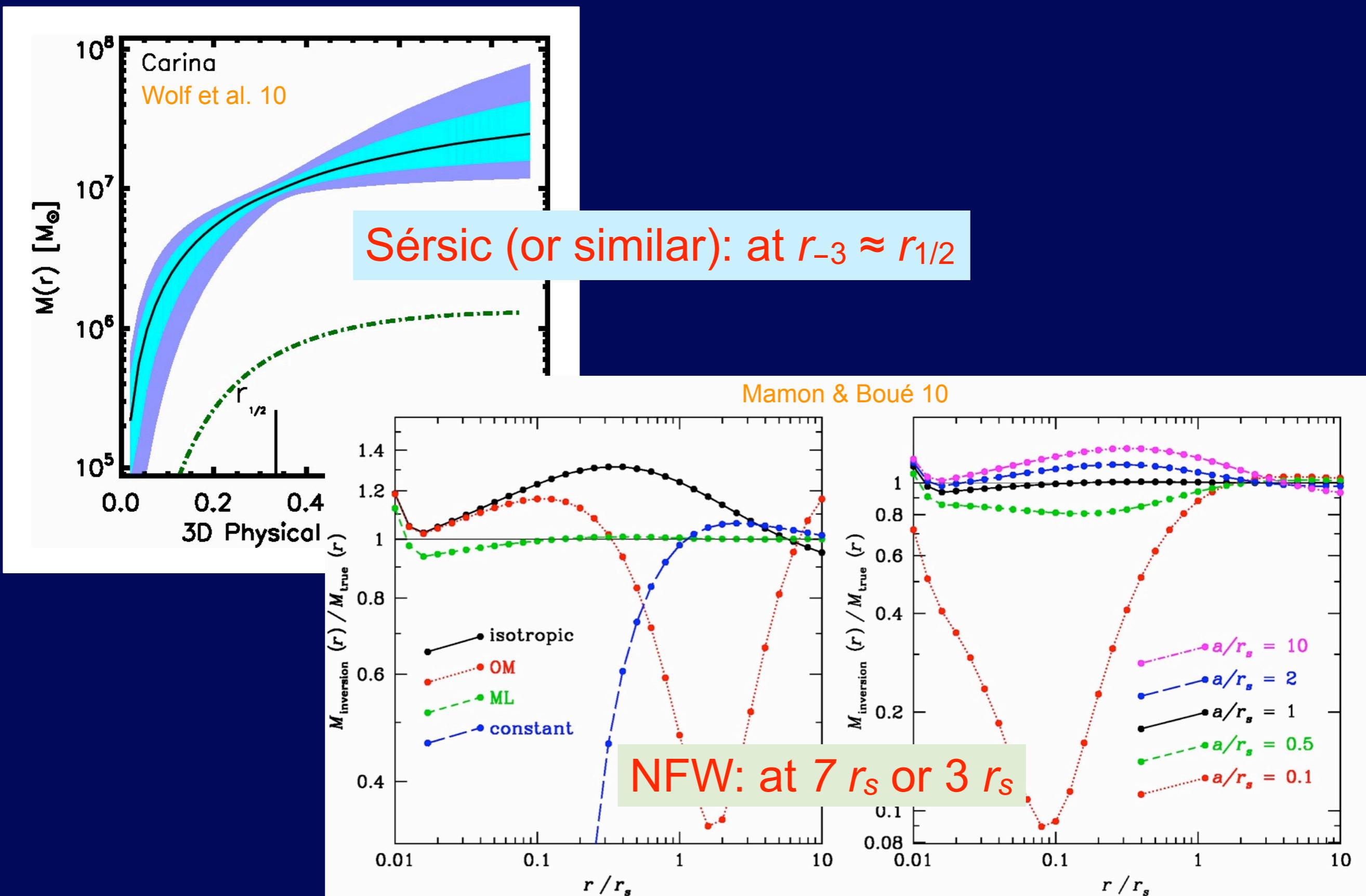
MB10:  $\downarrow \rightarrow$  simple  $\beta(r)$

$$= \int_r^\infty L_\beta(R, r) \frac{dP}{dR} dR$$

insert dynamical pressure into Jeans equation  $\rightarrow$  mass profile  
 simple  $\beta(r)$ : single integral!

$$v_c^2(r) = \frac{1}{\pi [1 - \beta(r)] \rho(r)} \int_r^\infty \left\{ \frac{R}{\sqrt{R^2 - r^2}} \frac{d^2 P}{dR^2} - C_\beta(R, r) \frac{dP}{dR} \right\} dR$$

# Radius where mass is independent of anisotropy



# *Exploratory Mass-Anisotropy Modeling*

1) Model-fitting of Line-of-Sight velocity dispersion profile

$$M_{\text{tot}}(r) + \beta(r) \longrightarrow \sigma_{\text{LOS}}(R)$$

2) Anisotropy inversion

$$\left\{ \begin{array}{l} \Sigma(R) \\ \sigma_{\text{LOS}}(R) \end{array} \right. + M_{\text{tot}}(r) \longrightarrow \beta(r)$$

3) Mass inversion

$$\left\{ \begin{array}{l} \Sigma(R) \\ \sigma_{\text{LOS}}(R) \end{array} \right. + \beta(r) \longrightarrow M_{\text{tot}}(r)$$

4) Fitting stars in Projected Phase Space **MAMPOSSt**

Mamon, Biviano & Boué 11, to be subm

$$v(r) + M_{\text{tot}}(r) + \beta(r) + \{v\}(r) \longrightarrow g(R, v_{\text{LOS}})$$

# Other popular modeling methods

gaussian  $\{v_{\text{LOS}}(R)\}$       lose info on  $\beta$   
 $v(r) + M_{\text{tot}}(r) + \beta[r] + \{v_{\text{LOS}}\}(R)$        $\longrightarrow$        $g(R, v_{\text{LOS}})$

dispersion-kurtosis      Łokas 02; see Łokas, Mamon & Prada 05  
 $M_{\text{tot}}(r) + \beta$        $\longrightarrow$        $\sigma_{\text{LOS}}(R) + \kappa_{\text{LOS}}(R)$   
must assume  $\beta = \text{cst}$

distribution function modeling      Dejonghe & Merritt 92; Merritt & Saha 93  
 $M_{\text{tot}}(\mathbf{r}) + f(E, J)$  or  $\{f_i(E, J)\}$        $\longrightarrow$        $g(R, v_{\text{LOS}})$   
don't know  $f(E, J)$ ; not sure that  $\{f_i(E, J)\} = \text{basis set}$

orbit modeling      Schwarzschild 79; de Lorenzi et al. 09  
 $M_{\text{tot}}(\mathbf{r}) + \{\text{orbits}\}$        $\longrightarrow$        $g(R, v_{\text{LOS}})$   
too slow for MCMC investigation of parameter space

