

# The Evolution of Stellar Triples



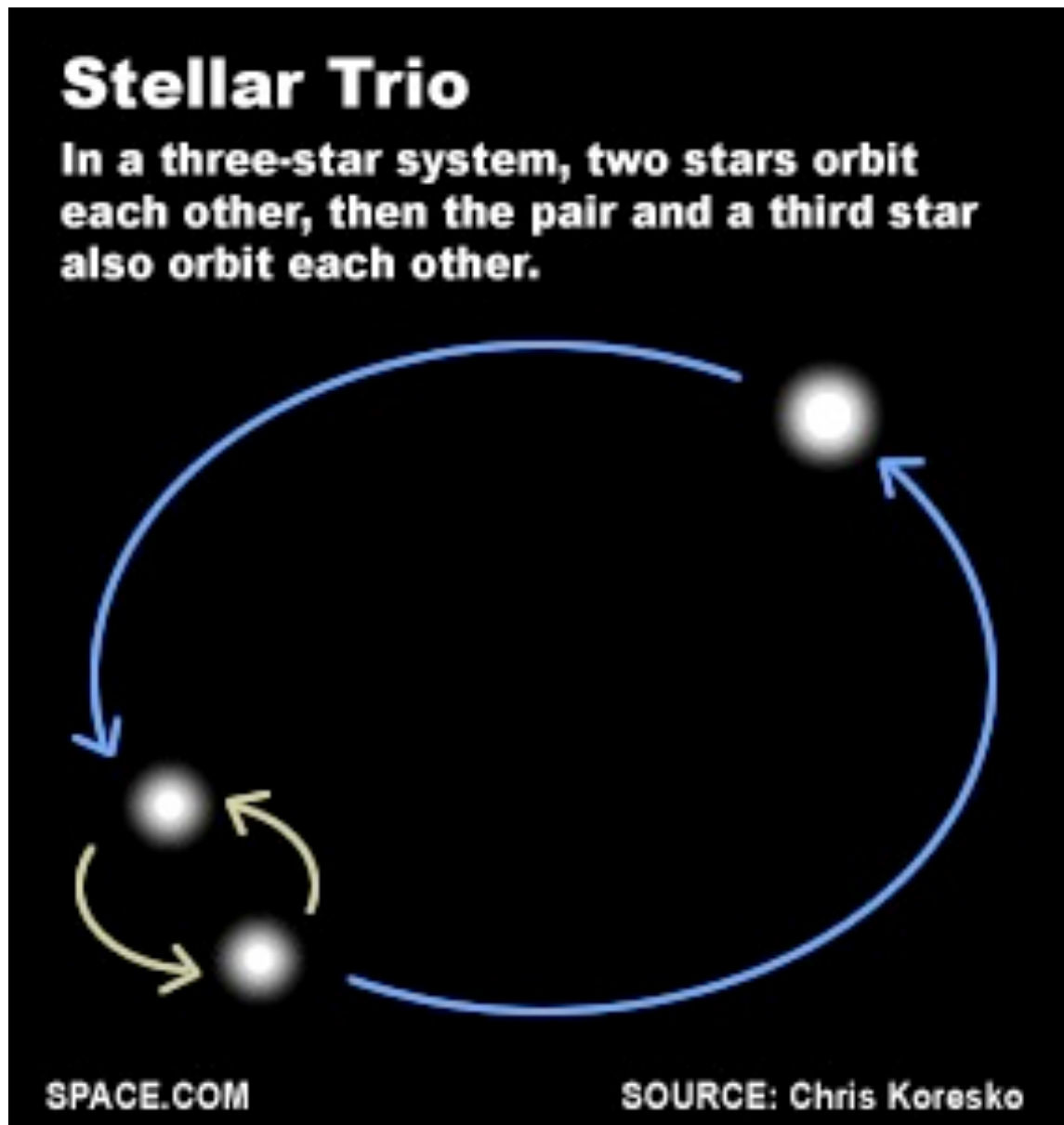
Silvia Toonen  
Veni-fellow at University of Amsterdam

Simon Portegies Zwart, Hagai Perets,  
Adrian Hamers, Fabio Antonini, Tjarda Boekholt

# Triples

- ✦ Fairly common

	Binary fraction	Triple fraction
Low-mass stars	40-50%	10-15%
High-mass stars	>70%	>30%
Refs	Raghavan+ '10, Tokovinin '08, '14, Remage Evans '11, Duchene & Kraus '13, Sana+ '14, Moe+ '17	



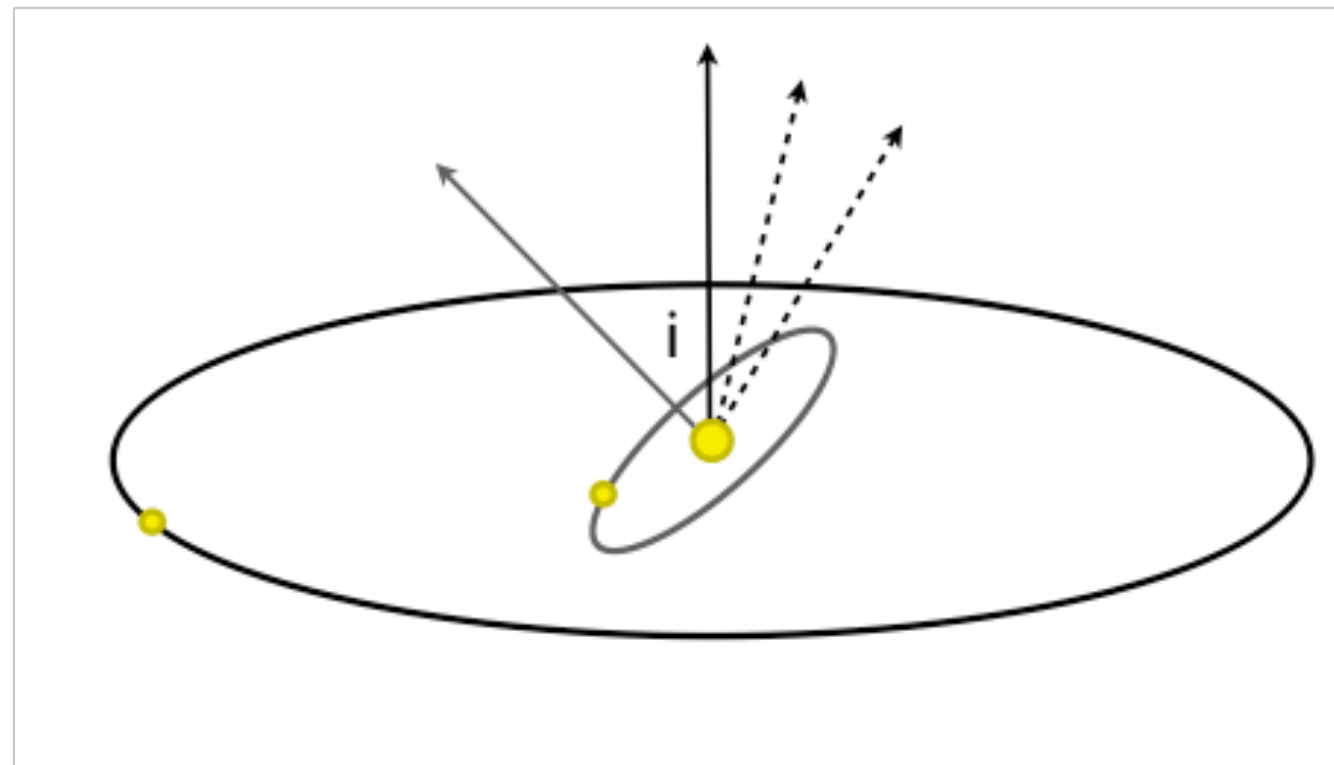
# Triple evolution

✓ Triple evolution provoked for:

- Gravitational wave sources, supernova type Ia progenitors, mergers, blue stragglers, low-mass X-ray binaries etc. etc.

✓ Unique evolution

- Three-body dynamics
- Stellar (& binary) evolution



✓ Impressive recent progress, but little coupling

- ❖ Rich interacting regime (Shappee+ '13, Hamers+ '13, Michaely+ '14, Antonini+ 17)

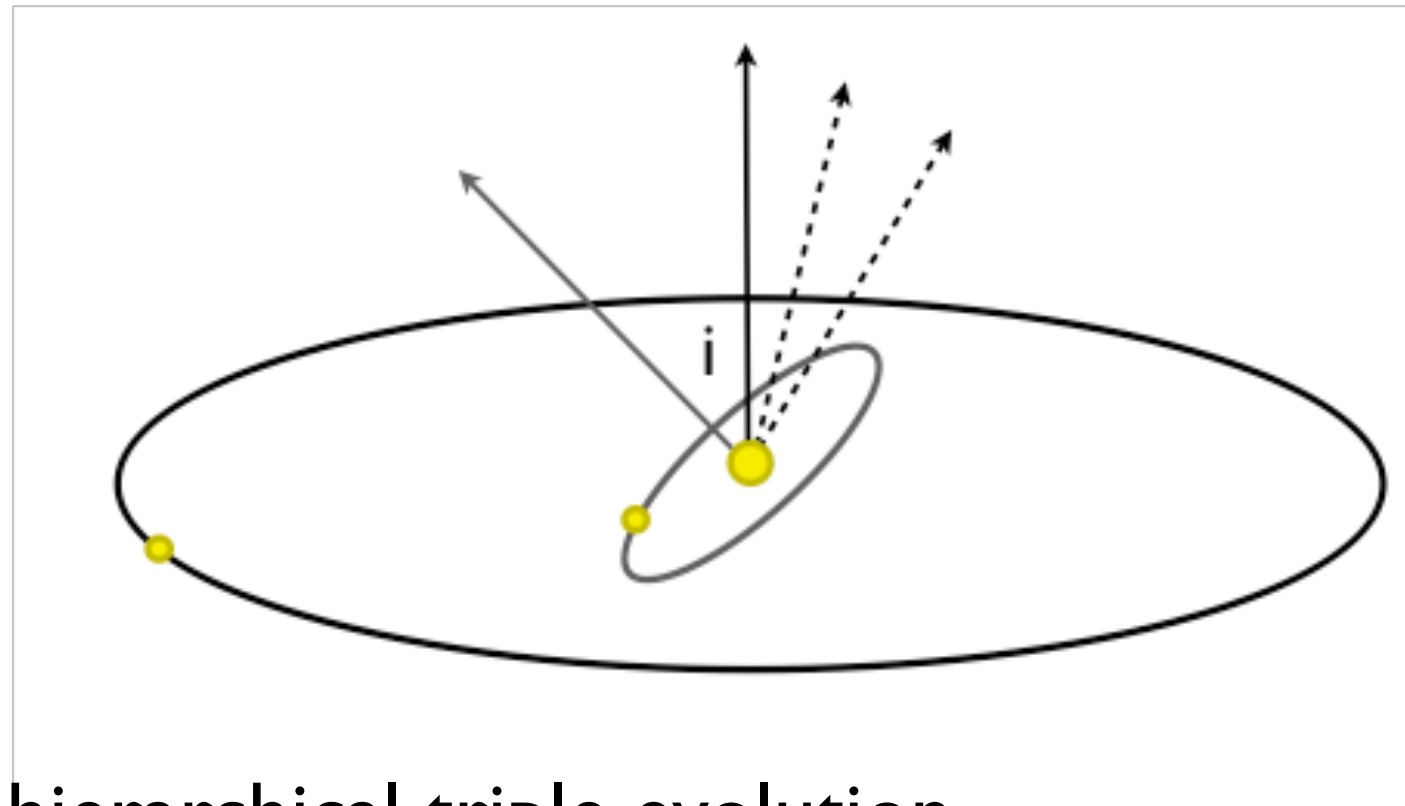
# Modelling Triple Evolution

New code **TRES** (Toonen, Hamers, Portegies Zwart 2016)

- ❖ Couples three-body dynamics with parametrized stellar evolution
  - ❖ Dynamics based on the secular-approach (quadrupole & octupole order included)
  - ❖ Stellar evolution tracks from SeBa (Portegies Zwart+ 96, Toonen+ 12,13)

❖ Including:

