Mathematical Algorithms and Software for ELT Adaptive Optics — The Austrian In-kind Contributions for Adaptive Optics

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The four-year Austrian in-kind project to provide improved algorithms and software for the correction of atmospheric turbulence in adaptive optics (AO) imaging, part of Austria's contribution to accession to ESO, has just concluded. The project work of the Austrian Adaptive Optics (AAO) team is summarised. Very fast algorithms, which are substantially quicker than previous implementations, have been developed in the fields of single conjugate, multi-conjugate, multi-object and extreme AO. As a result of this project, control of AO systems to achieve the necessary level of correction on extremely large telescopes can become managable with computers of reasonable size and cost.

On 22 October 2013, the final review of a four-year project, Mathematical Algorithms and Software for ELT Adaptive Optics, came to its conclusion. This project, part of Austria's in-kind contributions on its accession to ESO, was approved by the ESO Council in June 2008 and started officially in October 2009. The aim was the development of algorithms and software for the correction of images, degraded by atmospheric turbulence, using adaptive optics (AO), and faster than the traditional approach taken until now. This correction process is based on the reconstruction of the refractive index of the atmosphere from noisy measurements of the incoming wavefront; up until now, these corrections have been implemented with a matrix-vector multiplication. The challenging goal of this project was to invent a new and different approach to this problem while, at the same time, reducing the load on the computer that performs the computation.

The algorithms developed are very fast, of excellent quality and can provide enormous savings in the computing power required to reconstruct the wavefront on future AO systems for the European Extremely Large Telescope (E-ELT). But probably the most significant achievement of this study is to make the control of those AO systems manageable with computers of reasonable size and cost, and, in the case of eXtreme Adaptive Optics (XAO), to bring such a complex system into the domain of feasible implementations.

The team

Three institutes, all based in Linz, have contributed to this project: the Industrial Mathematics Institute of the Johannes Kepler University (JKU) Linz, with around ten applied mathematicians who specialise in inverse problems, regularisation theory and signal processing, among others; the Johann Radon Institute for Computational and Applied Mathematics (RICAM), a mathematical research institute belonging to the Austrian Academy of Sciences, with approximately 50 mathematicians; and MathConsult GmbH, a company that provides mathematical methods and software in industry and finance and consists of mathematicians, physicists and computer scientists. Thus, several applied mathematicians and physicists — from Masters and PhD students, to postdocs and professors — were combined in the Austrian Adaptive Optics (AAO) team[1]; part of the team is pictured in Figure 1.

Why adaptive optics?

Images from ground-based telescopes suffer from turbulence in the atmosphere, which lead to serious image degradation. AO is a technique for the correction of the phase of the incoming light which aims to compensate, in real time, for rapidly changing optical distortions in the atmosphere by deforming a mirror. The correction is based on the reconstruction of the turbulence in the atmosphere from measurements in the direction of one or several guide stars. A guide star is any star in the sky bright enough to be used as a sensor (natural guide star [NGS]) or an artificially generated light source using a laser (laser guide star [LGS]).

For each AO correction step, several subproblems have to be solved, where the most important is the severely ill-posed atmospheric tomography. This subproblem resembles limited angle tomography, as the approach to the solution requires the reconstruction of the atmospheric turbulence layers from data measured by wavefront sensors in several guide star directions, separated only by a fraction of a degree.

Many of these systems, such as multi-conjugate adaptive optics (MCAO), laser tomography adaptive optics (LTAO) and multi-object adaptive optics (MOAO) depend on a sufficient reconstruction of the turbulence profiles in order to obtain a good correction. Due to the steady

Figure 1. The Austrian Adaptive Optics (AAO) team photographed in the hallway at ESO Headquarters during the final project review. Back row, left to right: Roland Wagner, Andreas Binder, Daniela Saxenhuber, Ronny Ramlau, Iuliia Shatokhina; front row, left to right: Andreas Obereder, Mischa Yudytskiy, Günter Auzinger. Other team members not pictured: Tapio Helin, Sergiy Pereverzyev jun., Matthias Rosensteiner, Masha Zhariy.

growth in telescope size, there is a strong increase in the computational load for atmospheric reconstruction with current methods, in particular using the standard brute-force technique of matrix-vector multiplication (MVM).