# *Mass modeling of the Fornax dwarf spheroidal*

# Fornax data

$L_V = 1.9 \times 10^7 L_{\text{sun}}$

Irwin & Hatzimelitriou 95

$L_V = 0.9 \times 10^7 L_{\text{sun}}$

Walcher et al. 03

$m = 0.7$  Sersic distribution

Walcher et al. 03;  
Battaglia et al. 06

ellipticity:  $0.21 \rightarrow 0.36$

Battaglia et al. 06

main starburst: age = 5.4 Gyr

Saviane et al. 00



$M_{\text{stars}}/L_V = 4.8$

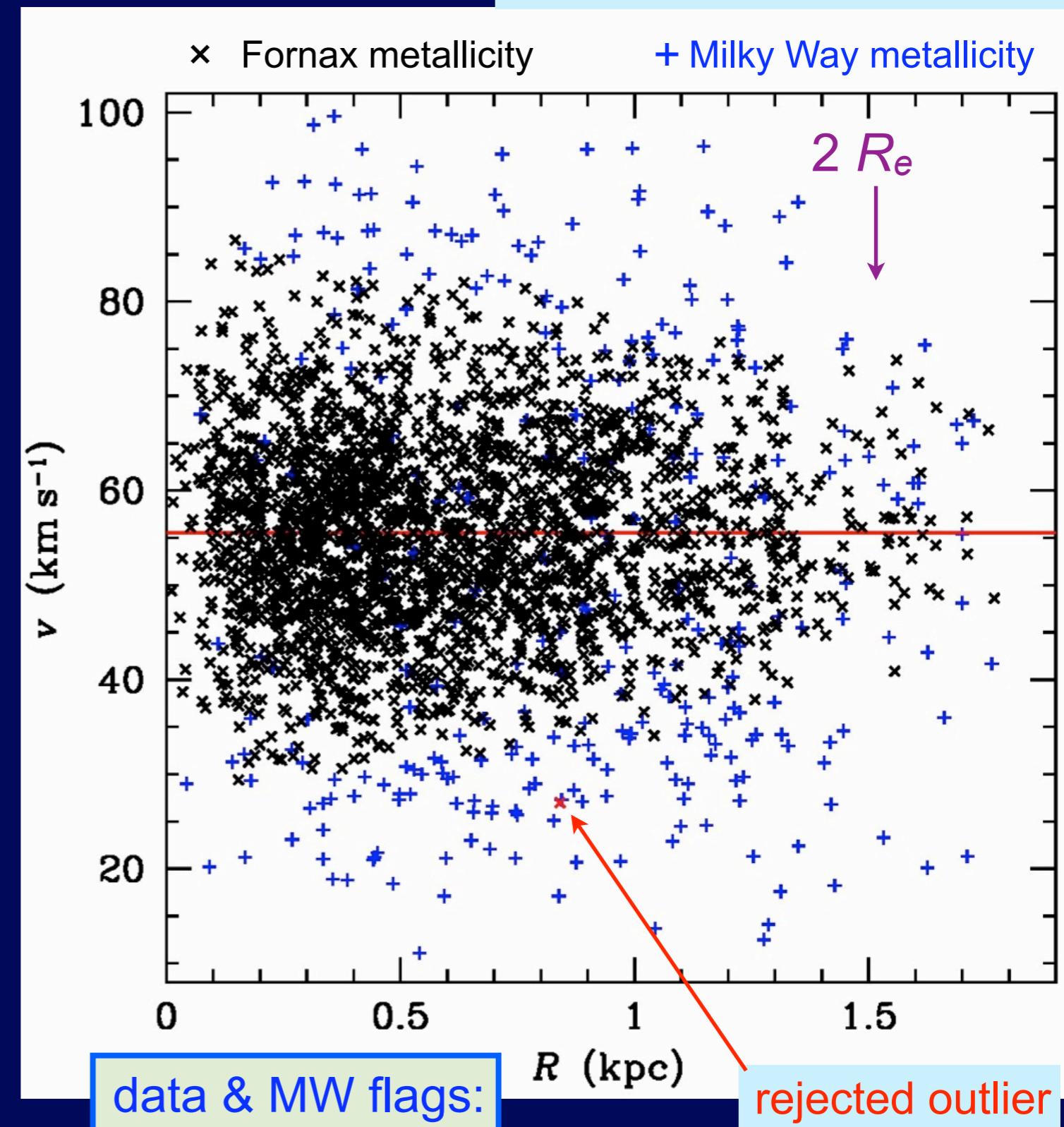
Walcher et al. 03

(uncertain) center:

Battaglia et al. 06

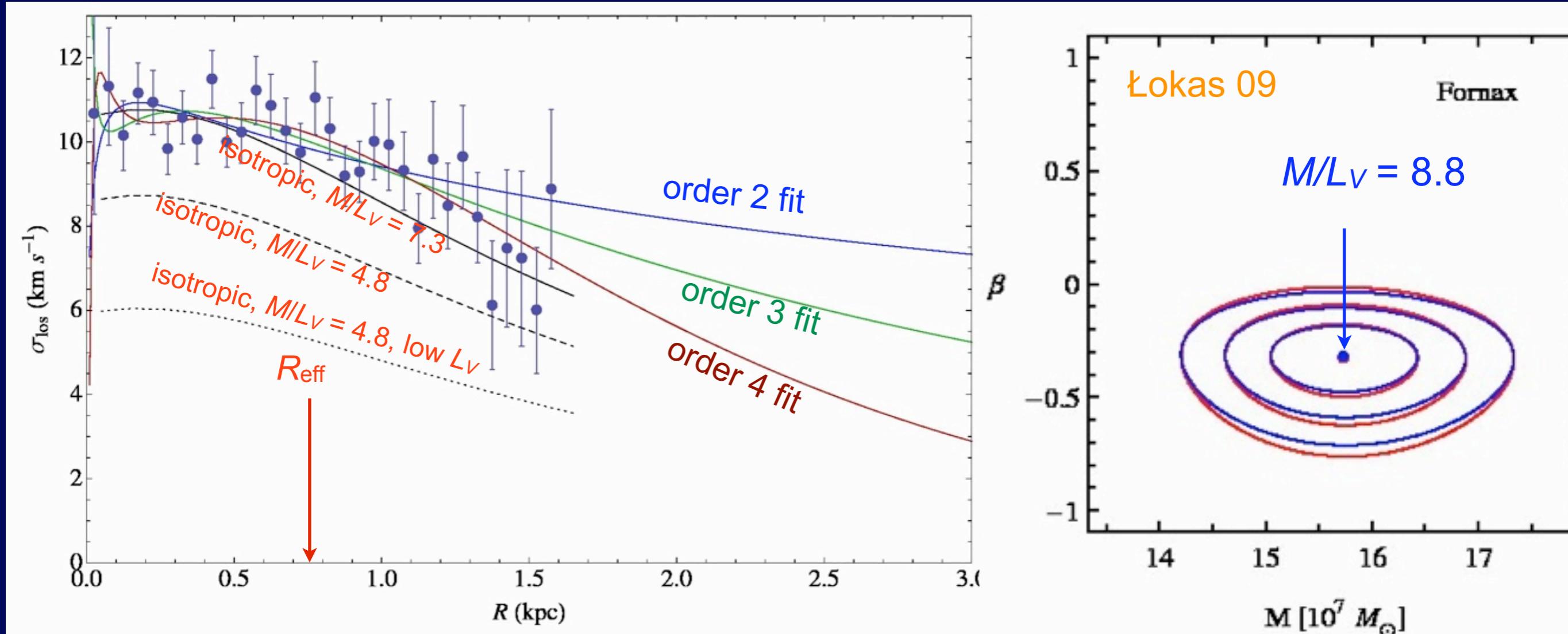
2633 velocities

2278 Fornax members



# Fornax: velocity dispersion profile

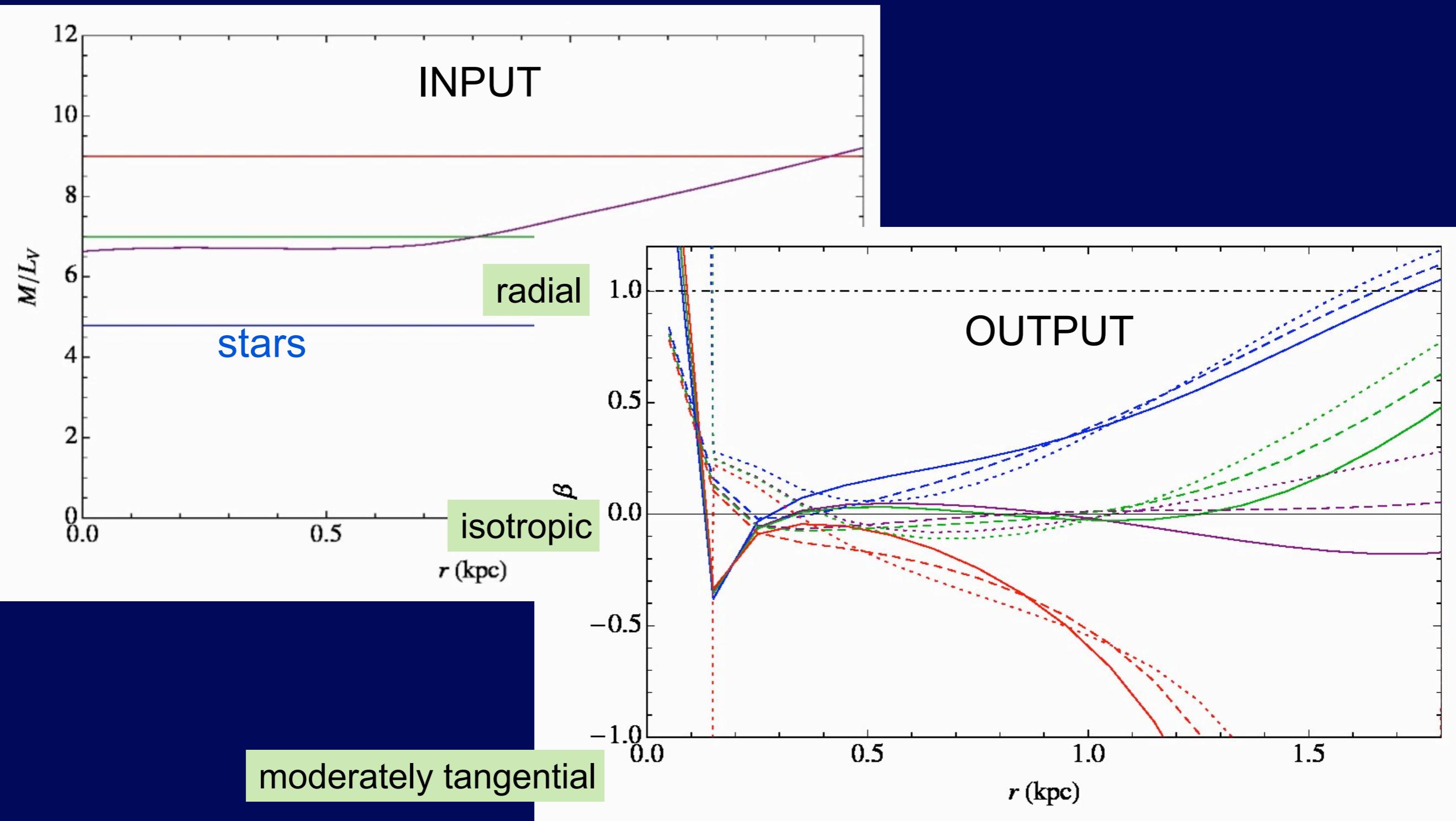
out to  $2 R_{\text{eff}}$



1/3 fraction of dark matter in inner regions? OR  $L_V$  underestimated by 40%?

