3.1.3 Photometric flats: the challenge of wide field photometry.

This is a non-standard procedure, but nevertheless necessary for the reductions of WFI data due to the variations of zero points with positions on the mosaic (Manfroid, Selman, and Jones 2001).

If the flat field screen illumination is uniform, then any observed variations in the flat field image are the result of variations in the transmission of the optics, and the response of the pixels. Such a flat field image can then be used to remove the variations of sensitivity from all science frames.

Nevertheless, it appears that for large area imagers with focal reducers even if the illumination of the flat field screen (or twilight sky) is uniform, there is redistribution of light by the optical system that corrupts the flat field images. In most cases the non-uniform illumination results in an increase of the amount of light reaching the central areas of the detector (*sky concentration*, Andersen et al. 1995). This *sky concentration* may fool us into believing that the relative response between the center and edge of the frame is larger than what it really is. Thus, when flat fielding with such incorrect flat field the fluxes of the stars near the center will be artificially reduced.

A flat field image which measures the sensitivity variations, as opposed to the sky level intensities as a function of position, is called a photometric flat.

Figure 3.5 illustrates the effect of reducing data with a flat field which suffers from sky concentration. The figure shows the color equations determined using observations of the stars in the Landolt field Rubin 149. The stars, which are very close together, were placed in each of the 8 chips of the mosaic. The images were flattened with flat field frames obtained during twilight. A full atmospheric extinction correction was obtained observing the same field at three different airmasses. The magnitudes are then extra-atmospheric, and should be independent of the position in the mosaic. The figure shows a large spread in zero-points, independent of color. Thus, the sky concentration effect is manifest as a spatial variation in the zero points of the photometry. A correction frame which equals this spatial variation of the zero points will be called a *photometric flat field correction*, or *zero-point variation map.*

We have designed a convenient procedure to obtain a zero-point variation map. The