- ❖ GW emission
- ❖ Tides
- ❖ Precession
- ❖ Stellar winds



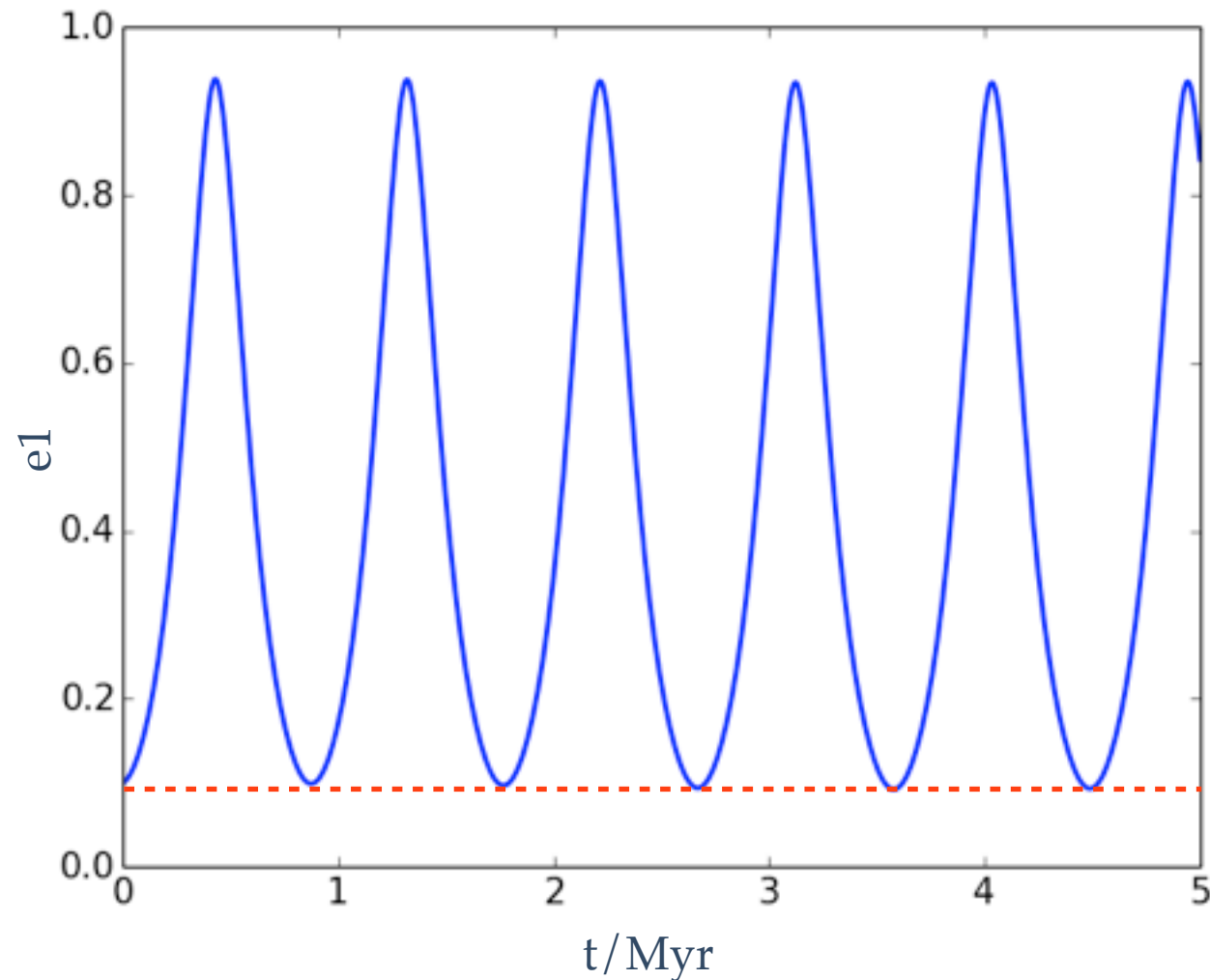
- ❖ Valid for isolated coeval stellar hierarchical triple evolution
  - ❖ Coupling possible with N-body code or detailed stellar evolution code



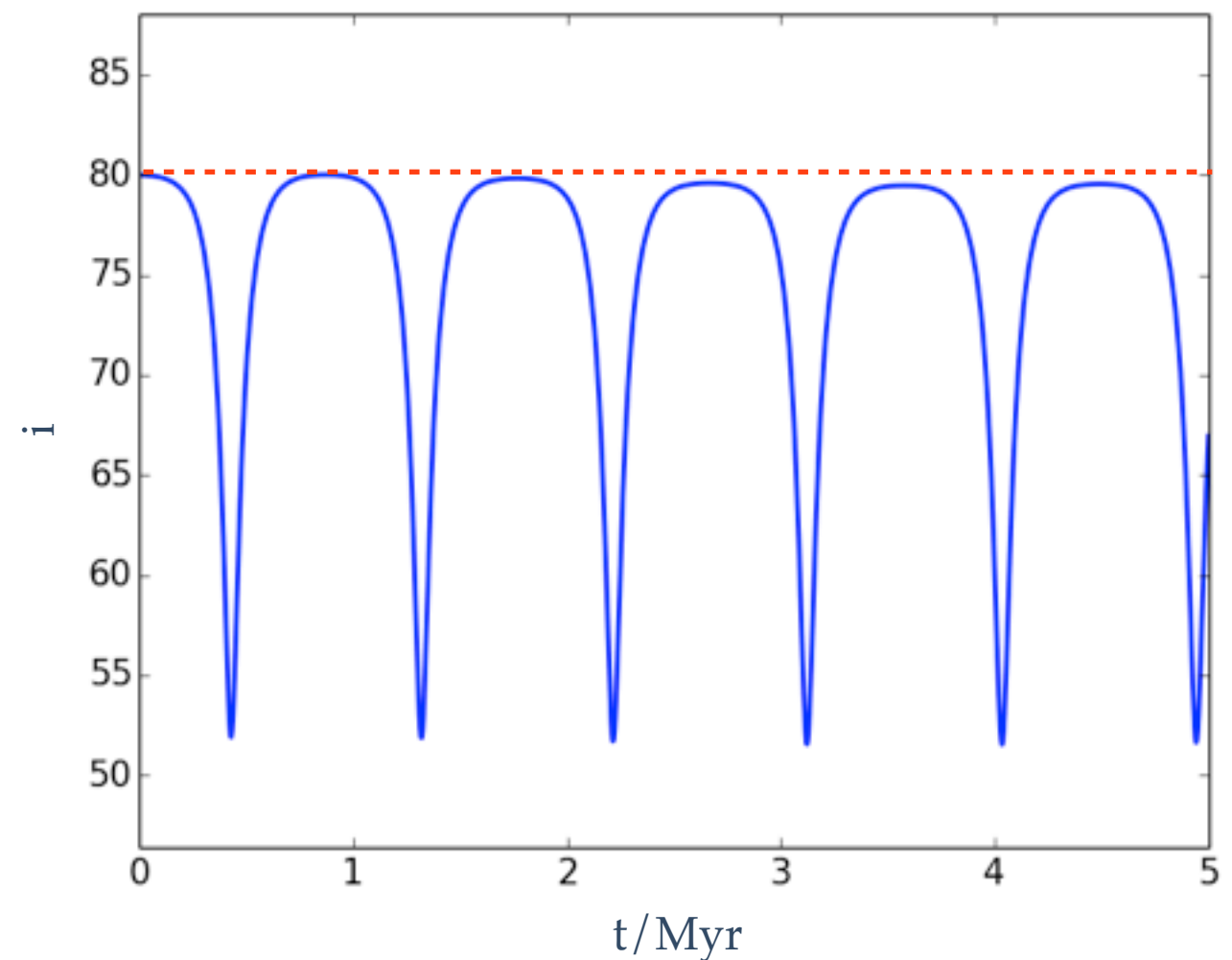
# Kozai-Lidov cycles

$M_1=1.3, M_2=0.5, M_3=0.5M_{\text{Sun}}, a_1=200, a_2=20000R_{\text{Sun}}, e_1=0.1, e_2=0.5, i=80, g_1=0.1, g_2=0.5$