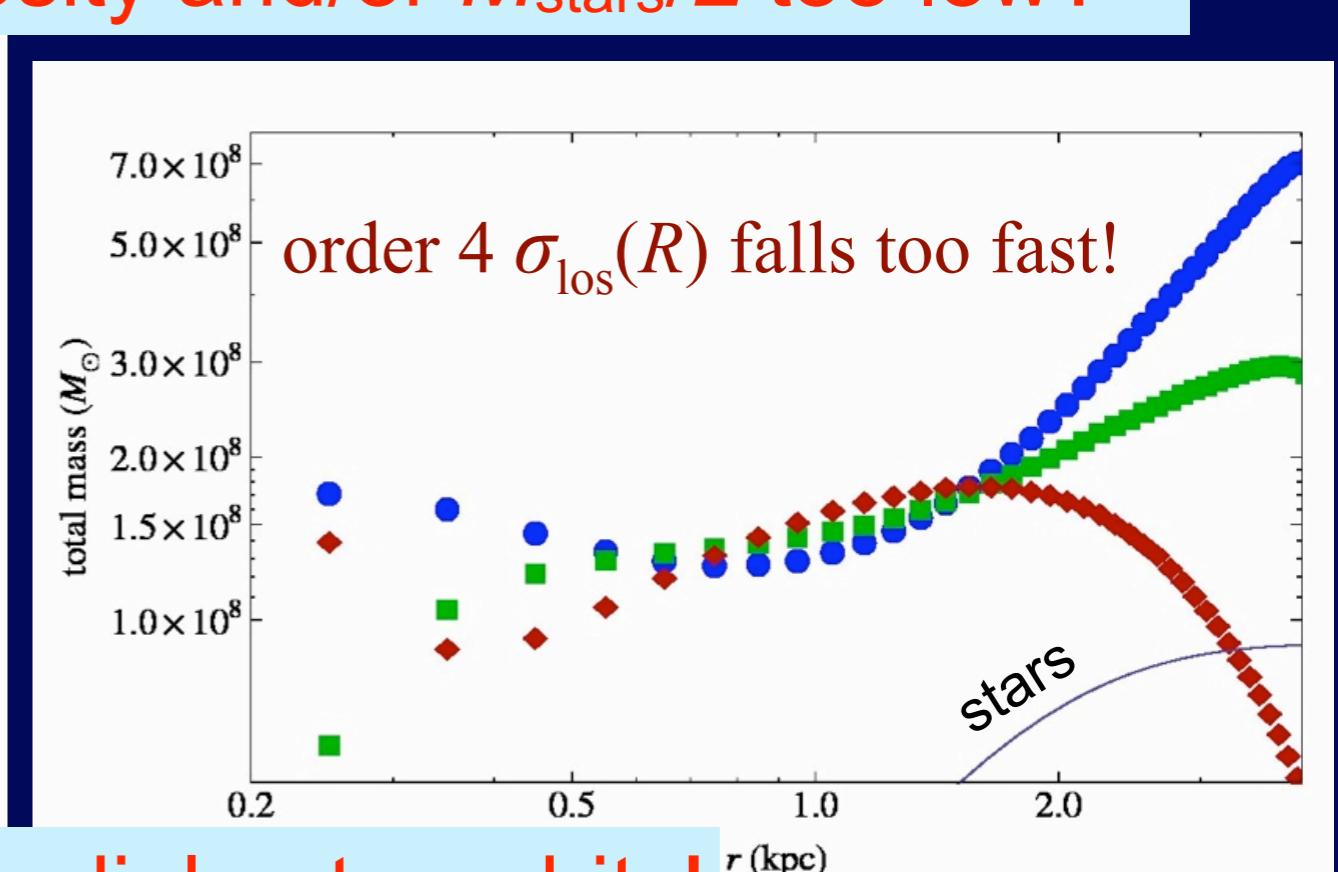
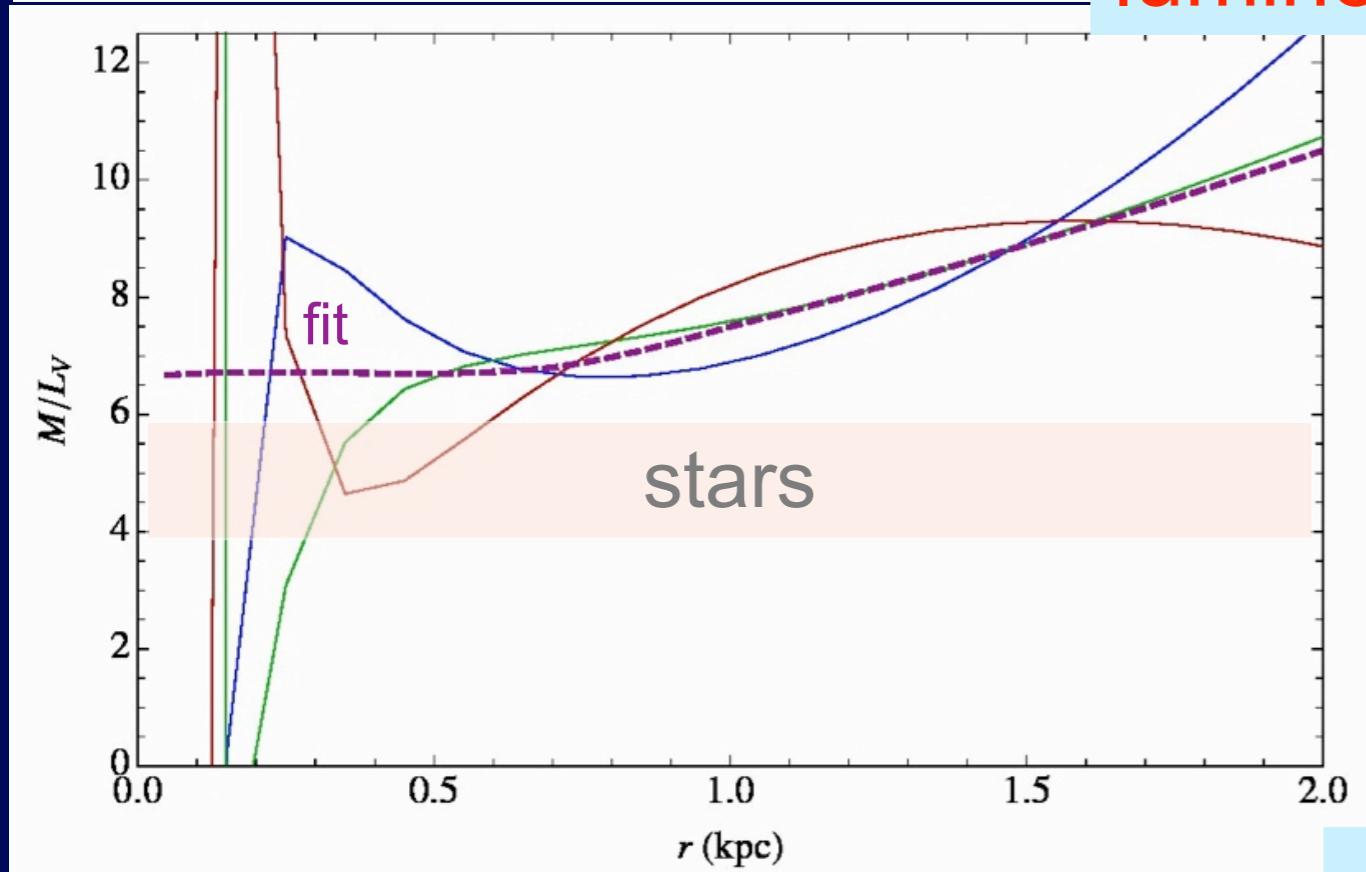
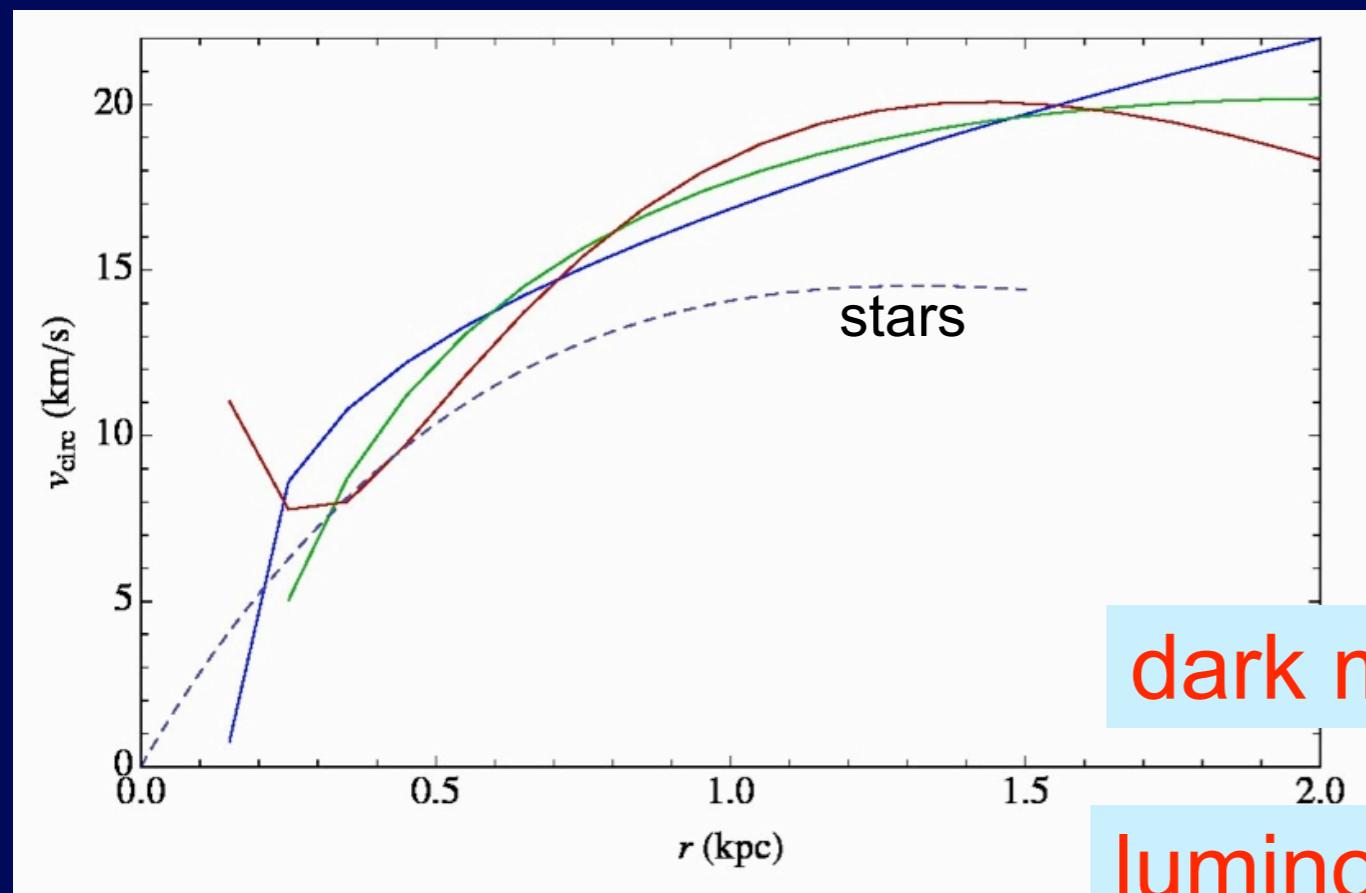
$M/L$  increases outwards? OR tangential outer orbits?