The specialist team from Linz had the challenging goal of inventing new approaches to this problem that delivered the same quality of correction as current techniques, while, at the same time, greatly reducing the computing load. This four-year project has recently successfully completed its final review. In order to test and compare the different methods in terms of reconstruction quality, as well as computational speed, the ESO end-to-end simulation tool OCTOPUS was used. For all the subprojects, the team provided technical reports on the algorithms that had been developed and the computational architecture in terms of conceptual design, performance verification and prototypes of the algorithms, including user and installation manuals. There were separate mid-term and final reviews for each subproject in order to verify the compatibility of the first deliverables with the technical specification.

Project organisation

The AAO project was divided into four subprojects. Within the first subproject, AO1, the team acquired the necessary knowledge in the field of AO. This phase lasted for nine months and ensured a mutual understanding of the requirements fixed by the technical specifications. In the introductory study to AO1, the Austrian team reviewed existing AO algorithms, methods and techniques and became acquainted with the state of the art in the field. The modelling of the influence of the turbulent atmosphere on wavefront sensor measurements also included a study of the physics of different sensors and the derivation of forward operators that describe this connection mathematically.

The subprojects AO2 and AO4 comprised the work on MCAO, single-conjugate adaptive optics (SCAO) and XAO and both began at the end of AO1, i.e., in July 2010. Through the next 27 months, the team worked on the development of algorithms and software prototypes. The last subproject — AO3 — began in October 2010 and finished in October 2013. Within the subproject AO3, the research was focused on ground layer adaptive optics (GLAO), MOAO and LTAO systems as a pre-study for the latter.

SCAO and XAO

The subproject AO4 was concerned with the development of reconstruction methods for SCAO. An SCAO system uses the measurements of the wavefront of an NGS in order to obtain the optimal shape for a deformable mirror, such that, after the wavefront conjugation, a sharp image of the science target is obtained. The main computational task for SCAO is the reconstruction of the incoming wavefront of the guide star from measurements of a wavefront sensor. As the sensors usually do not measure the wavefront directly, the reconstruction of the wavefront requires the solution of an inverse problem. For SCAO, different types of wavefront sensors are used. We considered systems that use a Shack–Hartmann sensor, which measures averages of the gradient of the incoming wavefront, and systems that use pyramid-type wavefront sensors. Pyramid wavefront sensors allow for a higher contrast of the corrected images as well as a higher resolution of the wavefronts and are therefore used for XAO systems.

For the reconstruction of the incoming phase from Shack–Hartmann wavefront sensor data, the cumulative reconstruction algorithm (CuRe) was developed (Zhariy et al., 2011). CuRe is based on one-dimensional line integration and a subsequent coupling of those lines in the orthonormal direction to obtain the full two-dimensional reconstruction. CuRe has been further improved in terms of noise propagation by the application of domain decomposition leading to the CuReD algorithm (CuRe with domain decomposition; Rosensteiner, 2011). Simulated results obtained with CuReD are shown in Figure 2.

CuReD has been tested on ESO's optical high order test bench (HOT) as well as on-sky (by Durham University on the William Herschel Telescope, La Palma). The CuReD algorithm performs with a computational complexity of O(n), that is, it scales linearly with the number of degrees of freedom and not quadratically (as in the standard matrix-vector multiplication), it can be parallelised in all guide star directions and is pipelineable. The algorithm fulfills the requirements of the project both with respect to quality and