Eccentricity inner orbit



Mutual inclination

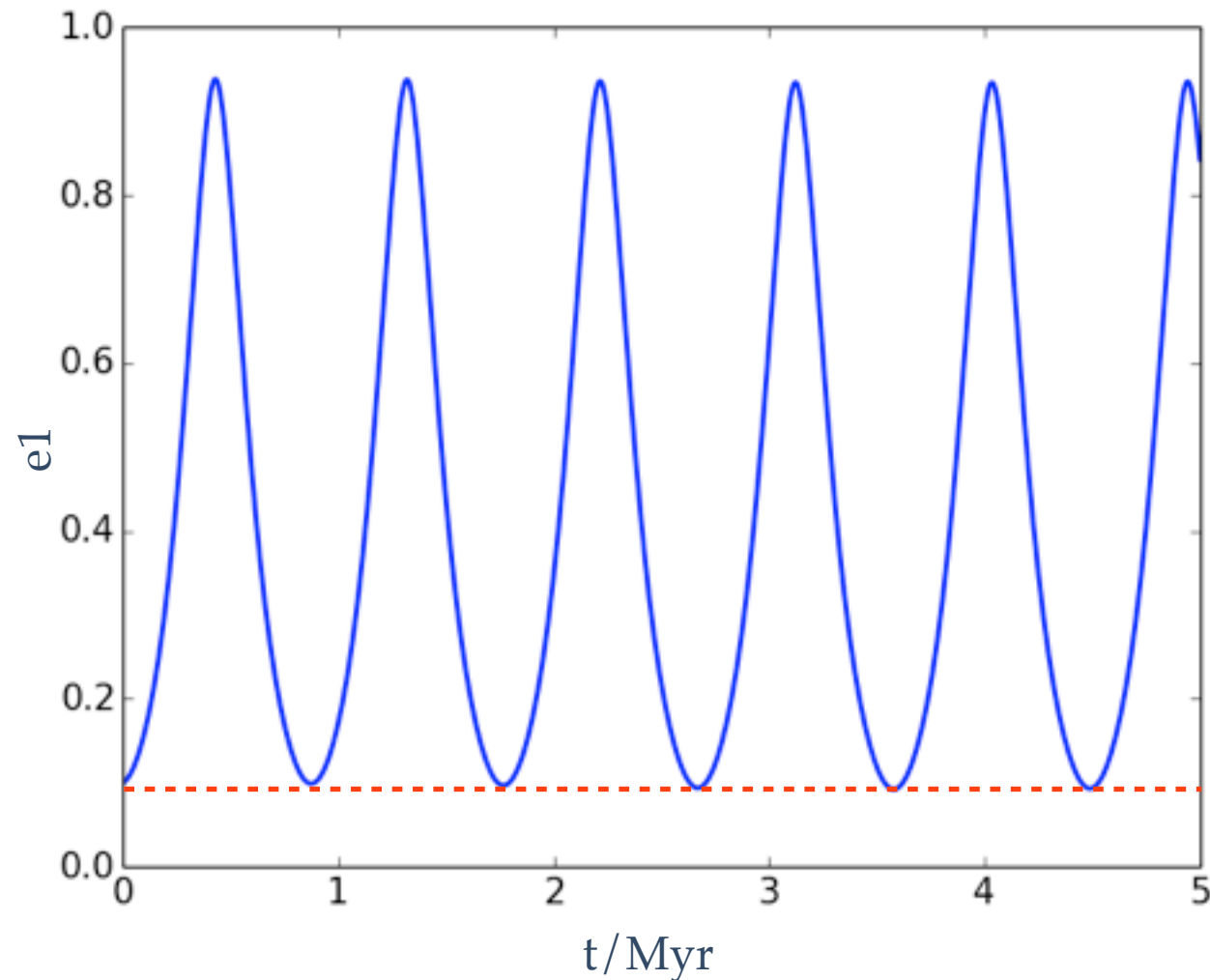


----- Binary case  
— Triple case

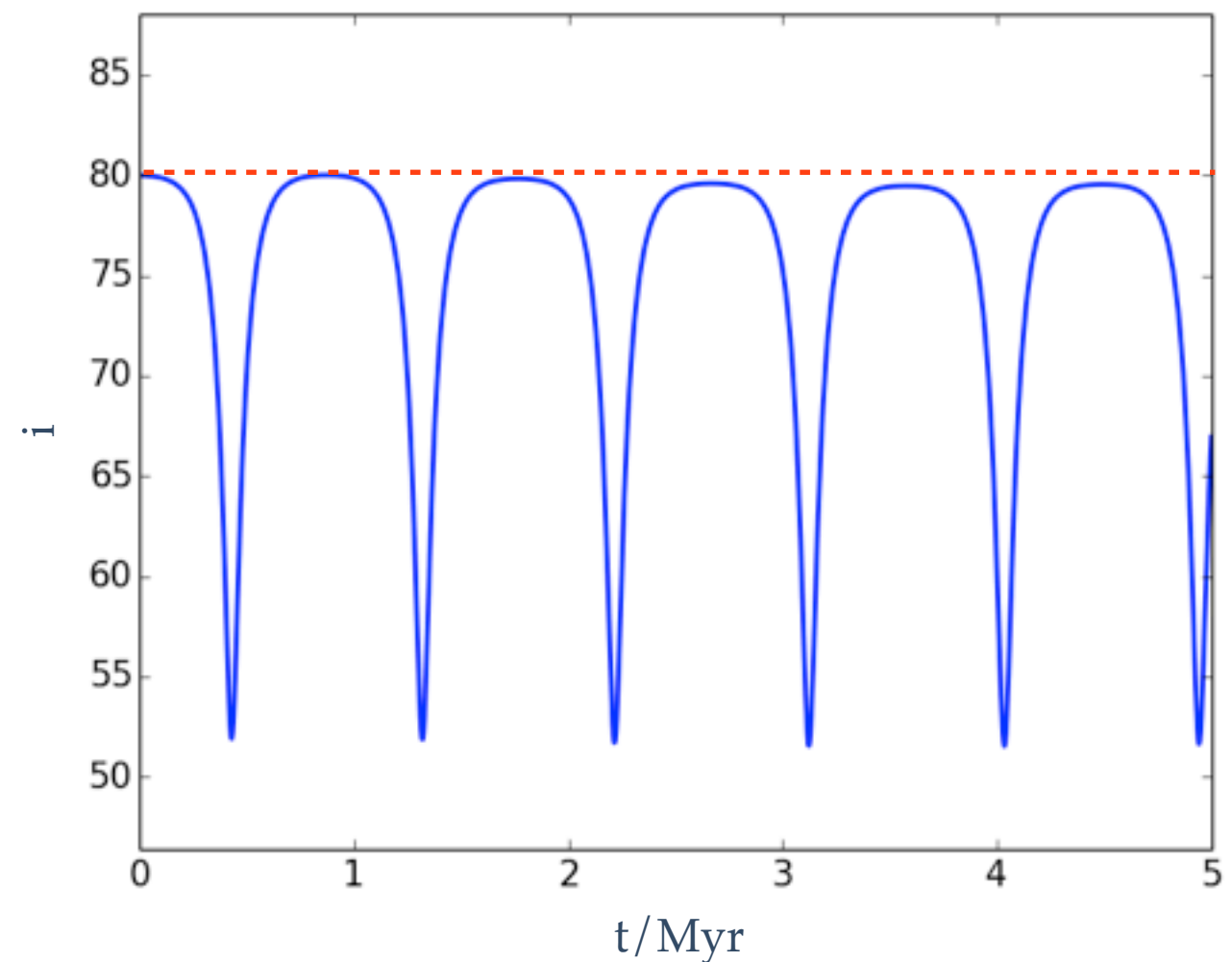
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Eccentricity inner orbit



Mutual inclination



$$P_{\text{kozai}} = \alpha \frac{P_2^2}{P_1} \frac{m_1 + m_2 + m_3}{m_3} (1 - e_2^2)^{3/2}$$

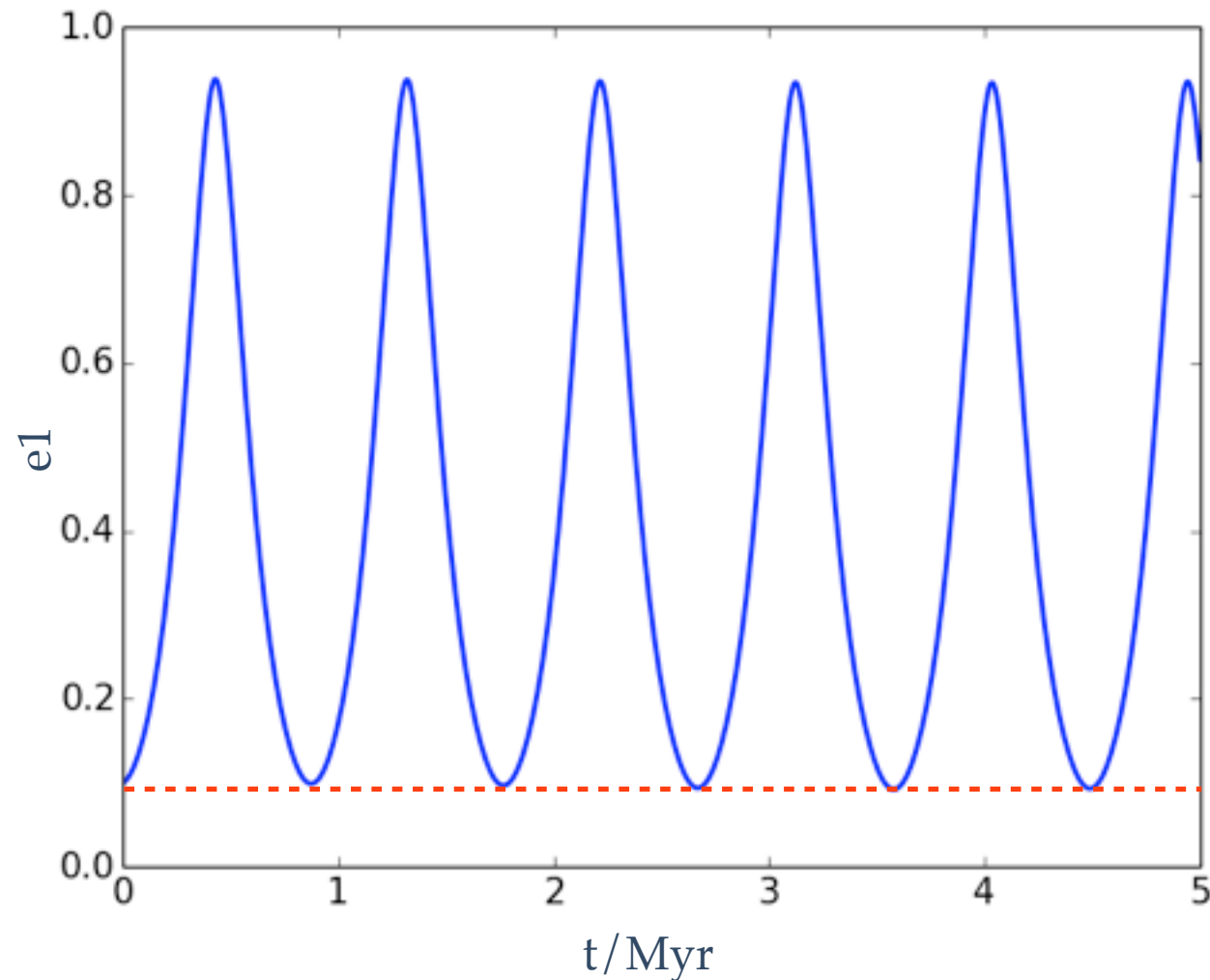
$$e_{\text{max}} = \sqrt{1 - \frac{5}{3} \cos^2(i_i)}$$

--- Binary case  
 — Triple case

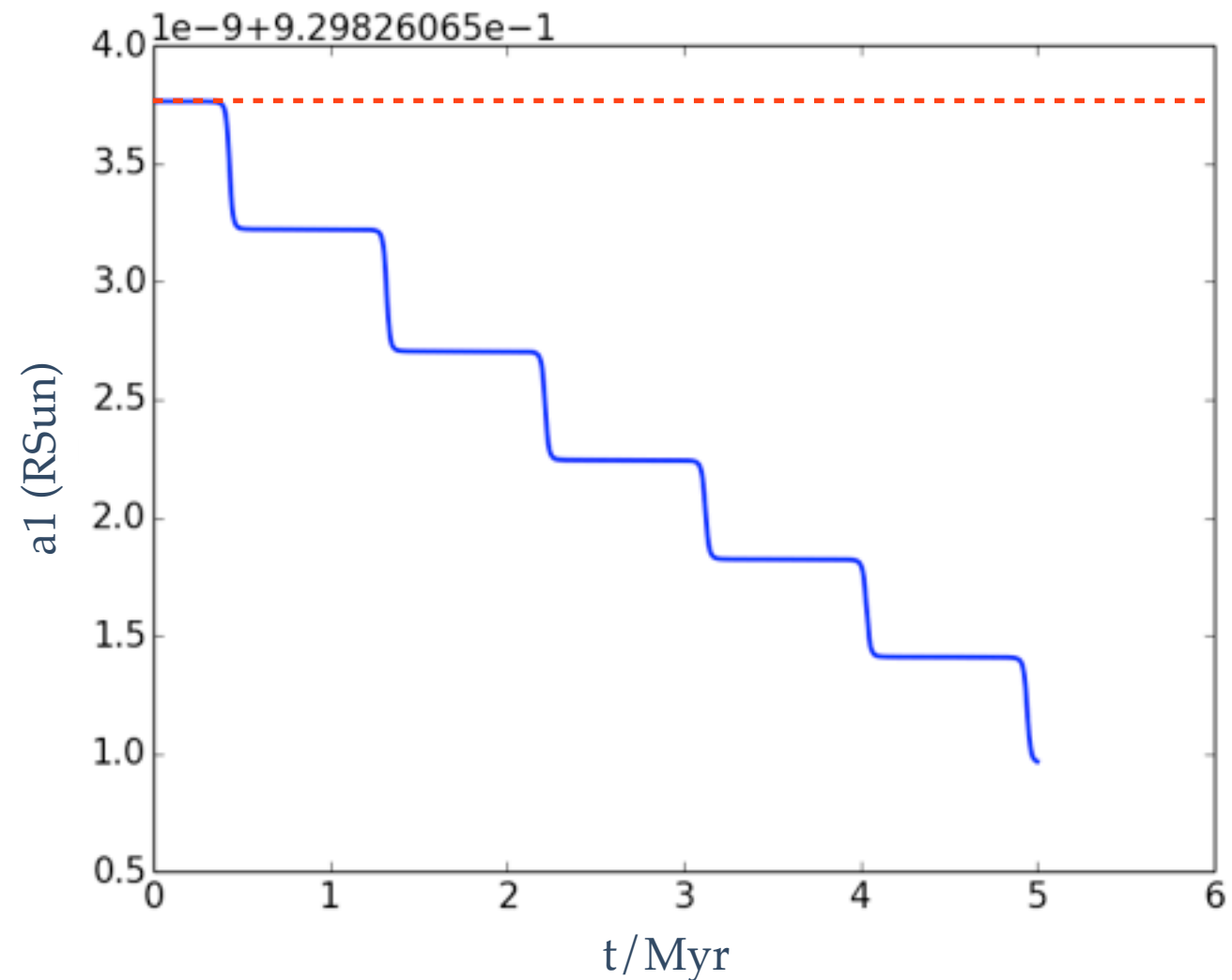
# ... with dissipation

$M_1=1.3, M_2=0.5, M_3=0.5 M_{\text{Sun}}, a_1=200, a_2=20000 R_{\text{Sun}}, e_1=0.1, e_2=0.5, i=80, g_1=0.1, g_2=0.5$

Eccentricity inner orbit



Semimajor-axis inner orbit



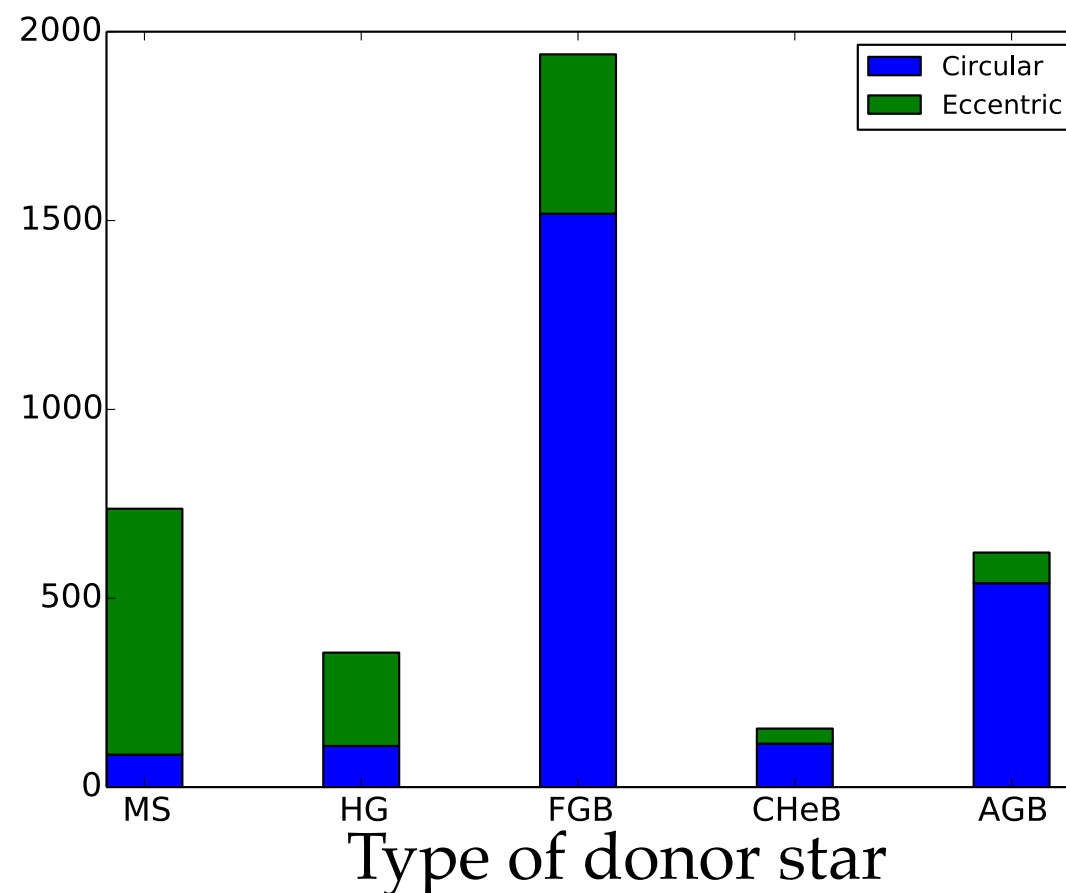
Dissipation:

- Tides, gravitational wave emission

----- Binary case  
----- Triple case

# Triple evolution leads to...

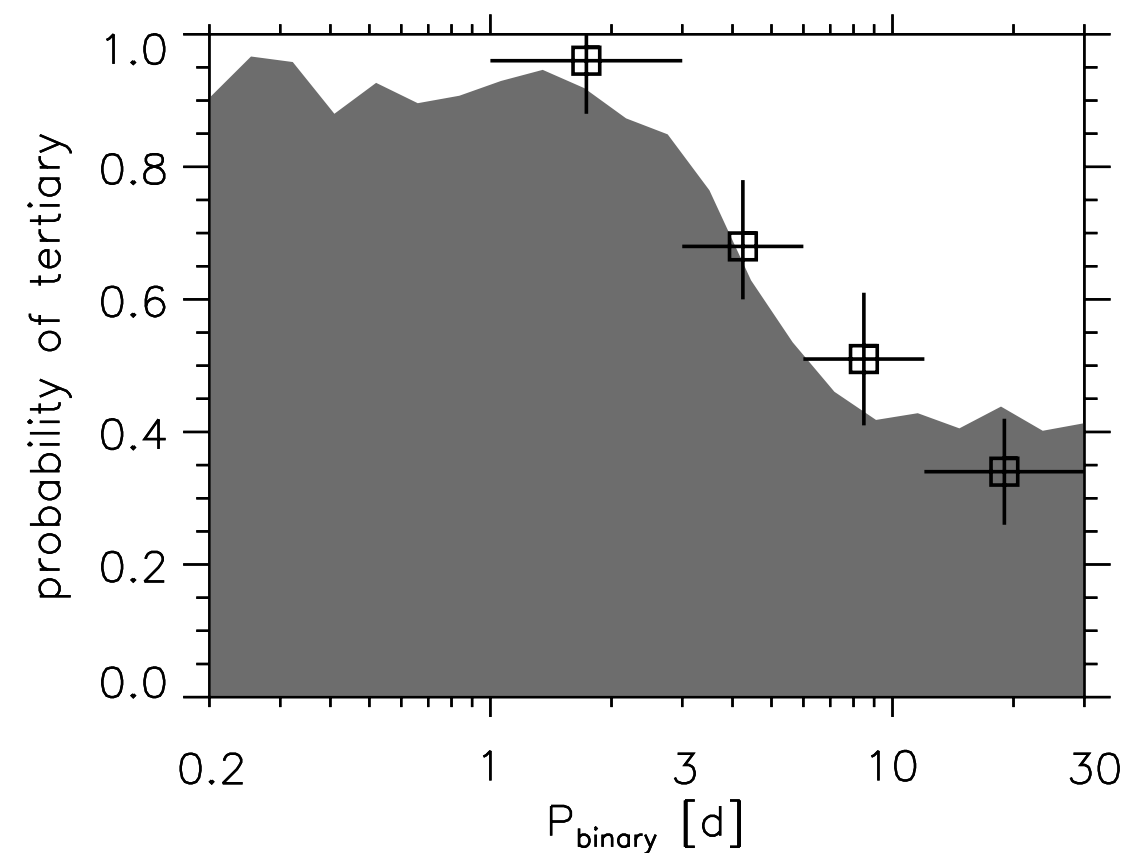
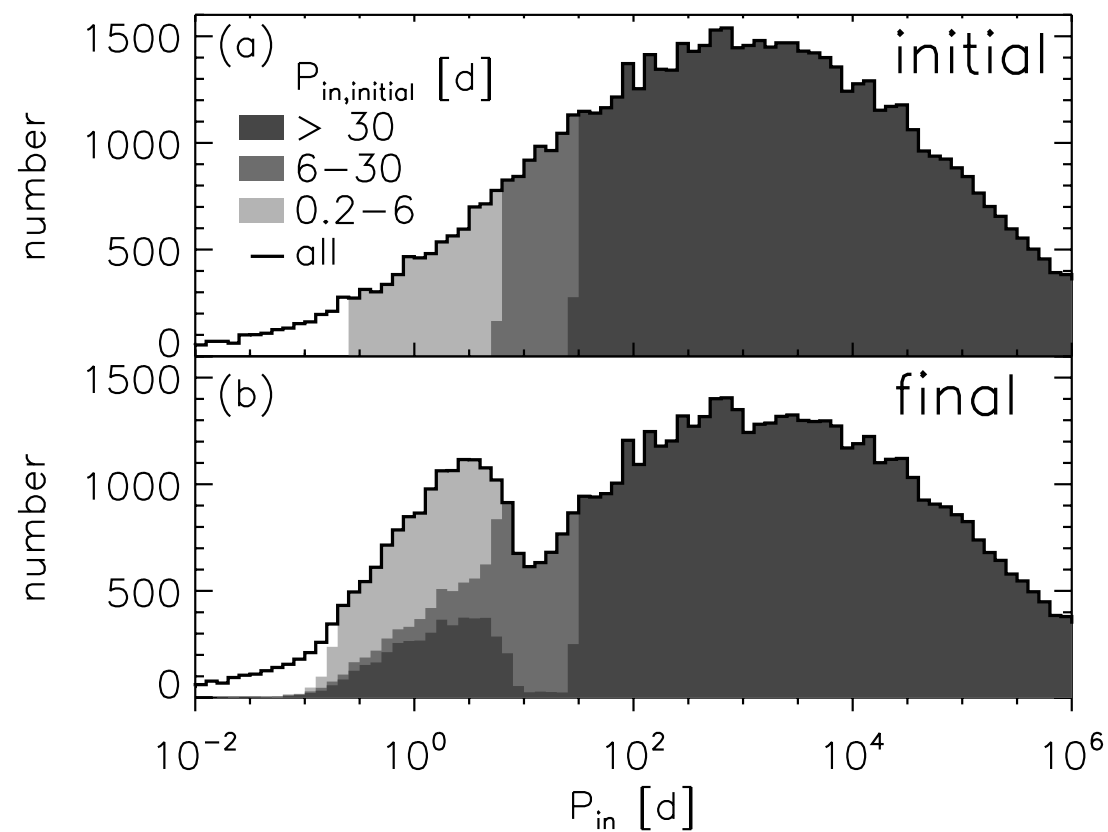
- ➔ Enhanced occurrence rate of mass transfer
  - ❖ ~1.5x more often mass transfer compared to binaries (Toonen+ in prep.)
  - ❖ ~40% of Roche lobe overflow in an eccentric orbit (Toonen+ in prep.)



- Interesting for
  - Blue stragglers (Perets & Fabrycky '09)
  - Great eruption of Eta Carinae (Portegies Zwart & van den Heuvel '16)

# Triple evolution leads to...

- ➔ Enhanced occurrence rate of mass transfer
- ➔ Enhanced formation rate of compact binaries
  - ❖ Excess of close MS-MS (Fabrycky & Tremaine '07, Naoz+ '11)
    - ❖ 96% is part of a triple (Tokovinin+ '06)



# Triple evolution leads to...

- ➔ Enhanced occurrence rate of mass transfer
- ➔ Enhanced formation rate of compact binaries
- ➔ Enhanced merger rate of compact objects

BH-BH mergers

# Triple evolution leads to...

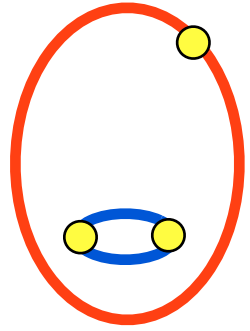
- ➔ Enhanced occurrence rate of mass transfer
- ➔ Enhanced formation rate of compact binaries
- ➔ Enhanced merger rate of compact objects

## BH-BH mergers

Conventional formation channels:

- ❖ Dynamical interactions in dense environments (e.g. Rodriguez+ 15)
- ❖ Isolated massive binaries in the field (e.g. Belczynski+ 10, 16)
- ❖ Chemically homogeneous evolution of compact binary (e.g. Mandel & de Mink '16)

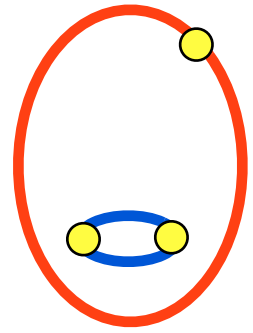
# Evolutionary channel



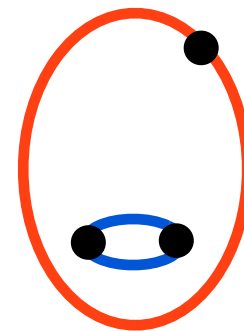
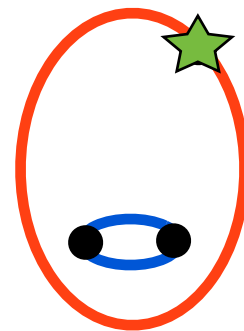
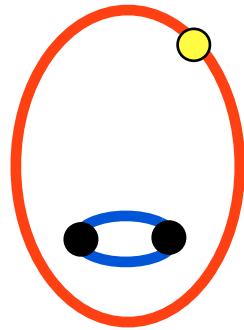
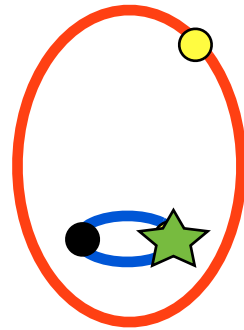
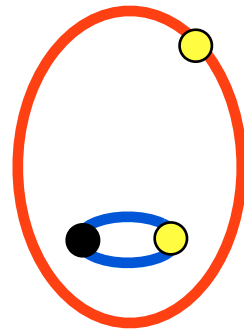
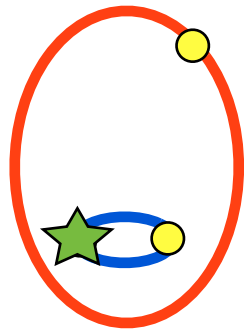
I) Three massive stars in wide orbits



# Evolutionary channel

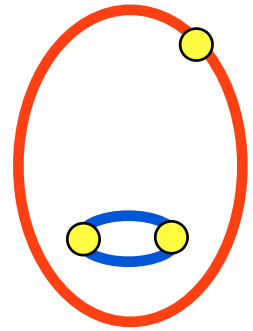


1) Three massive stars in wide orbits

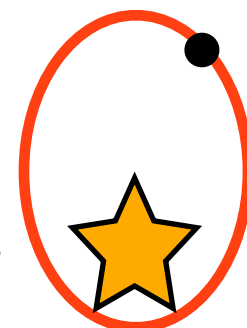
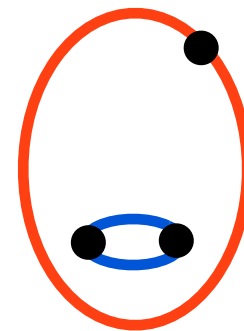
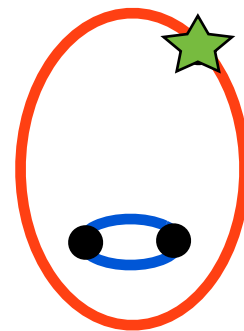
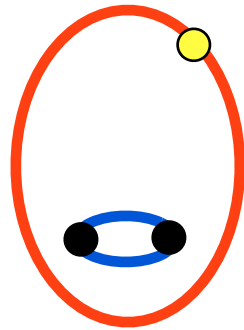
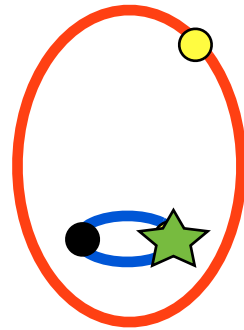
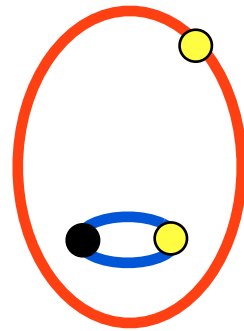
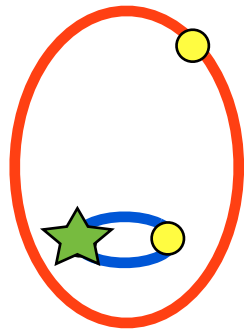