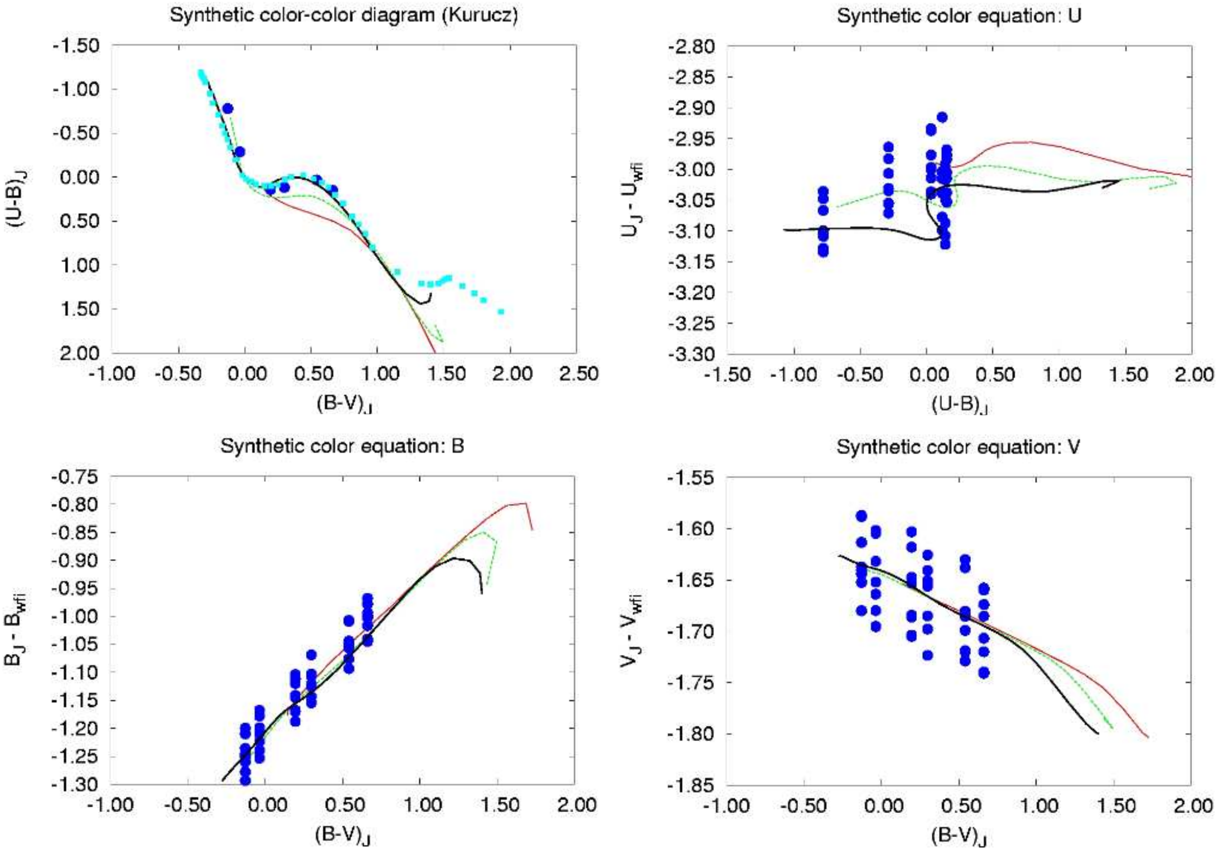

Figure 3.5: Same as Figure 3.4 but with the data of the observations of Ru149 in all chips. The rms residuals from linear colour equations are 0.034, 0.029, and 0.040 for V, B, and U respectively.

main idea is that by imaging a reasonably dense stellar field three times, with two small perpendicular offsets between exposures, any errors in the flat field procedure will result in differences in the flux measured for stars in different parts of the image.

The zero point variations can be modeled with a set of two dimensional basis functions (i.e. Cosine, or Chebyshev), and then use a least squares procedure to determine the coefficients that best fits the data.

Denote the magnitude of the same set of n stars, in three frames taken in rapid succession, following a cross pattern,

$$\{m(x_i, y_i), m(x_i + h_x, y_i), m(x_i, y_i + h_y)\}_{i=1}^{n}.$$

The variations in zero points can be modeled as

$$zp = zp(x, y) = \sum_{j=1}^{l} b_j f_j(x, y)$$

where the set $\{f_j(x, y)\}$ is a set of l, 2-dimensional, basis functions. Then, the following set of equations can be written as,

$$\begin{aligned} \delta m_i = m(\mathbf{x}_i + \delta\mathbf{x}_i) - m(\mathbf{x}_i) &= \sum_{j=1}^{n} b_j((f_j(\mathbf{x}_i + \delta\mathbf{x}_i) - f_j(\mathbf{x}_i)) \\ &= \sum_{j=1}^{n} \delta f_{ij} b_j, \end{aligned}$$

where $m(\mathbf{x}_i)$ is the magnitude of star i, measured at position $\mathbf{x}_i$; and $m(\mathbf{x}_i + \delta\mathbf{x}_i)$ is the magnitude of the same star measured at $\mathbf{x}_i + \delta\mathbf{x}_i$ ($\delta\mathbf{x}_i = (h_x, h_y)^T$). The previous system of equations in matrix form is,

$$\begin{pmatrix} \delta m_1 \\ \vdots \\ \delta m_n \end{pmatrix} = \begin{pmatrix} \delta f_{11} & \dots & \delta f_{1l} \\ \vdots & \ddots & \vdots \\ \delta f_{n1} & \dots & \delta f_{nl} \end{pmatrix} \times \begin{pmatrix} b_1 \\ \vdots \\ b_l \end{pmatrix} \Leftrightarrow \mathbf{\Delta M} = \mathcal{A} \times \mathbf{b} \qquad (3.1)$$

where δm_i is the i-th magnitude difference; δf_{ik} represents the k-th basis function evaluated at the positions for the i-th magnitude difference. $\mathcal{A}$ is called the design matrix. If we use 25 basis functions, and 1000 magnitude differences, then the design matrix has

25 columns and 1000 rows. Typically, the number of basis functions used is less than 100, and the number of magnitude differences is several thousands. Thus, the simultaneous equations represented by the design matrix must be solved using *least squares.*

Denote a set of estimates of the parameters by $\tilde{b} = (\tilde{b}_1, \ldots, \tilde{b}_l)^T$. These parameters lead to the fitted values $\tilde{\mathbf{\Delta M}} = (\tilde{\delta m}_1, \ldots, \tilde{\delta m}_n)^T = \mathcal{A} \times \tilde{\mathbf{b}}$. The residual errors are given by $\tilde{e} = (e_1, \ldots, e_n)^T = \mathbf{\Delta M} - \mathcal{A} \times \tilde{\mathbf{b}}$. The *residual sum-of-squares* $E = \sum_{i=1}^{n} e_i^2$ is the sum of the square differences between the actual and fitted values, measuring the fit of the model with these parameter estimates. The *least squares* estimates are the parameter estimates which minimize the residual sum-of-squares,

$$E = \sum_{i=1}^{n} (\delta m_i - \delta f_{i1}\tilde{b}_1 - \ldots - \delta f_{il}\tilde{b}_l)^2.$$