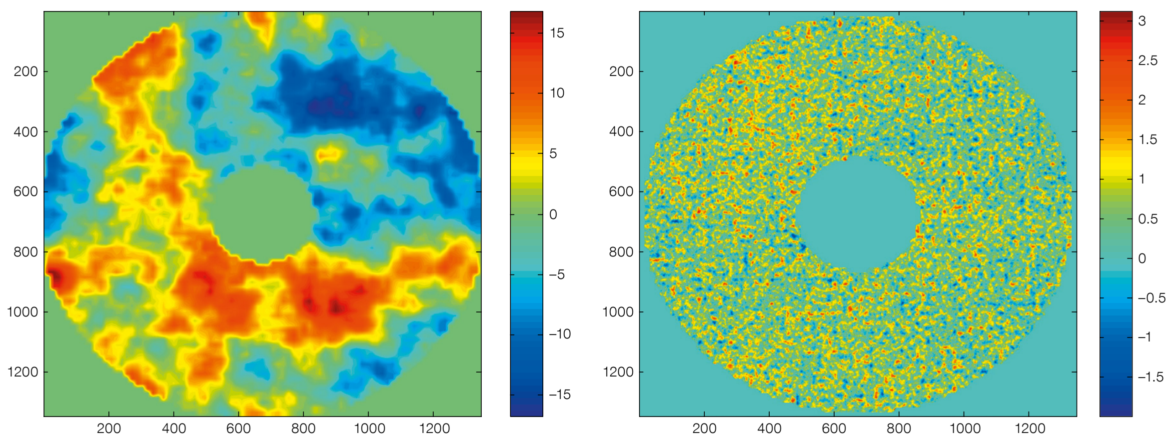

Figure 2. Reconstructions with CuReD for a 42-metre telescope with an 84 × 84 sensor are shown. Left: the wavefront as reconstructed with CuReD. Right: the residual between the reconstructed wavefront and the simulated one. Colour bars show units in radians.

speed. The speed-up factor reached, for example, for XAO was 1100 (see Table 1 for a summary of the results obtained). In combination with a data-preprocessing step, CuReD is also well suited for the XAO system with a modulated pyramid wavefront sensor (Shatokhina et al., 2013).

Besides giving the required quality, both methods (CuRe and CuReD) provide a significant increase in the reconstruction speed compared to the MVM method. More methods for wavefront reconstruction from Shack–Hartmann data have been developed, such as the wavelet method or an algorithm based on singular value decomposition. Further methods for the pyramid sensor were developed: the CLIF (convolution with linearized inverse filter) and the CGNE (conjugate gradient for the normal equation) for the modulated and the HTMR (Hilbert transform with mean restoration) for the non-modulated sensor. See Table 1 for their performance figures.

MCAO

The subproject AO2 was concerned with the development of reconstruction methods for MCAO, with the aim of achieving a uniform image quality over a larger field of view, and, thus, allowing the observation of different astronomical objects at the same time. To this end, many MCAO systems use a combination of laser and natural guide stars. A wavefront sensor is assigned to each guide star and several deformable mirrors, optically conjugated to different altitudes, are used for image correction. The shape of the deformable mirrors is computed based on measurements from the wavefront sensors.

Owing to the rapidly changing atmosphere, the mirror shapes of the telescope have to be updated with a frequency of 500 Hz, i.e., each reconstruction has to be performed within a millisecond. The algorithms developed were the finite element–wavelet hybrid algorithm (FEWHA), see (Yudytskiy et al., 2014), and three-step approach methods that solve the atmospheric tomography problem using the Kaczmarz iteration (Rosensteiner & Ramlau, 2013), the gradient-based method or the conjugate gradient (CG) algorithm.

Algorithm	Applicable System	Speed-up
Cure (CuReD, Cure w/ preprocessing)	SCAO/XAO	100–1000
CLIF for pyramid sensor	XAO	200
Multi-Cure for GLAO	GLAO	100–1000
Kaczmarz, Gradient-based, CG	MCAO/LTAO/MOAO	10–200
Wavelets (FEWHA) with PCG	MCAO/LTAO/MOAO	10–200

Table 1. Portfolio of algorithms developed by the Austrian team for in-kind contributions for AO and the speed-up with respect to the reference case (ESO MVM implementation).