2) Three supernovas

# Evolutionary channel



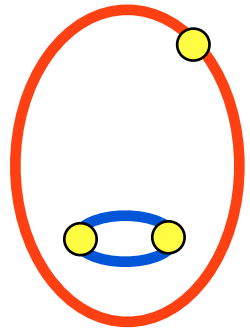
1) Three massive stars in wide orbits



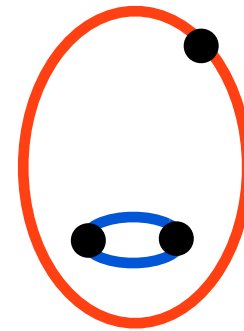
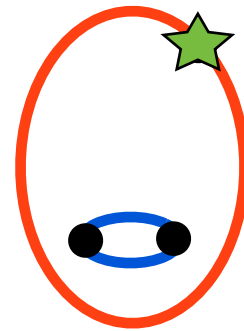
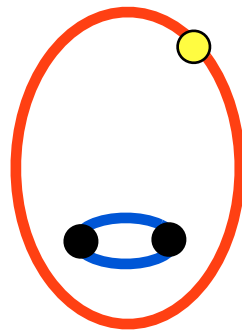
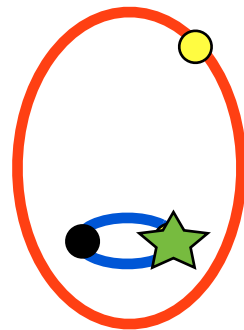
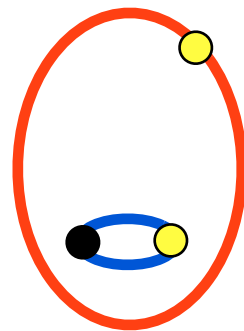
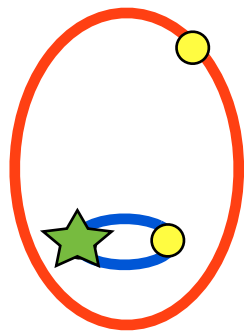
2) Three supernovas

3) Merger due to secular dynamics

# Difficulties



1) Three massive stars in wide orbits



2) Three supernovas

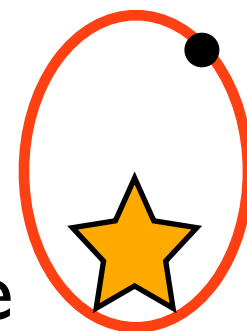
Avoids: a) Mass transfer

b) Dynamical instabilities

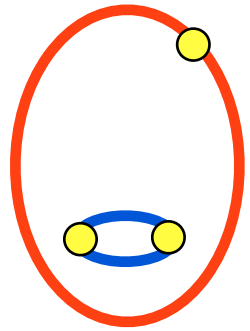
c) Dissolution

3) Merger due to secular dynamics

Get a merger, but not at earlier stage

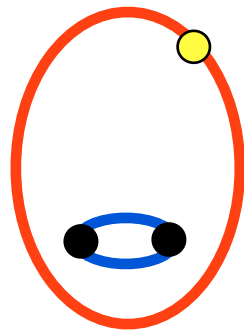
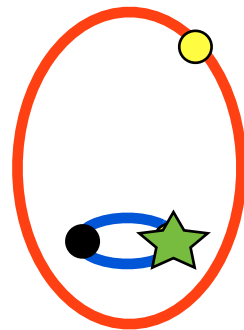
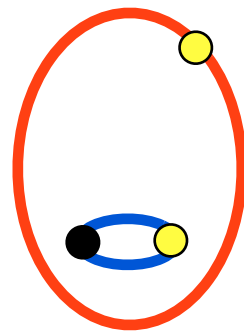
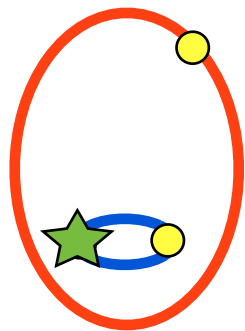


# Difficulties



1) Three massive stars in wide orbits

But no mass transfer or common-envelope needed!

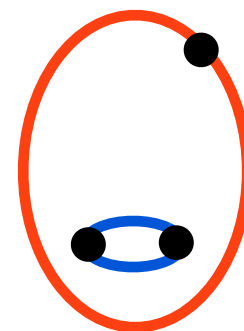
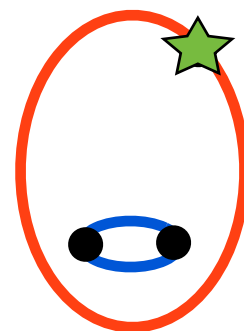


2) Three supernovas

Avoids: a) Mass transfer

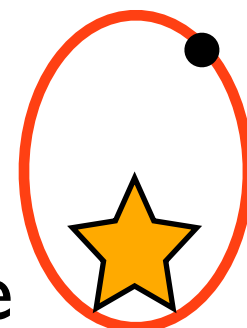
b) Dynamical instabilities

c) Dissolution



3) Merger due to secular dynamics

Get a merger, but not at earlier stage

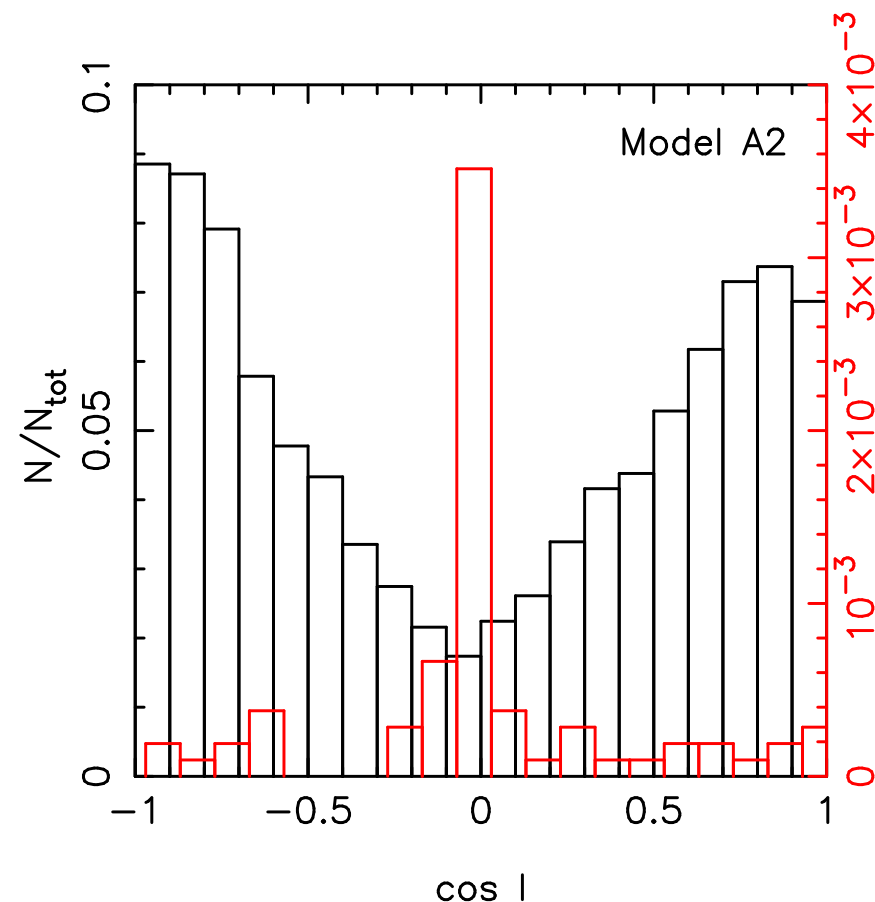


# BH-BH mergers

- ➔ Enhanced occurrence rate of mass transfer
- ➔ Enhanced formation rate of compact binaries
- ➔ Enhanced merger rate of compact objects
  - ❖ Observed rate (Abbott+16): 2-600 per year per  $\text{Gpc}^3$
  - ❖ With natal-kick:  $\sim 0.4$  per year per  $\text{Gpc}^3$
  - ❖ Only Blaauw-kick:  $\sim 1.2$  per year per  $\text{Gpc}^3$

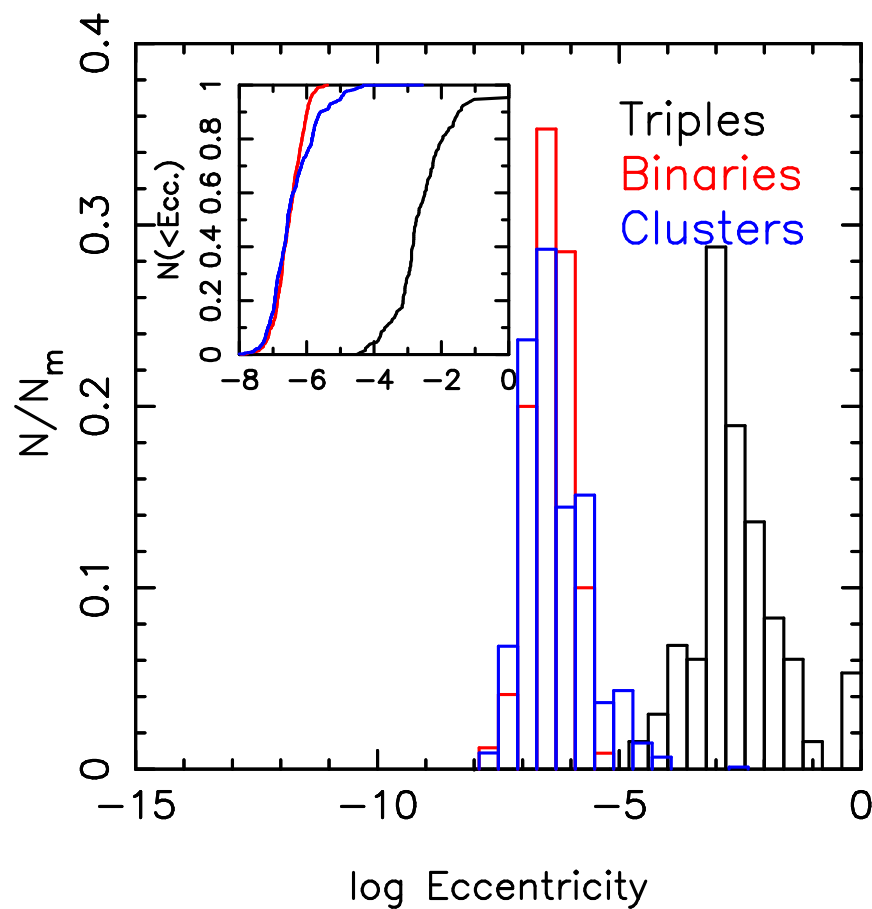
Hobbs / Arzoumanian, momentum-conserving kicks, direct collapse for  $M > 40 M_{\text{sun}}$ ,

- ➔ Triple BHs formed
- ➔ **Merging BHs**
- ❖ Important to model formation of BH-BH in triples consistently