E is minimized when

$$\frac{\partial E}{\partial \tilde{b}_k} = 2\sum_{i=1}^{n} (-\delta f_{ik})(\delta m_i - \delta f_{i1}\tilde{b}_1 - \ldots - \delta f_{il}\tilde{b}_l) = 0,$$

or in matrix form,

$$\mathcal{A}^T \mathbf{\Delta M} = (\mathcal{A}^T \mathcal{A})\tilde{\mathbf{b}}.$$

These equations are called the *normal equations.* If $(\mathcal{A}^T\mathcal{A})$ is invertible, then the least squares estimates are

$$\tilde{\mathbf{b}} = (\mathcal{A}^T \mathcal{A})^{-1} \mathcal{A}^T \mathbf{\Delta M}.$$

Of all the possible unbiased estimates of the parameters $\tilde{\mathbf{b}}$, if the errors are Gaussian, the least-squares estimates have the smallest variance (Gauss-Markov theorem; Brandt 1977).

It is very common for the design matrix to have linearly dependent columns, that is, the problem is *overparameterized,* and the matrix $(\mathcal{A}^T\mathcal{A})$ is singular and has no inverse. In this case one can use the Moore-Penrose pseudo inverse which chooses a particular set of parameters from the infinitely many possibilities. I have used the method of singular value decomposition to solve the problem. For the details see Brandt (1998).

Because of a well known theorem from information theory the effective resolution of a flat determined with this procedures does not depend on the size of the spatial offset between exposures, but instead depends only on the effective density of stars.[1]

[1] Consider a 1-dimensional function $f(x)$, which we want to reconstruct if we are only given the difference

The method in practice

The main complications arise from variations in extinction between exposures. To understand the problem better it is useful to consider a case in which there is no zero point variations in the mosaic. After taking two dithered exposures the difference in magnitudes of the same stars measured in both frames will only be a reflection of the change in extinction between the two exposures. If we do not recognize this, we will wrongly conclude that there is a linear gradient in the zero points variations across the mosaic. The solution is to subtract the median change in magnitude from all stellar magnitudes. But, when we allow for zero point variation this procedure is no longer valid because the group of stars used to determine the median magnitude difference will be located in a different part of the mosaic in the second exposure and will thus have a different median zero point. The problem is particularly troublesome when we maximize the signal by using a large dither amplitude (δx). In principle it is possible to disentangle changes in atmospheric transparency from spatial zero point variations if the images are obtained quickly on a photometric night and with small offsets. In practice, however, this is not possible because some filters, e.g. U, require long integrations; and if the offsets are small the signal (δm_i) is also small. To solve this we need an independent way to determine the change in extinction between the

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function $dif(x, h) = f(x+h) - f(x)$. The Fourier transform of the latter is

$$\begin{aligned}
\mathcal{F}(dif) &= \int_{-\infty}^{\infty} e^{ikx}(f(x+h) - f(x))dx \\
&= \int_{-\infty}^{\infty} e^{ikx} f(x+h)dx - \int_{-\infty}^{\infty} e^{ikx} f(x)dx \\
&= (e^{-ikh} - 1)\mathcal{F}(f) \\
&= \tilde{\mathcal{F}}(f)
\end{aligned}$$

From the above equation we can see that the Fourier transform are related by the multiplication of an oscillatory term, which does not suppress any information. If $\mathcal{F}(f)$ is band-limited, then $\tilde{\mathcal{F}}(f)$ also is band-limited. The sampling theorem still holds, thus the assertion of the previous section, that the effective resolution is only determined by the density of stars, and not by the size of the dither amplitude is justified.

With this we can determine that the optimal conditions for the dither are such that the magnitude differences must be maximized. That is, h should be on the order of half the field size, and the field should be as crowded as possible, but still adequately sparse for aperture photometry.

two frames, or to use small enough offsets so that the changes in zero point is small but large enough for the signal to be detectable. We have implemented the second solution, with mixed results.

The three frames of each field are processed in two stages. The first is a preparation stage in which we find stars in one frame and do aperture photometry in all three. This procedure is encapsulated in the python gluescript `zpPreparedData.py`, whose output is a file with the difference in magnitudes versus position.