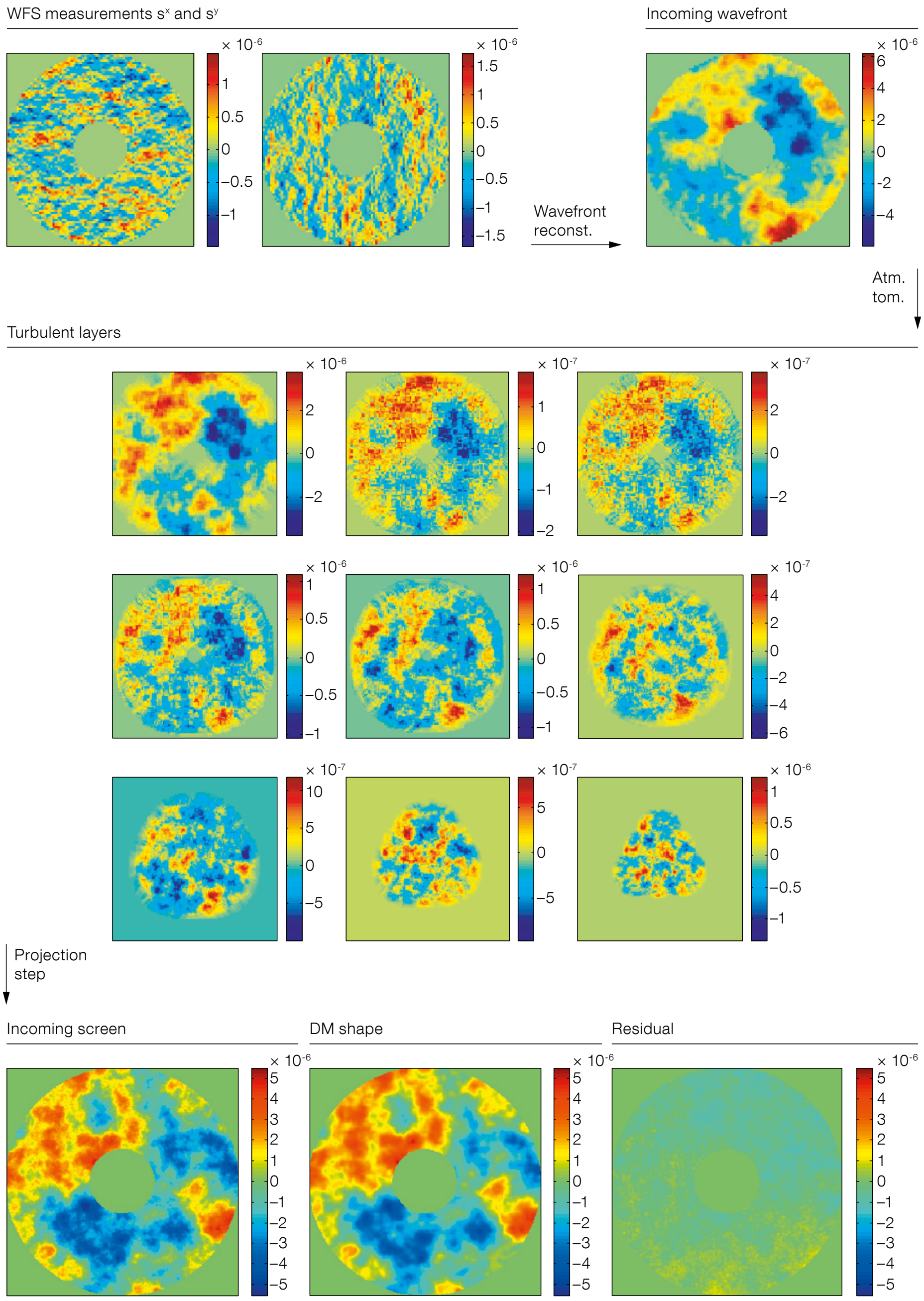

Figure 3. A visualisation of the three-step approach. From the wavefront sensor measurements, the wavefront is reconstructed using CuReD (step 1, upper). This is performed in parallel on all wavefront sensors. The reconstructed wavefronts are used in the second step to perform the atmospheric tomography using, for instance, the Kaczmarz algorithm (step 2, middle). Finally the reconstructed layers that have been computed in step 2 are used in the projection step to compute the best configuration of the available deformable mirrors (step 3, lower). All colour bars show optical path difference in metres.