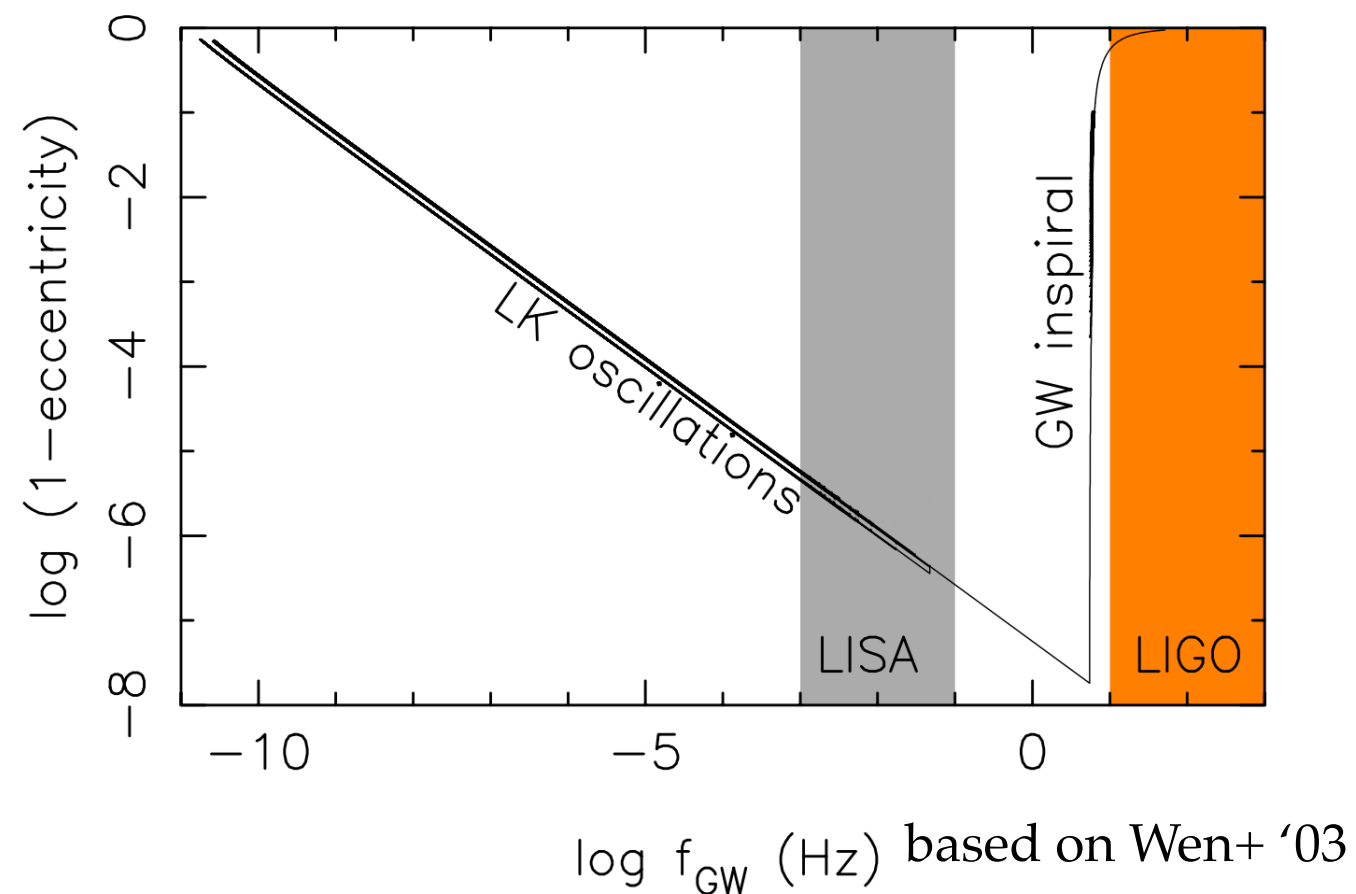
# Distinct characteristics

- ❖ Eccentricity upon entering the LIGO band



Taken from  
Breivik+ '16

- ❖ High eccentricities in the LISA band!



- ❖ Chen & Amaro-Seoane+17:
  - ❖  $e \gtrsim 5e-3$  hard to detect with LISA
  - ❖ ~40% of our systems

# Summary

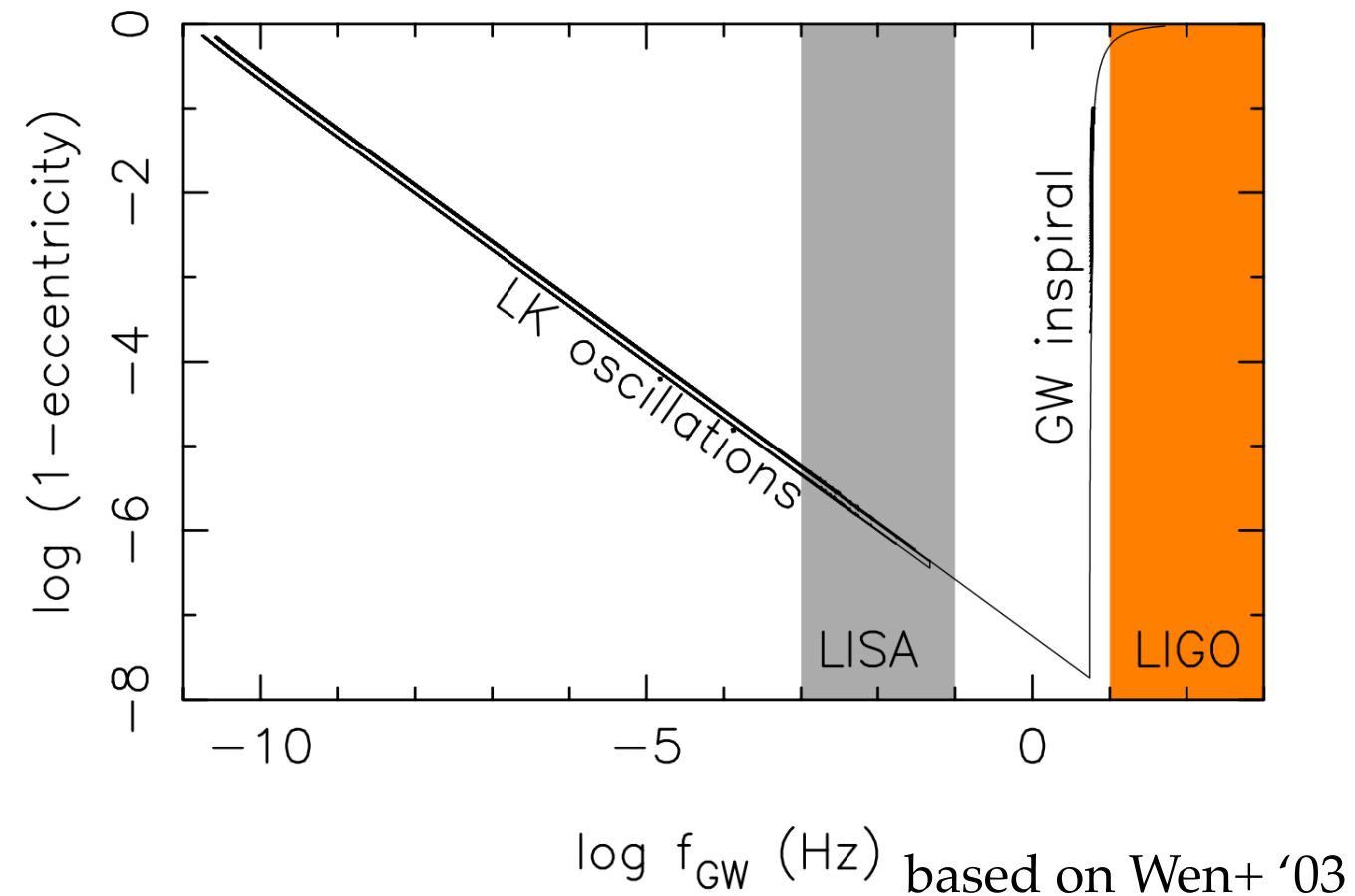
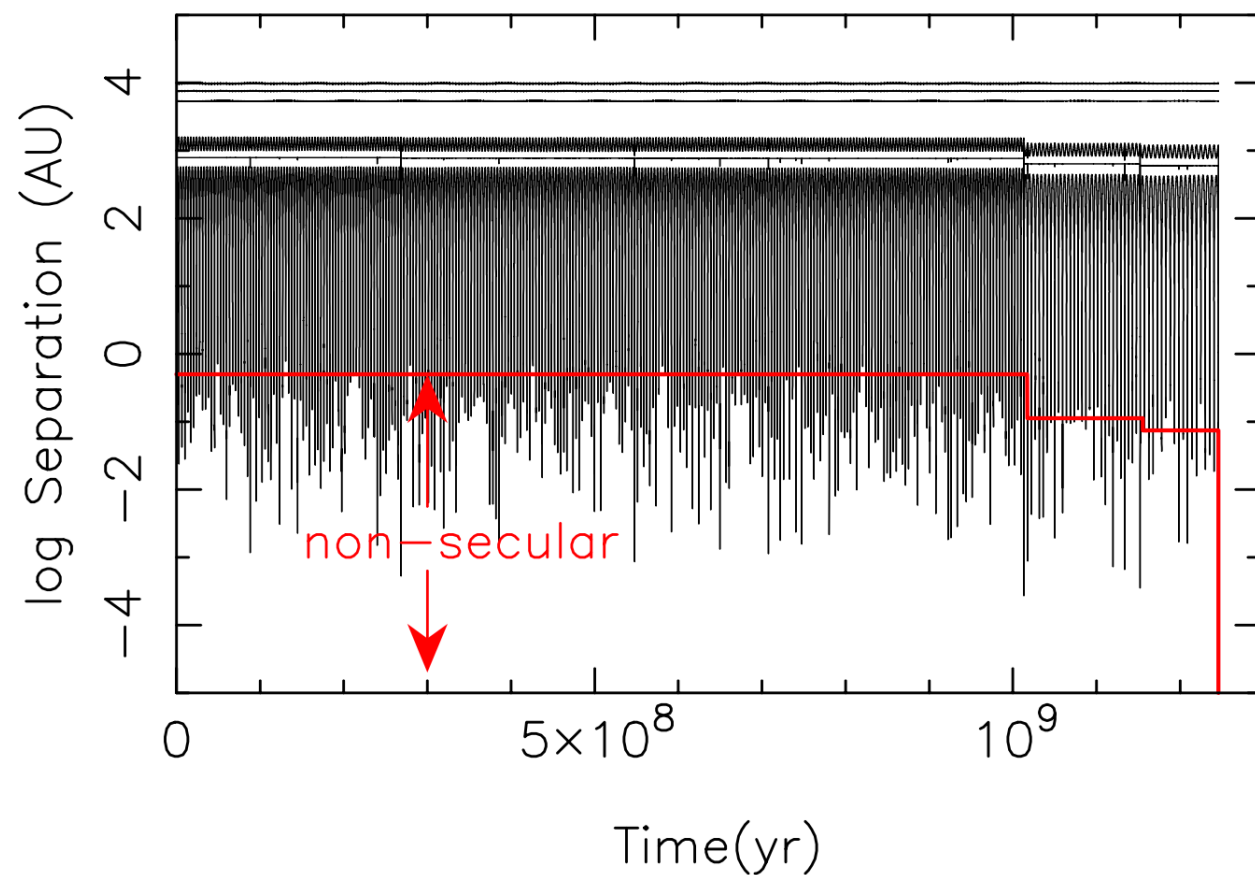
- ❖ The presence of a third star can have a strong effect on the evolution of the inner binary
- ❖ Evolution: Three body dynamics + stellar evolution
  - ❖ Rich interacting regime (Shappee+ '13, Hamers+ '13, Michaely+ '14, Toonen+ '16, Antonini, Toonen & Hamers '17)
- ❖ New code **TRES** for (coeval stellar hierarchical) triple evolution (Toonen+ 2016 => also for review on triple evolution in stellar systems)
- ❖ BH-BH mergers from isolated triples (Antonini, Toonen & Hamers '17)
  - ❖ Rate: 0.3-1.3 per year per  $\text{Gpc}^3$
  - ❖ Few detections per year with aLIGO, harder to detect with LISA
  - ❖ High eccentricities due to 3body dynamics

# Extra slides



# Evolutionary channel

Example: (inner) binary:  $M_1 = 8.96$ ,  $M_2 = 7.51 M_\odot$ ,  $a_{in} = 1727$  AU,  $e_{in} = 0.65$ ,  $g_{in} = .61$  rad  
tertiary star  $M_3 = 8.35 M_\odot$  on an orbit with  $a_{out} = 16571$  AU,  $e_{out} = 0.29$ ,  $i = 93^\circ$ ,  $g_{in} = -2.82$  rad