# Fornax: anisotropy inversion

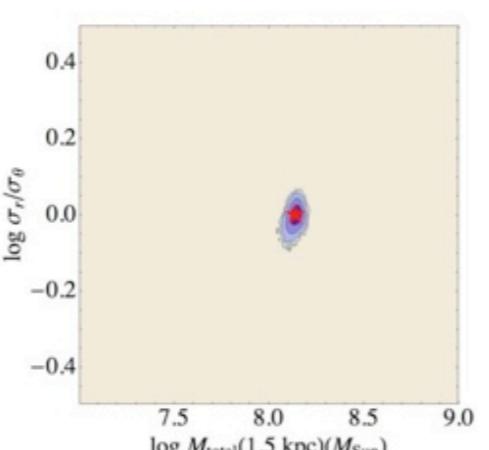
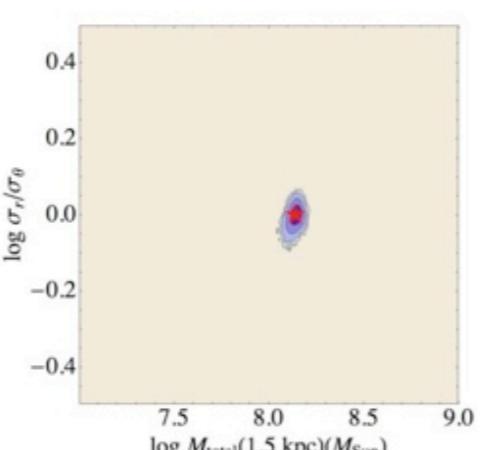
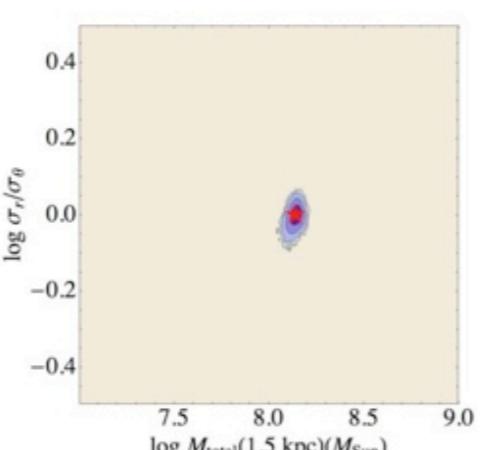
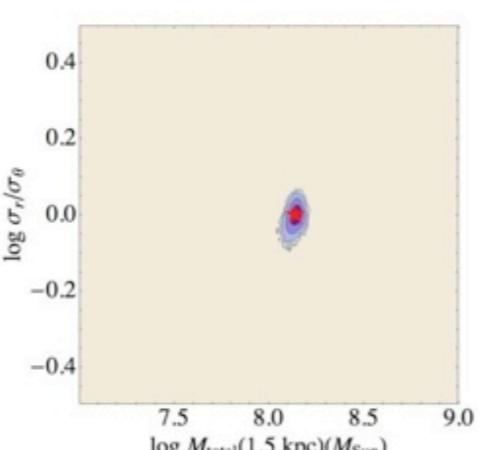
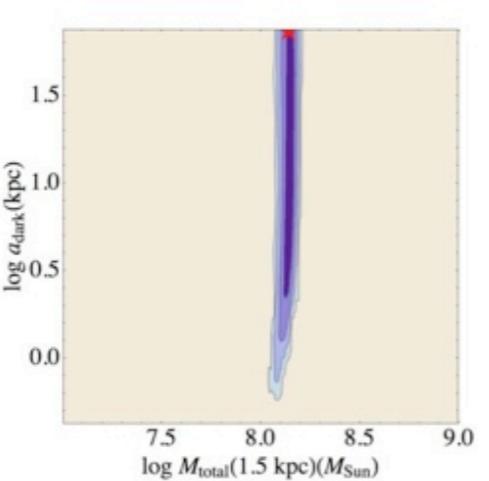
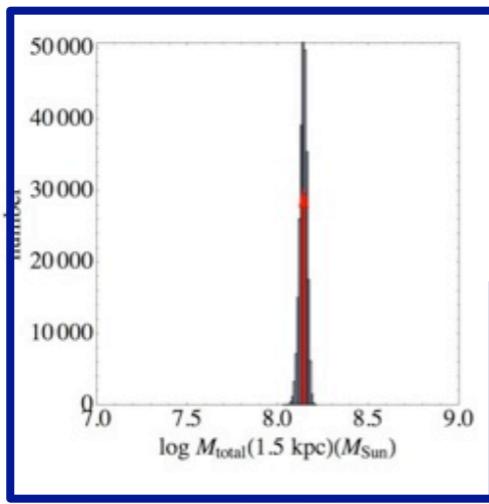
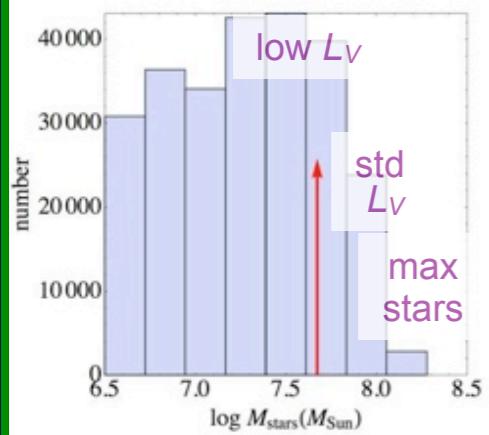