The second stage is the analysis of the data using the python gluescript `zpAnalyze.py`. Here we assume, following the previous section, that there is a function of the form $zp = zp(x, y)$ which represents the change in zero point across the mosaic. We parametrize that function as an expansion in 2D Cartesian Chebyshev polynomials. Then we find the coefficients through the least squares procedures outlined above.

To make the method robust we have decided to make it interactive in order to do sigma-clipping of deviant points. The output of this procedure is a file with the coefficients of the 2D Chebyshev polynomial that best fits the data. Both gluescripts use the library `zplib.py` found in Appendix .2. Figure 3.6 shows the 9th-order Chebyshev basis typically used to analyze the data.

A screen shot of zpAnalyze.py at work can be seen in Figure 3.7. The three plots show the positions of the stars used in the fit, the goodness of the fit, and the magnitude difference as a function of position for all stars used in the fit (the signal). The surface plot shows $zp = zp(x, y)$. Notice that the mosaic range is from -1 to +1 in both directions. These graphs allow the user to keep total control over the procedure.

The zero-point polynomial thus determined for the V filter is shown in Figure 3.8 which is a 9th order 2D polynomial (91 coefficients) that best represented the variations in zero points for the V filter. This zero point map was obtained, after some experimentation, with spatial offsets of 100".

Figure 3.9 shows the colour equations determined after applying the zero-point correction map to the V filter. The V filter correction works better even for the U and B filters, probably due to problems associated with atmospheric transparency variations affecting the data in the other filters. Using the V zero-point correction map for all filters the 1σ residuals from the standard stars transformations were reduced from 0.034 to 0.009 in V,

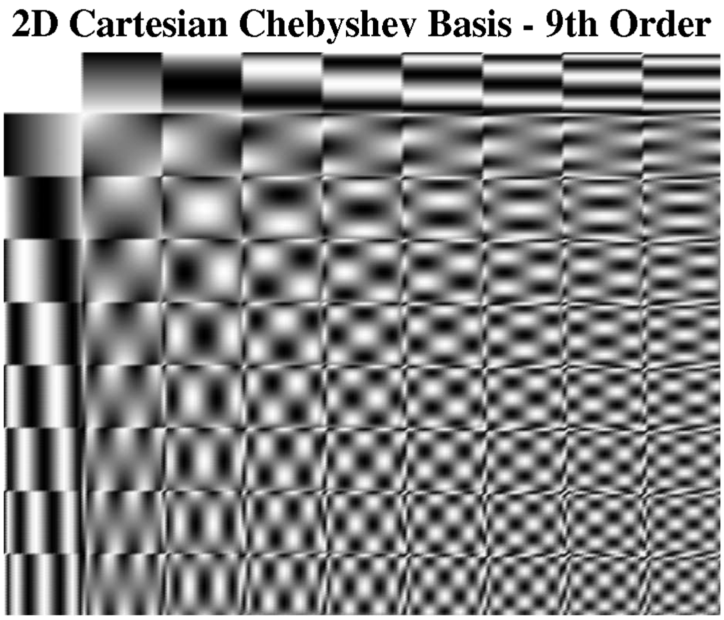

Figure 3.6: The figure displays the 81 basis functions used in the least squares procedure outlined in the text.