FEWHA is a two-step method, i.e., it reconstructs turbulent layers from Shack–Hartmann data and then performs a projection step. It tackles the problem of atmospheric tomography reconstruction by computing the Bayesian maximum *a posteriori* estimate with a preconditioned conjugated gradient algorithm coupled with a multi-scale strategy. In the hybrid algorithm, the turbulence layers of the atmosphere are made discrete using a finite element and a wavelet basis simultaneously. A wavelet representation of layers has several advantages. First, wavelets have useful properties in the frequency domain, which allow turbulence statistics to be represented efficiently. Secondly, the operation of transforming the coefficients between the bilinear finite element basis and the wavelet basis, called the discrete wavelet transform (DWT), has a linear complexity and can be parallelised. Finally, wavelets have a good property of approximation, which means a turbulence layer can be well approximated using only a few wavelet coefficients. In contrast, the finite element representation of layers allows the atmospheric tomography operator to be represented very efficiently. Due to these properties, the resulting finite element–wavelet hybrid algorithm delivers good qualitative performance, requires only a few iterations, and is computationally cheap compared to MVM, due to FEWHA's linear complexity and high level of parallelisation.

The three-step approach, shown in Figure 3, decouples the problem into the reconstruction of the incoming wavefronts from the wavefront sensor data, the reconstruction of the turbulent layers (atmospheric tomography) from the reconstructed incoming wavefronts and the computation of the optimal mirror shape (fitting step) from the reconstructed atmosphere. The first step has been very efficiently solved by the CuReD algorithm, developed within the subproject AO4. For the atmospheric tomography problem, the Kaczmarz algorithm, the gradient-based method and the CG method have been used. All three perform with linear complexity and parallelise well. An efficient preprocessing step has been developed to counter noise due to spot elongation. The three-step approach allows each of the three subproblems to be solved independently and, thus, guarantees a fast and flexible reconstruction. Moreover, it can be easily used with the wavelet wavefront reconstructor, instead of CuReD, and also for the pyramid wavefront sensor with the CuReD algorithm with preprocessing (P-CuReD).

Both methods, FEWHA and the three-step approach, fit the atmosphere onto the deformable mirrors as the final step. Atmospheric reconstruction can be performed, either on artificial layers at the altitudes to which the deformable mirrors are conjugated or on more layers. Then, by means of the Kaczmarz iteration or the conjugate gradient method, the mirror shapes can be optimised in multiple directions. See Table 1 for the respective performance figures.

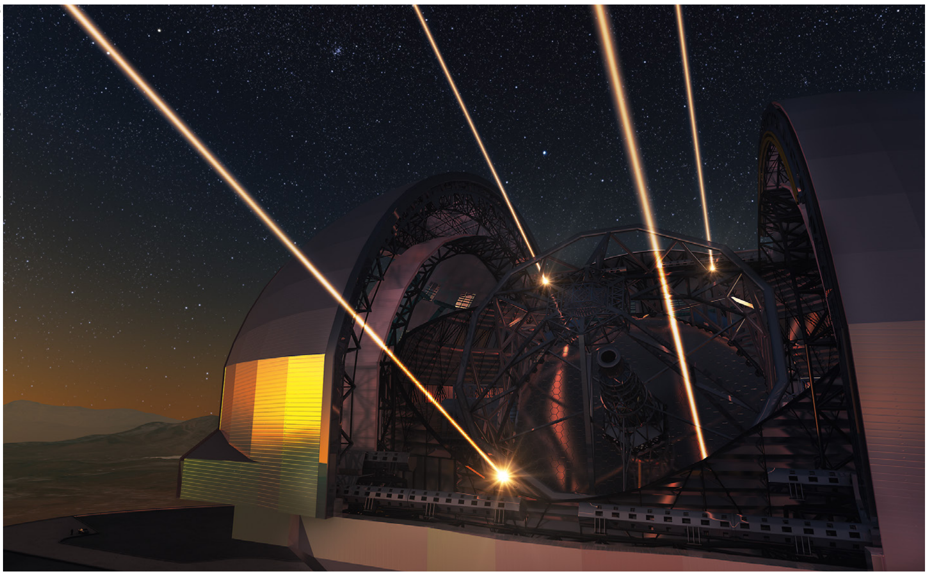
ESO/L. Calçada/N. Risinger (skysurvey.org)