cst  $M/L \rightarrow$  radial (low  $M/L$ ) or tangential (high  $M/L$ ) orbits

# Fornax: isotropic mass inversion



# MAMPOSSt with MCMC



gaussian 3D velocities

cst  $\beta$

cst  $M_{\text{stars}}/L$

Kazantzidis DM:  $\rho \sim \exp(-r/a) / r$

MCMC: 9 chains of 30 000

well-defined  
 $M_{\text{tot}}(2R_e)$

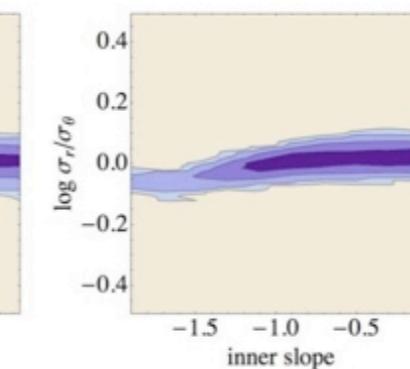
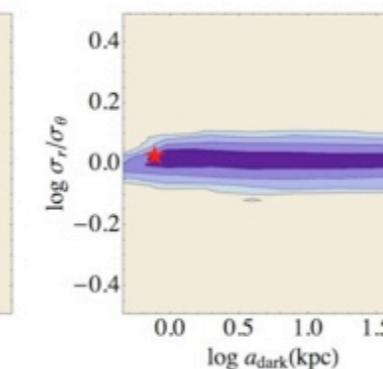
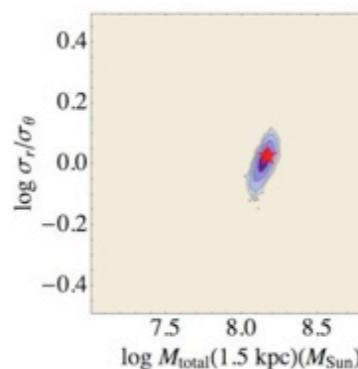
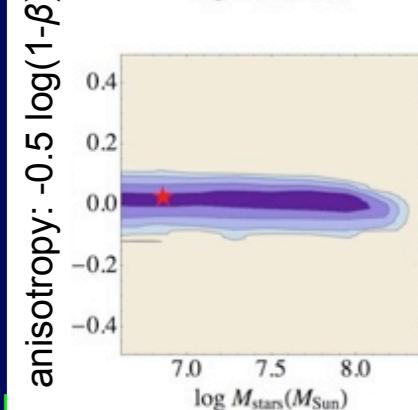
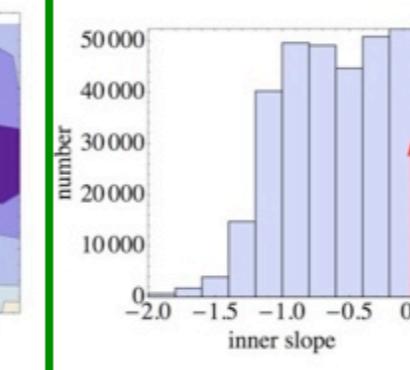
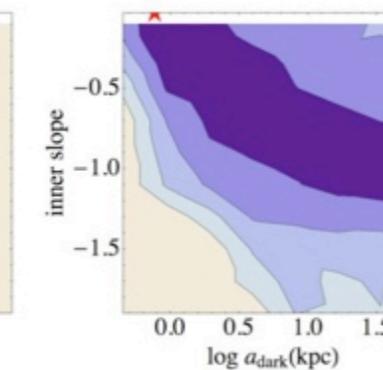
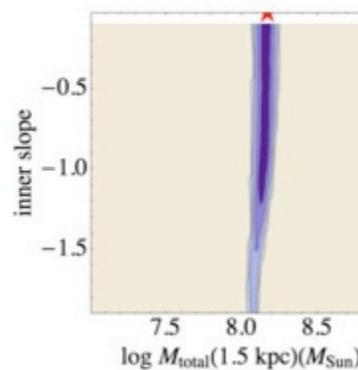
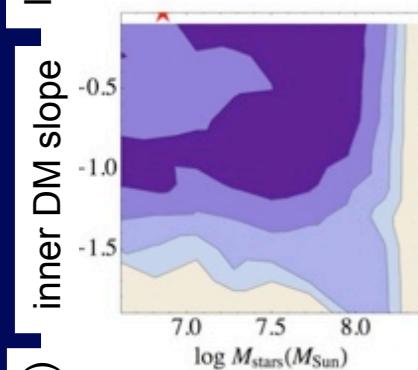
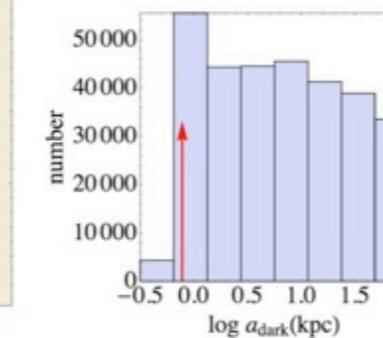
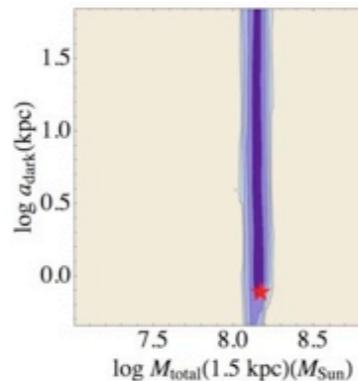
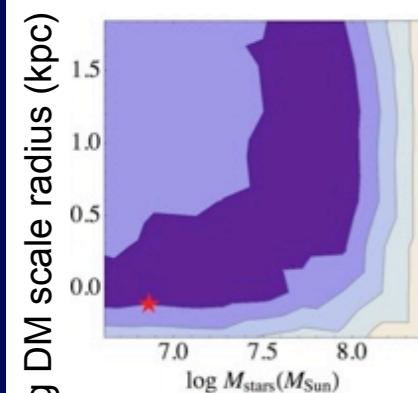
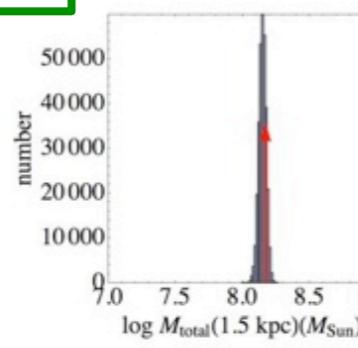
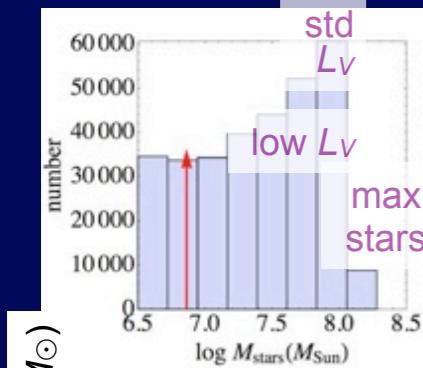
$a > 2 \text{ kpc}$   
(95% conf.)