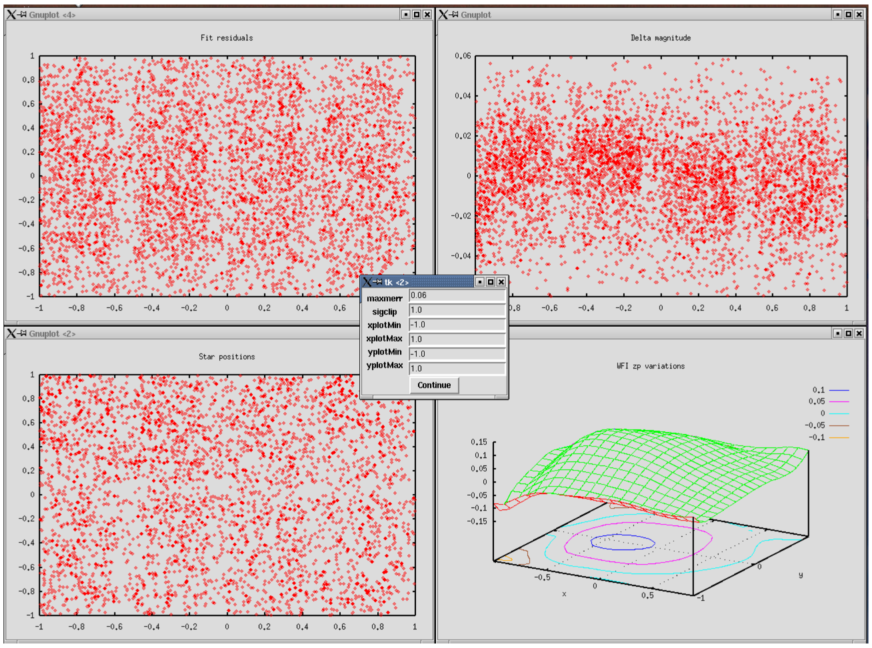

Figure 3.7: Screen shot of zpAnalyze.py at work. See text for details.

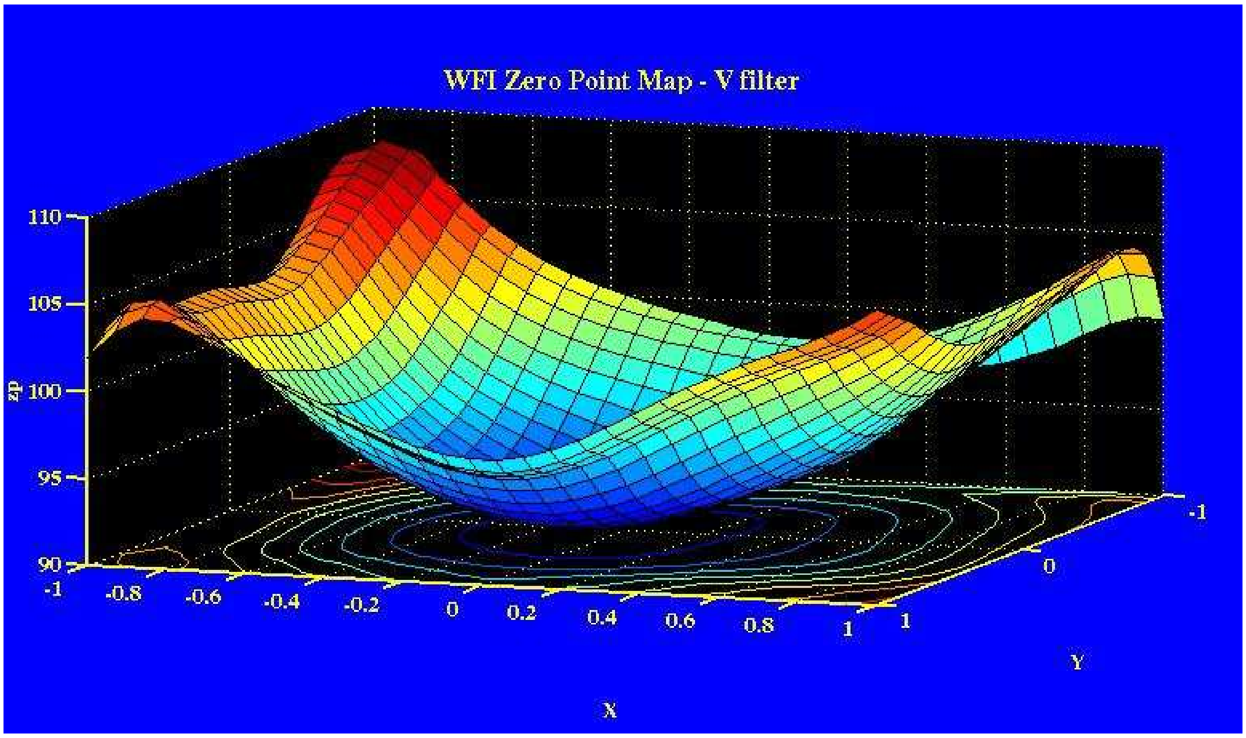

Figure 3.8: Zero point variation of the WFI mosaic for the V filter. The surface represents the percentual variation of the zero point as a function of position. Lower values means that the magnitudes of stars which were measured at those positions (near the center of the mosaic) needs to be made more negative (brightened).

from 0.029 to 0.010 in B, and from 0.040 to 0.014 in U.

3.1.4 Analysis of Internal Errors.

3.1.5 Comparison with External Photometry.

3.2 The Final UBV Catalogue.

In the previous sections the method used for the photometry was presented together with an analysis of the precision achieved. Table 3.2 presents the first few entries of the