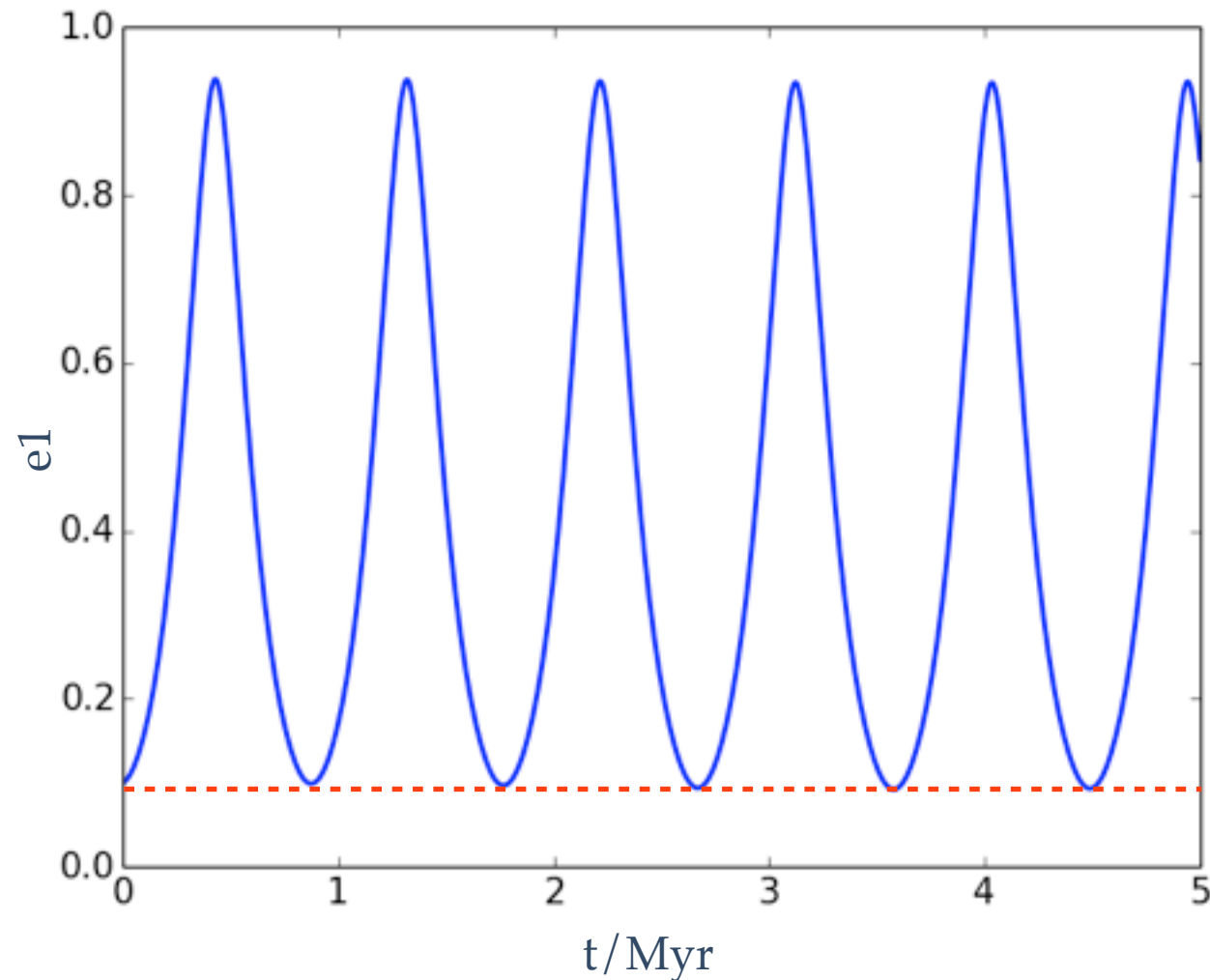


- ❖ **High-eccentricity behaviour: jump in  $J_{inner\_orbit}$  by order unity** (Antonini & Perets 12, Katz & Dong '12)
- ❖ **Step 3): high-precision direct integrator for systems with weaker hierarchies** (Mikkola & Merritt 08)

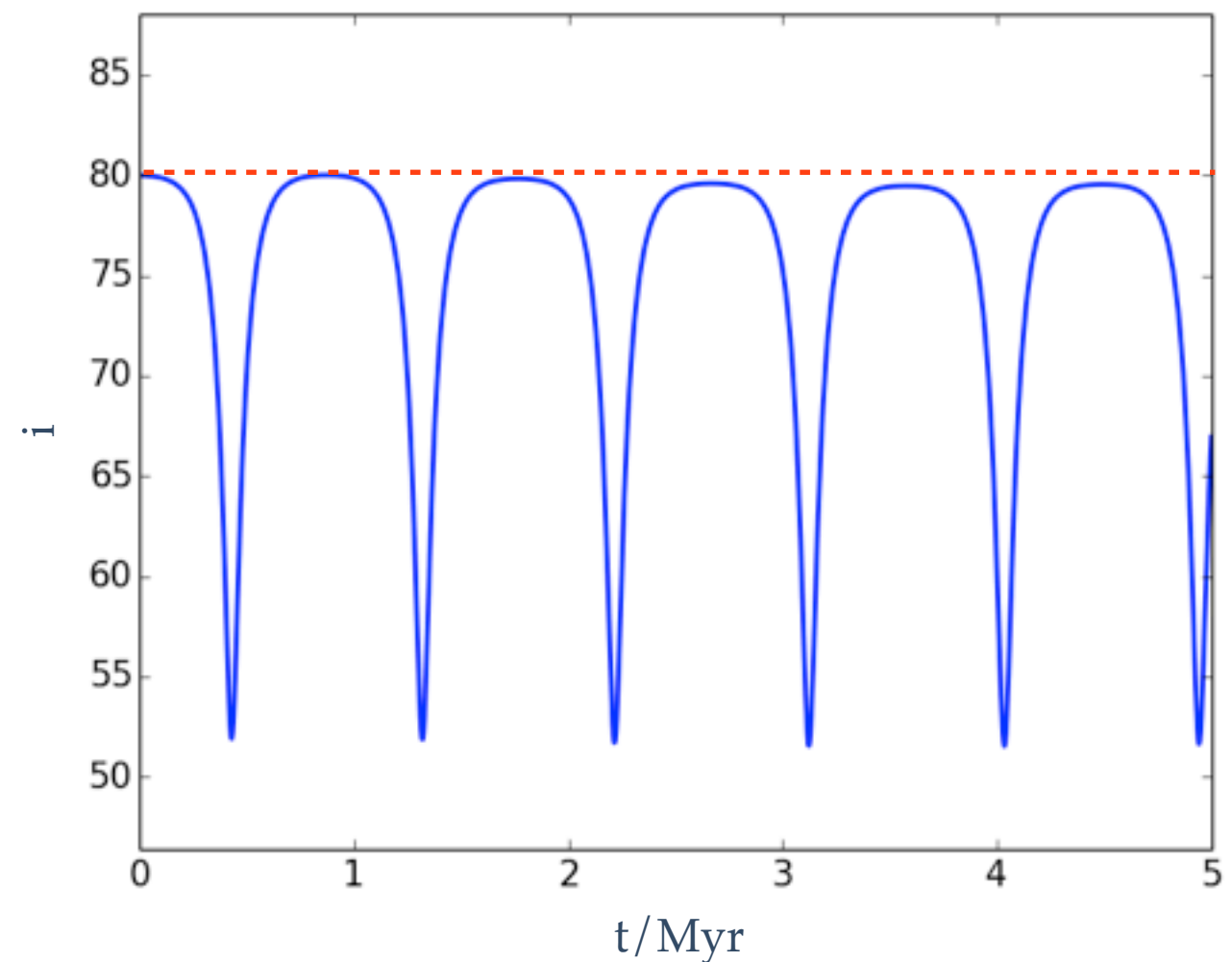
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Eccentricity inner orbit



Mutual inclination



Regular Kozai: 
$$P_{\text{kozai}} = \alpha \frac{P_2^2}{P_1} \frac{m_1 + m_2 + m_3}{m_3} (1 - e_2^2)^{3/2}$$

Eccentric Kozai: 
$$P_{\text{oct}} \sim P_{\text{kozai}} / \epsilon \quad \epsilon \equiv \frac{m_1 - m_2}{m_1 + m_2} \frac{a_1}{a_2} \frac{e_2}{1 - e_2^2}$$

----- Binary case  
 ——— Triple case

# MIEK

Shappee & Thompson (2013) studied the case of mass-loss from a component in the inner binary, which leads to a transition from a more regular Kozai-Lidov secular behavior to the regime where octupole level perturbations become significant, and the amplitude of eccentricity changes become significant; a behavior

## MIEK - Mass-loss induced eccentric Kozai (Shappee & Thompson 2013)

- ❖ Mass-loss from the inner binary causes a transition from regular quadrupole Kozai behaviour to where the octupole becomes significant

$$P_{\text{oct}} \sim P_{\text{kozai}} / \epsilon \quad \epsilon \equiv \frac{m_1 - m_2}{m_1 + m_2} \frac{a_1}{a_2} \frac{e_2}{1 - e_2^2}$$

In other words:

1. Primary star becomes compact object without RLOF

2. Afterwards, secondary fills RL

- ❖ Special evolutionary channel: accreting compact objects without common-envelope phase! (see also: Shappee & Thompson '13, Michaely & Perets '14)

- ❖ How often does this happen in triples?
  - ❖ a few in a 1000 systems for all models (Toonen+ in prep.)

# MIEK

Shappee & Thompson (2013) studied the case of mass-loss from a component in the inner binary, which leads to a transition from a more regular Kozai-Lidov secular behavior to the regime where octupole level perturbations become significant, and the amplitude of eccentricity changes become significant; a behavior

- ❖ Standard example:  
 $m_1=7M_{\text{Sun}}, m_2=6.5M_{\text{Sun}}, m_3=6M_{\text{sun}}; a_1=10\text{AU}, a_2=250\text{AU}, e_1=0.1, e_2=0.7, g_1=0, g_2=180, i=60$
- ❖ Varying  $i, e_1, e_2, g_1$ :
  - ❖ 2 up to 7% of systems go through MIEK (Shappee & Thompson 2013)
- ❖ However...
  - ❖ Even if the inner binary was isolated, RLOF when  $a_1 < 15\text{AU}$
  - ❖ Slightly wider orbits affected by Kozai-Lidov induced-RLOF and wind-induced dynamical instabilities

# Roche Lobe Overflow

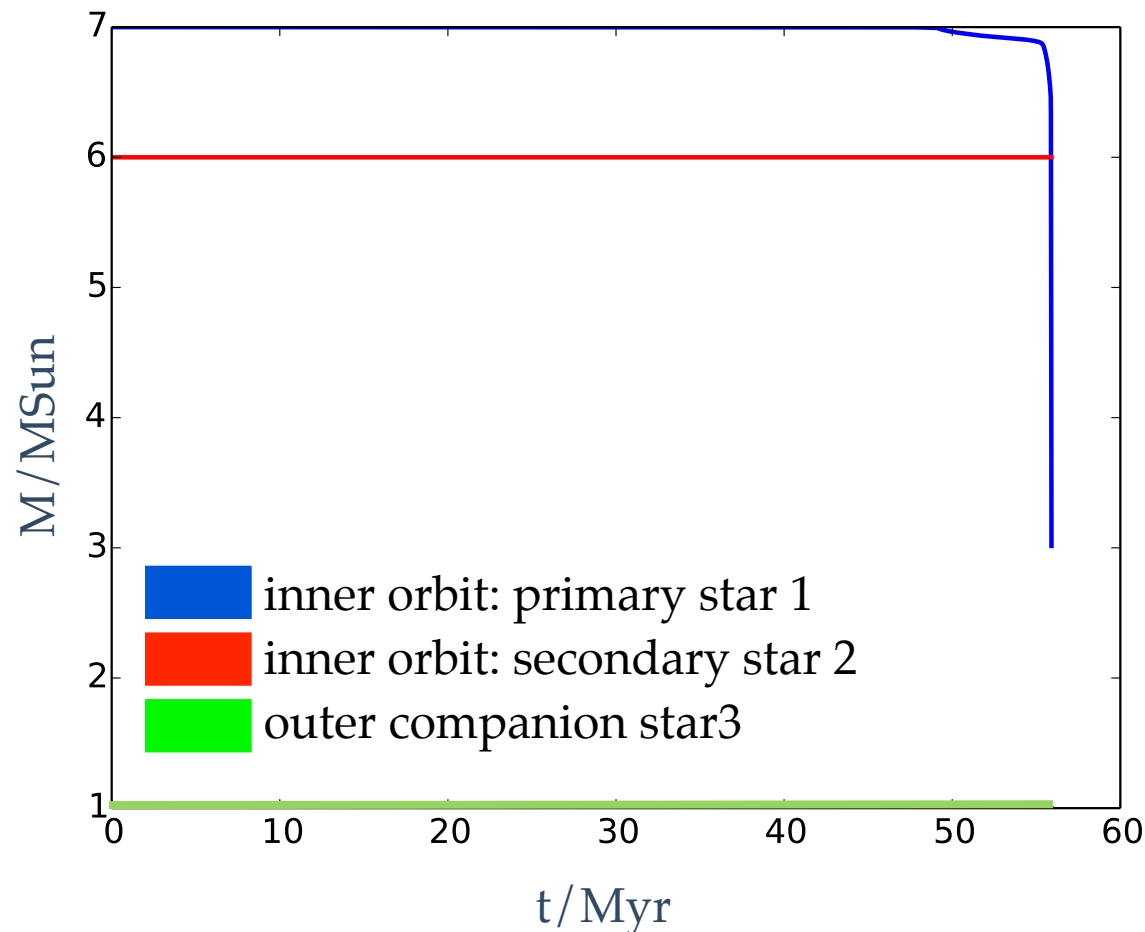
- ❖ In the outer binary from the outer companion
- ❖ How often does this happen in triples?
  - ❖ 0.5% for model uncorrelated binaries I
  - ❖ 1% for model uncorrelated binaries II (Tokovinin)
  - ❖ 0.9% for model Eggleton
- ❖ In good agreement with de Vries et al. '13
  - ❖ For 1% of triples in the Tokovinin catalogue (full primary mass range), the outer companion initiates RLOF before any of the inner stars leave the main sequence
  - ❖ Predominantly evolved (AGB) donor stars
  - ❖ From SPH simulations for  $\xi$  Tau and HD97131