Isotropic orbits!  
 $\beta = -0.03 \pm 0.09$

# Free inner dark matter slope...

std  $L_V$

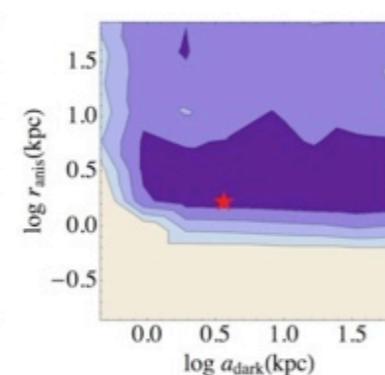
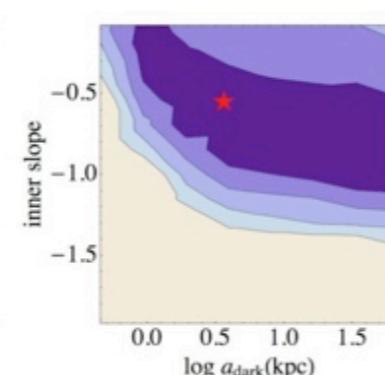
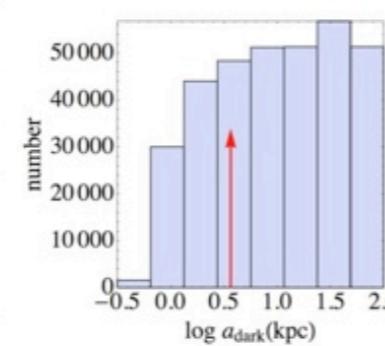
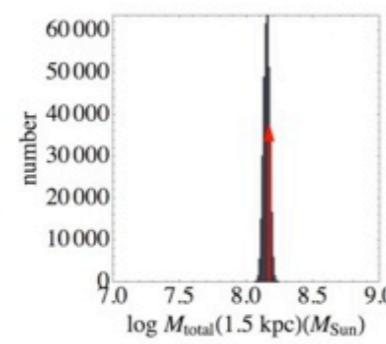
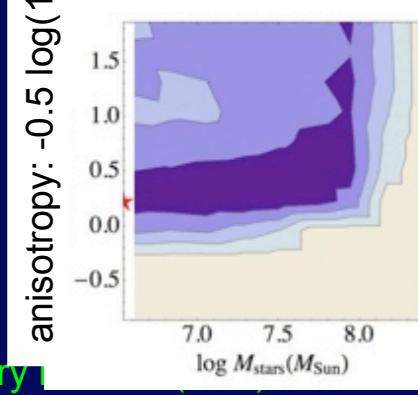
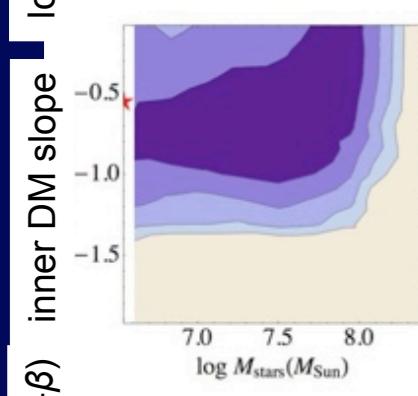
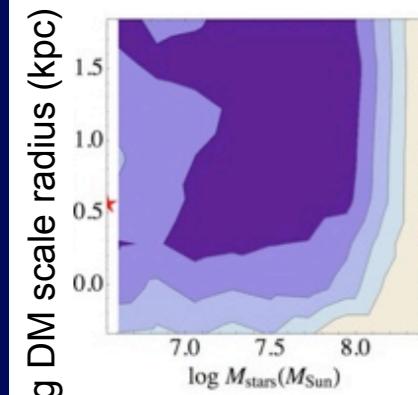
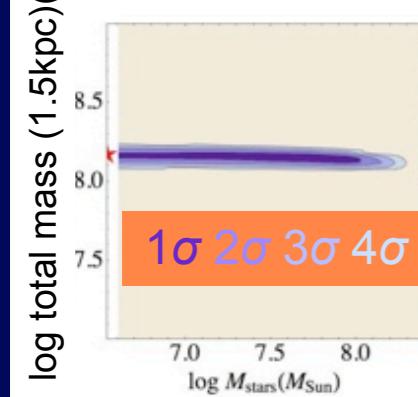
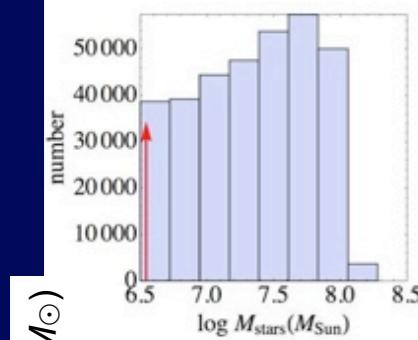
gaussian 3D velocities  
 cst  $\beta$   
 cst  $M_{\text{stars}}/L$   
 gen'l Kazantzidis DM:  $\rho \sim r^\gamma \exp(-r/a)$   
 MCMC: 11 chains of 30 000



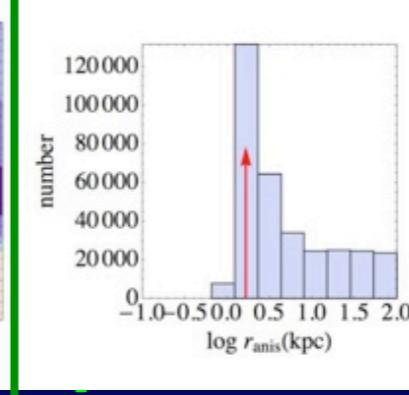
inner DM slope:  
 not too steep!

# Increasing anisotropy...

gaussian 3D velocities  
 $\beta = r^2/(r^2+a^2)$    Osipkov-Merritt  
 cst  $M_{\text{stars}}/L$   
 gen'l Kazantzidis DM:  $\rho \sim r^\gamma \exp(-r/a)$   
 MCMC: 11 chains of 30 000