Figure 4. An artist's impression of a laser guide star system in action on the E-ELT, showing how the laser-based MCAO, LTAO or MOAO systems studied in the course of this project could be operated.

GLAO, LTAO and MOAO

For GLAO, the CuReD algorithm was used to reconstruct each of the incoming wavefronts from the guide stars separately. Then the wavefronts were averaged to make an estimate of the ground layers. LTAO and MOAO systems — similar to MCAO — require fast and efficient algorithms for the atmospheric tomography subproblem. However, in an MOAO system, several deformable mirrors are used for wavefront correction in the directions of each of the multiple objects of interest. A sufficiently accurate reconstruction of atmospheric turbulence over a wider field of view is vital to obtain a good wavefront correction for these systems.

FEWHA was the method of choice for the LTAO and MOAO parts of subproject AO3. This algorithm was able to overcome the new challenge of a wide field of view (7.5 arcminutes) combined with the problem of spot elongation introduced by the LGS. FEWHA was compared to the ESO version of the state-of-the-art algorithm and has been shown to surpass the reference provided in almost all test cases. In terms of speed, a real-time computing (RTC) prototype of the algorithms showed that the method performs well on off-the-shelf hardware and has the potential to achieve the required reconstruction time of 1 millisecond on a dedicated RTC system. FEWHA outperforms ESO's MVM method, which serves as a benchmark in terms of speed, by a significant margin and is significantly more compact in terms of the required RTC hardware configuration for MOAO.

All algorithms can tackle the problem of tip/tilt indetermination and the cone effect efficiently. All the developed methods are formulated matrix-free, i.e., operate in a purely functional setting, which reduces computational complexity considerably. Furthermore, they scale linearly and yield a speed-up factor for MCAO/LTAO/MOAO of between 10 and 200, as shown in Table 1.

Future work

The next steps following on from this work are the analysis and inclusion of real-world effects such as spiders, dead actuators, mis-registration, different NGS asterisms or sodium layer variability for LGS, among others. The AAO team also would appreciate being able to test the developed reconstructors on more simulation systems, optical benches and on sky.

Project completion

Throughout the whole project, the AAO team analysed six AO-system types and published more than 17 scientific papers and attended several AO-related conferences and workshops. Four to eight mathematical reconstruction algorithms were analysed extensively for each of the considered systems, for a total of ~ 30 algorithms or customisations of an algorithm to a particular case, and the AAO project produced about 50 mathematically dense technical reports.

The benefits of the project are many-sided: easy implementable algorithms have been developed, which reduce parameter tuning considerably and do not require time-consuming pre-computations. Major speed-up factors were achieved, which could substantially reduce hardware costs. These new adaptive optics algorithms are very fast, as shown in Table 1, and provide excellent performance results. This leads to enormous savings in the computing power required to handle AO data from a telescope like the E-ELT. The most significant achievement of this study was to make the control of those systems manageable with computers of reasonable size and cost and, in the case of more extreme kinds of adaptive optics, to bring such a complex system into the realm of feasible implementation.

Acknowledgements

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Links

[1] Austrian Adaptive Optics Team webpage: http://eso-ao.indmath.uni-linz.ac.at/

ESO/J. Girard

Time-lapse view of the Paranal Observatory when the Laser guide star was tracking a target for 30 minutes. See Picture of the Week 1234 for more details.