anisotropy radius:  
 $1.6 < r_{\text{anis}} < 25 \text{ kpc}$   
 $(1\sigma)$



# Radial variations

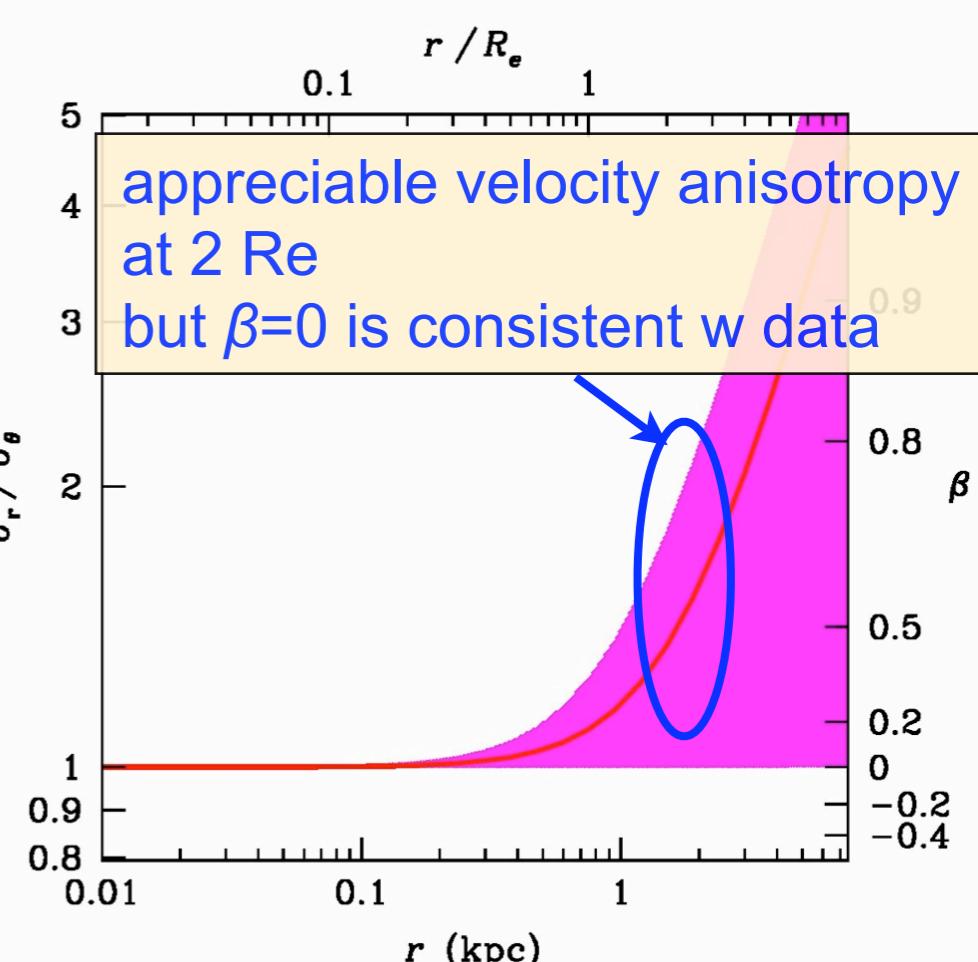
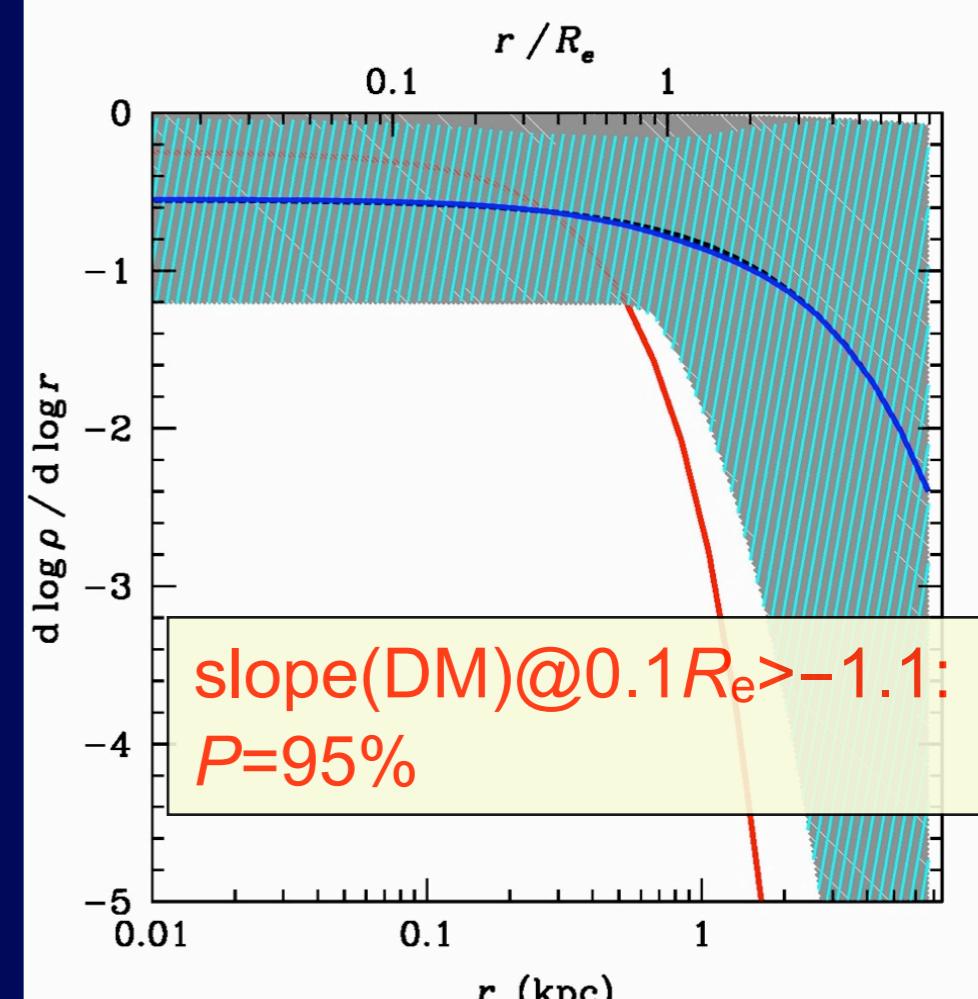
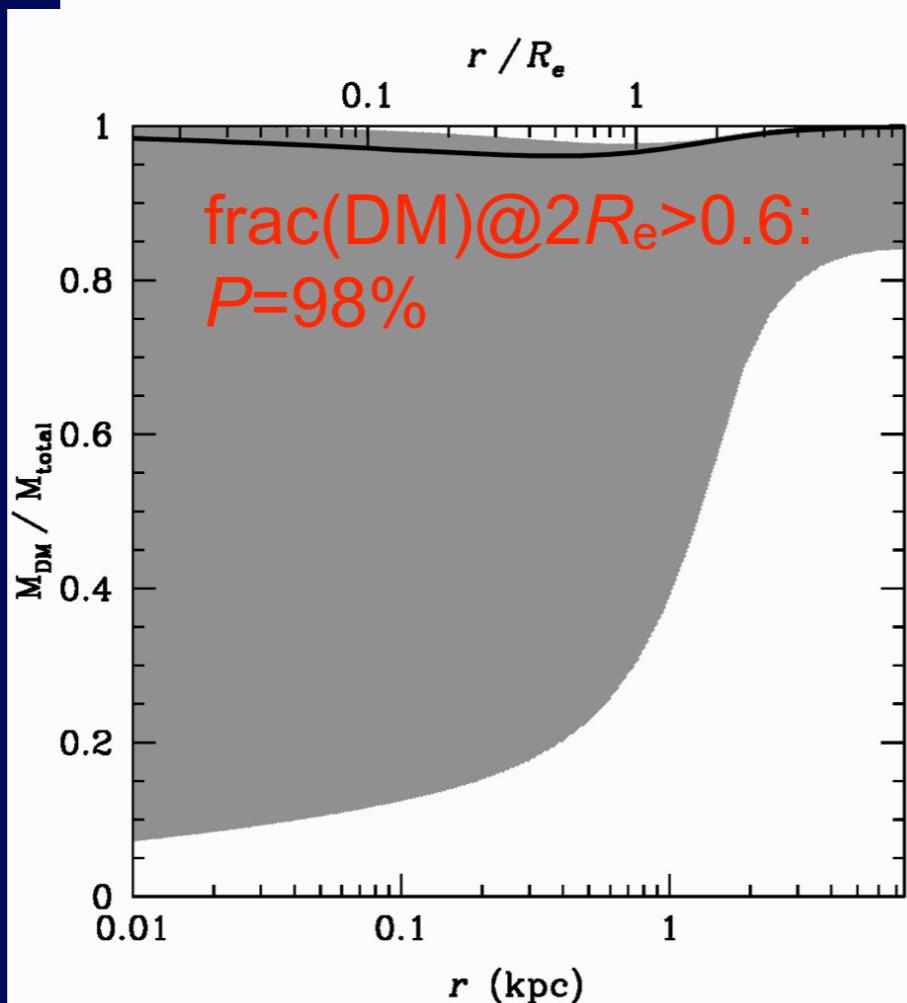
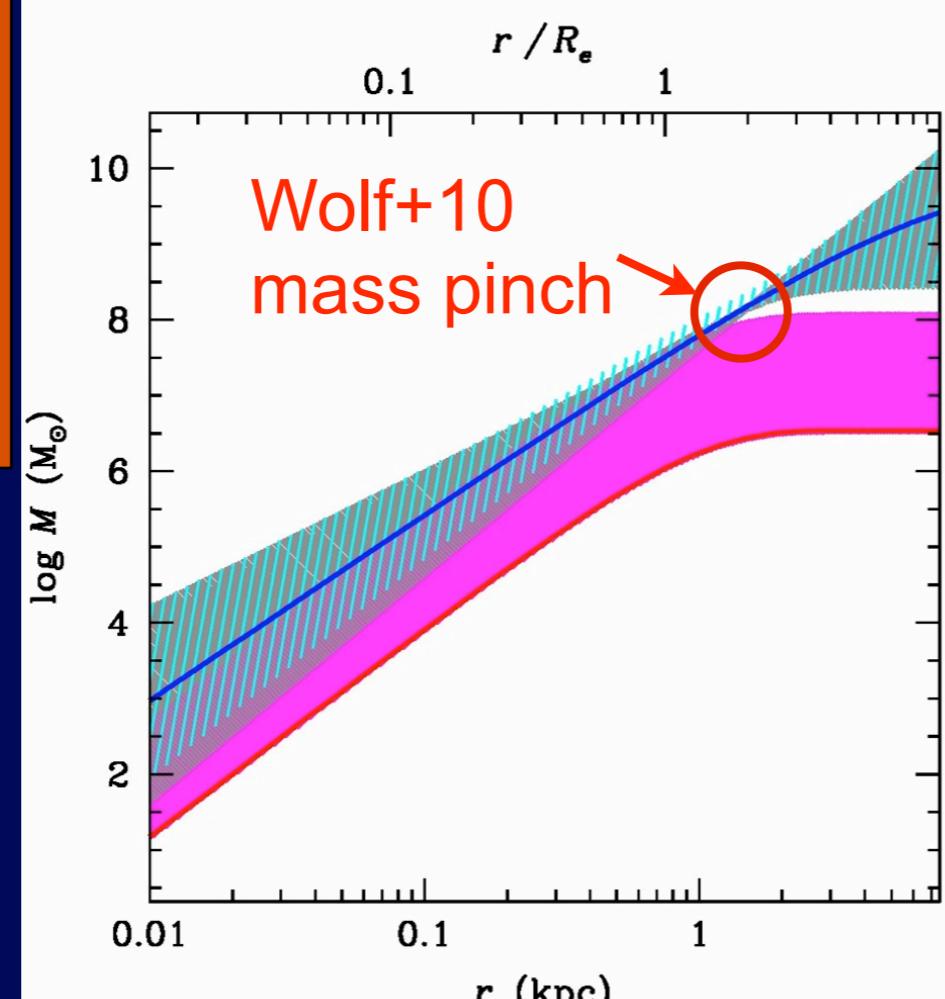
stars



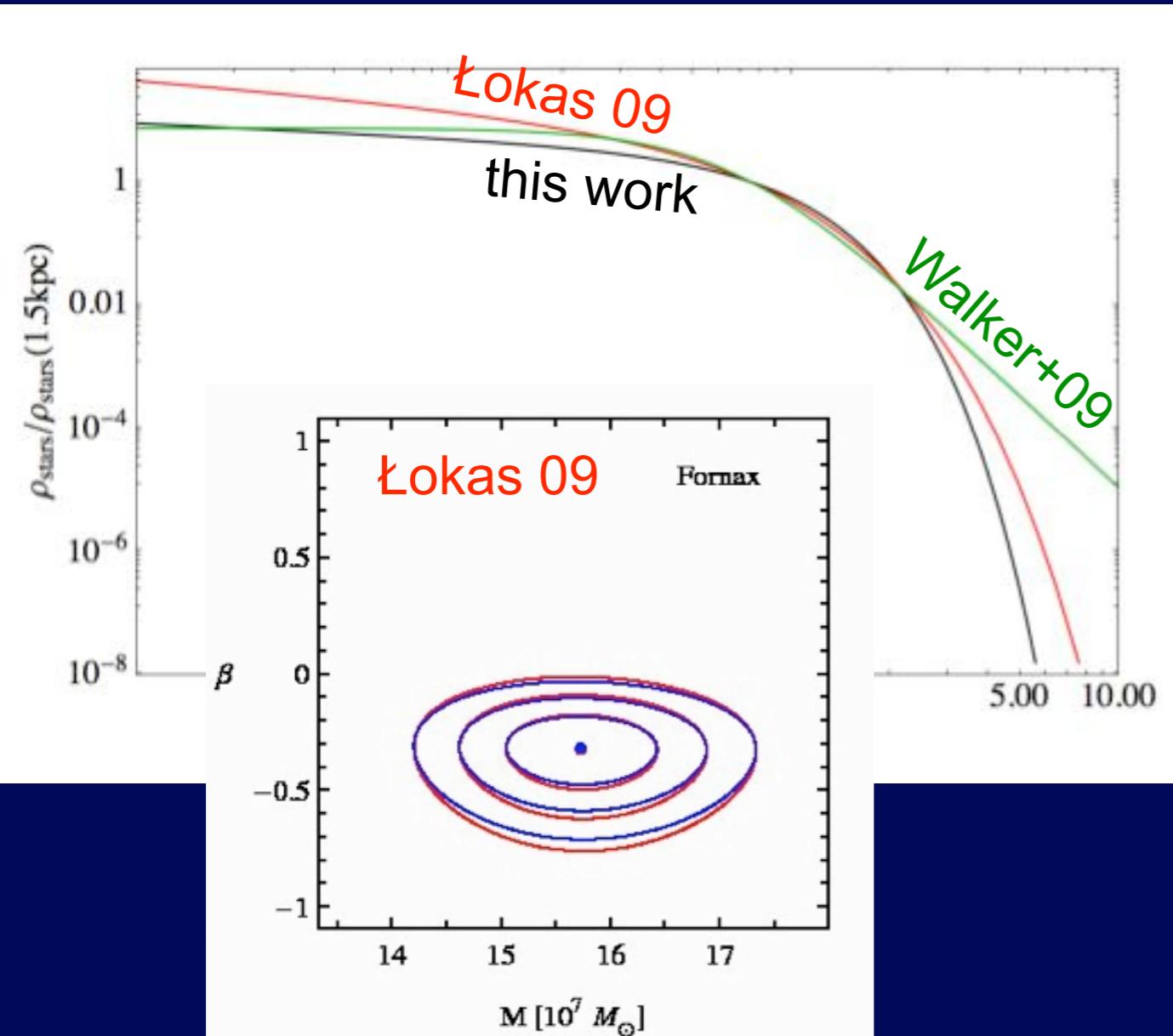
DM



total



# Comparison with previous work



anisotropy poorly constrained:

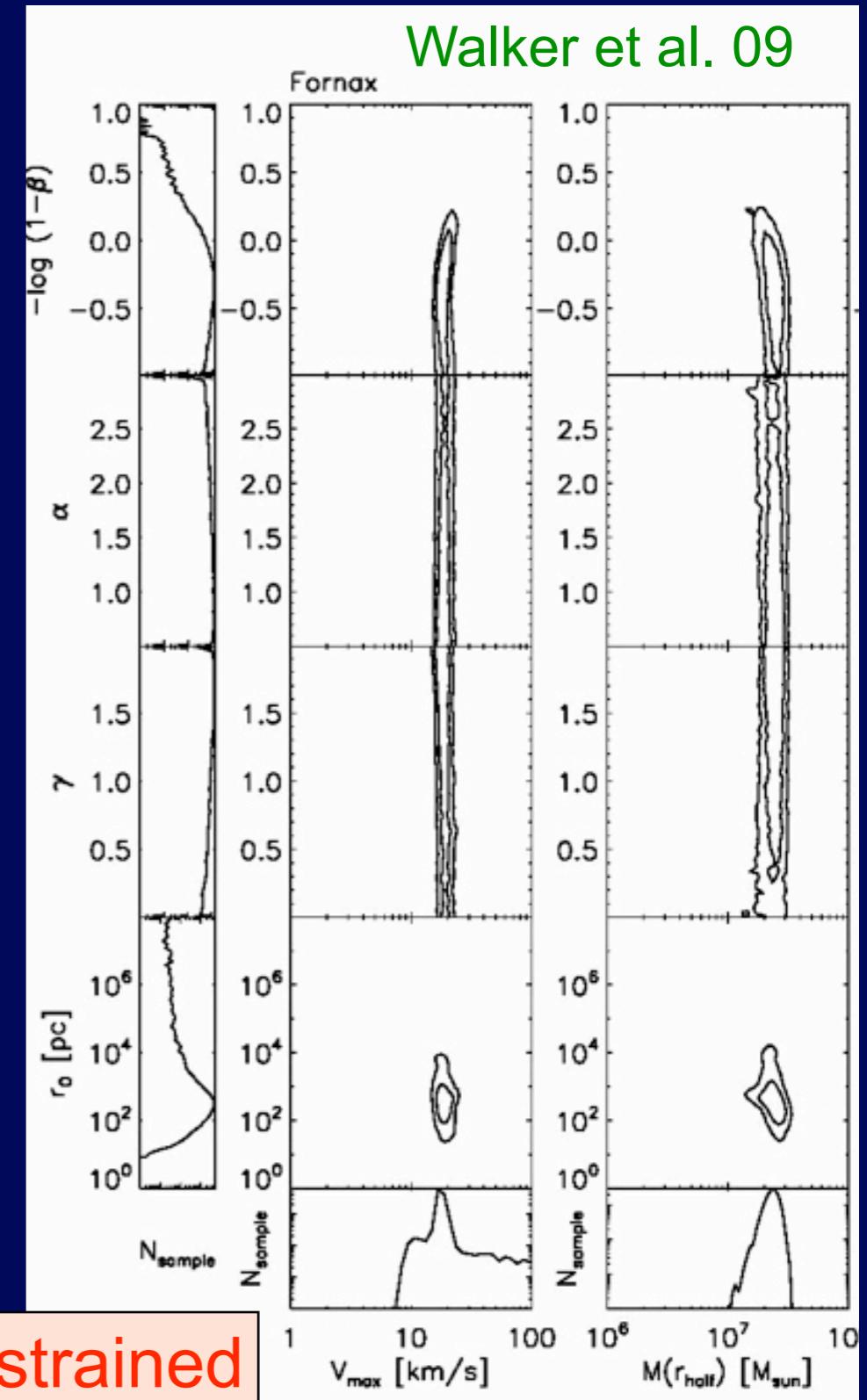
$$-0.42 < \beta < -0.18$$

$$\beta < 0.17$$

$$\text{vs. } -0.05 < \beta < 0.13$$

1 $\sigma$  limits

inner slope not constrained  
 $\gamma < -0.3$  vs.  $\gamma > -1.0$



# *Conclusions*

Mass modelling = *exploratory data analysis*:  
try  $\neq$  methods: mass &  $\beta$  inversions, & MAMPOSSt

## Fornax

- MAMPOSSt → stronger constraints
- Isotropic inner orbits,  $\approx$  radial outer orbits
- Dark matter present:
  - likely  $> 50\%$  at all radii  
seen in simulations: Klementowski+07; Lokas+10
- Dark matter inner slope shallower than  $-1.1$   
see Pennarubia for tidal effects on GCs

# *Perspectives*

- MAMPOSSt with non-gaussian 3D velocity distributions
- model rotation & metallicity effects Battaglia+06 in Fornax
- higher-parameter MCMC fits