$$\frac{(\dot{a}_{in}/a_{in})}{(\dot{a}_{out}/a_{out})} \simeq 1$$

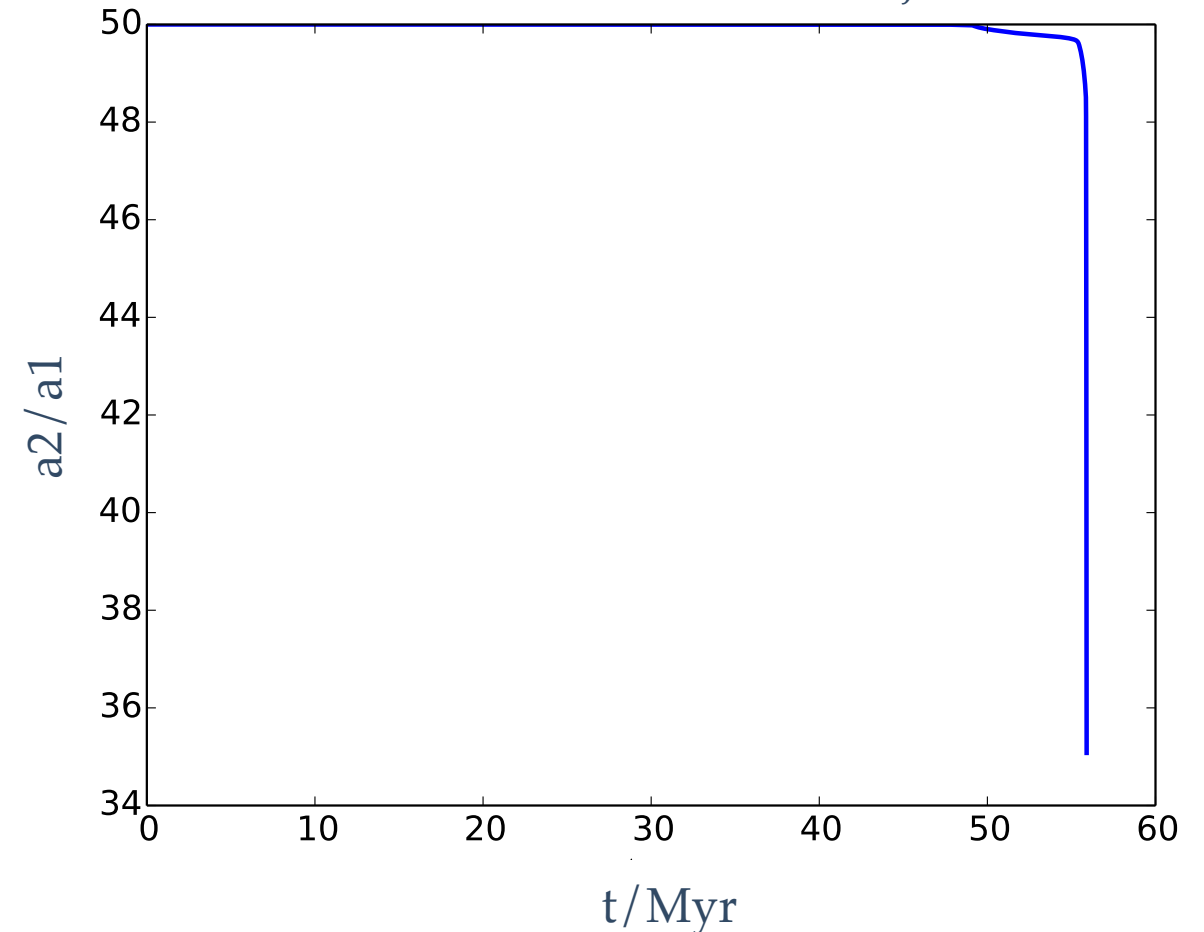
# Dynamical instability

$M_1 = 7, M_2 = 1, M_3 = 6 M_{\text{Sun}}, a_1 = 1e4, a_2 = 5e5 R_{\text{Sun}}, e_1 = 0.1, e_2 = 0.8, i = 0, g_1 = 0.1, g_2 = 0.5$

Stellar mass



Ratio of outer and inner semimajor-axis



Effect of wind mass-loss in inner binary:

❖ Orbits widen, inner orbit widens more

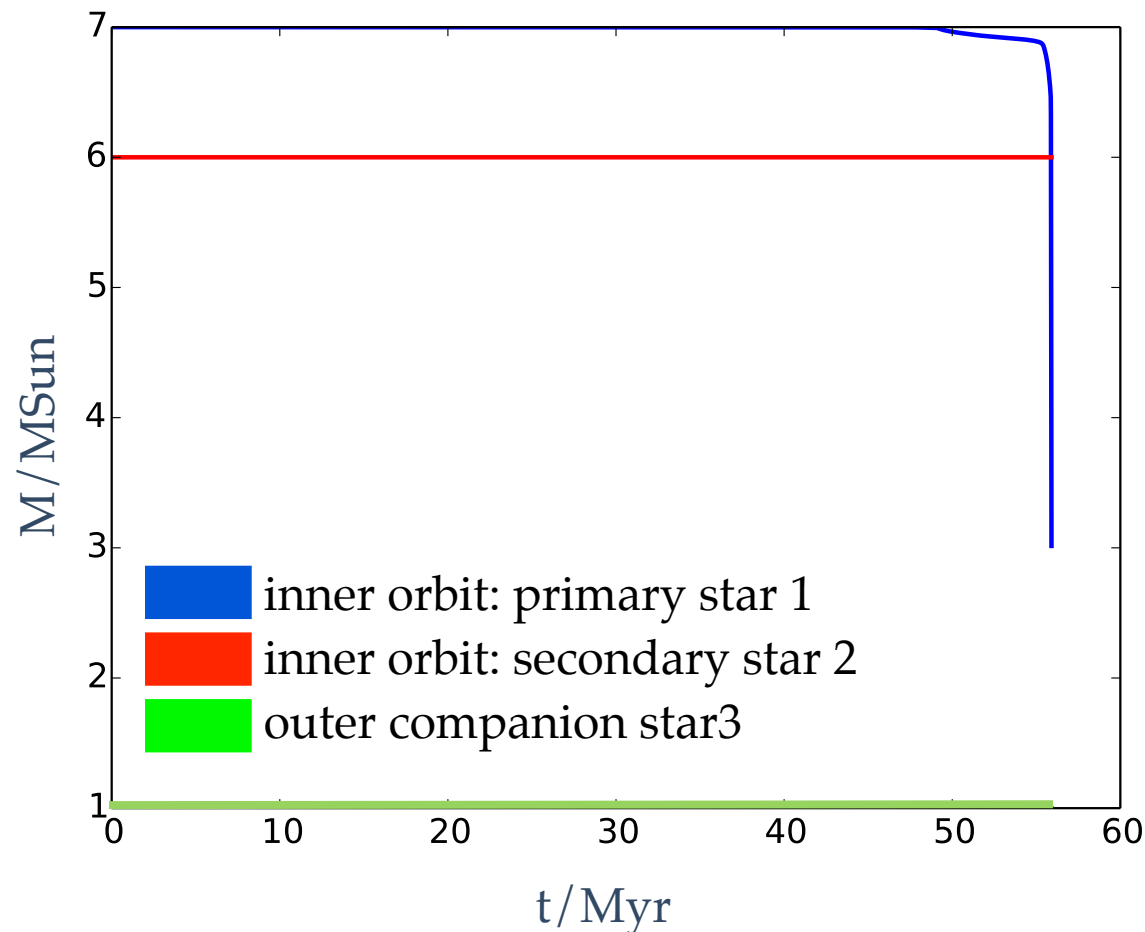
❖ Orbits come closer to each other  $\Rightarrow$  possible dynamical instability

$$\frac{a_f}{a_i} = \frac{M_i}{M_f} = 1 - \frac{\Delta M}{M}$$

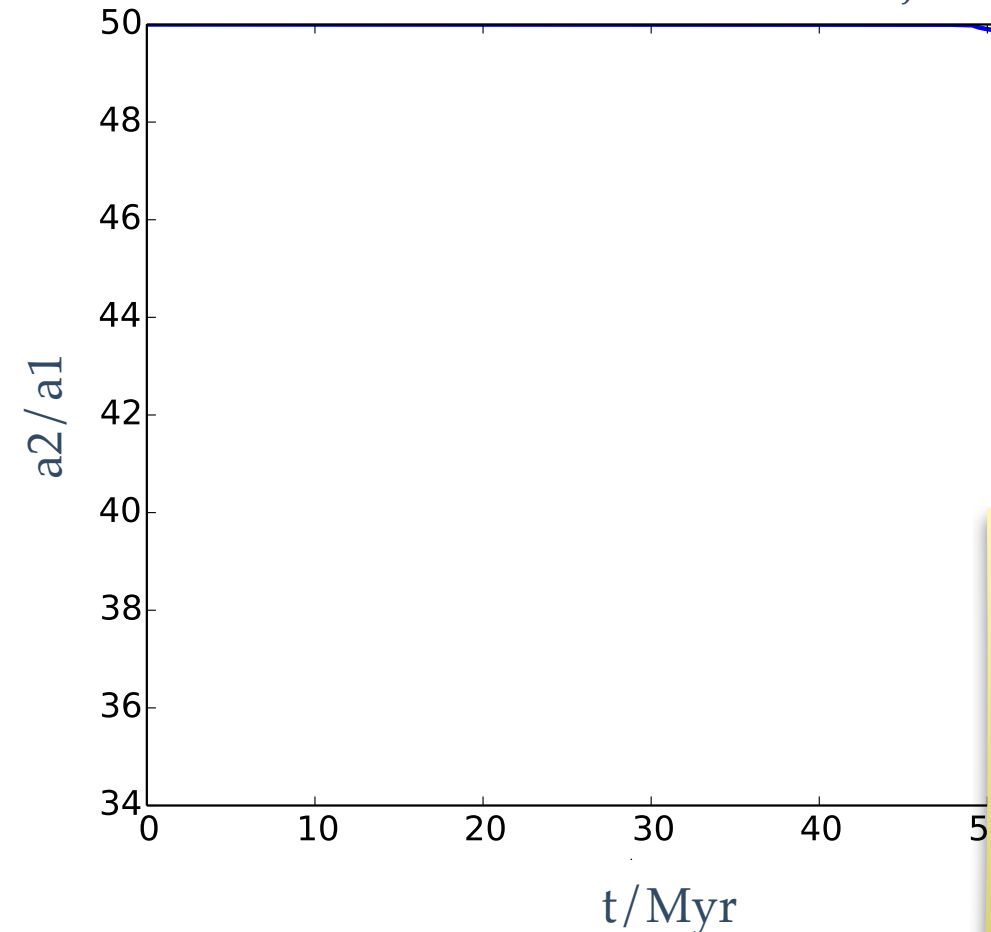
# Dynamical instability

$M_1 = 7, M_2 = 1, M_3 = 6 M_{\text{Sun}}, a_1 = 1e4, a_2 = 5e5 R_{\text{Sun}}, e_1 = 0.1, e_2 = 0.8, i = 0, g_1 = 0.1, g_2 = 0.5$

Stellar mass



Ratio of outer and inner semimajor-axis



Moreover, we find that the dominant type of stellar collision is qualitatively different; most collisions involve asymptotic giant branch stars, rather than main sequence, or slightly evolved stars, which dominate collisions in globular clusters.

we demonstrate that the rate of stellar collisions due to the TED is approximately  $10^{-4} \text{ yr}^{-1}$  per Milky-Way Galaxy, which is nearly 30 times higher than the total collision rate due to random encounters in the Galactic globular clusters.

## Triple dynamical instability (e.g. Kiseleva+ '94, Iben & Tutukov '99)

- ❖ Rate: 3% of all triples (Perets & Kratter '12, Hamers+ '13, Toonen+ in prep.)
  - ❖ close encounters, collisions, stellar exchanges, eccentric binaries
  - ❖ high collision rate, involving AGB stars (Perets & Kratter '12)

# Summ

At the same time, triple evolution is often invoked to explain exotic systems which cannot be explained easily by binary evolution. Examples are low-mass X-ray binaries, supernova type Ia progenitors and blue stragglers.

- ❖ The presence of a third star can have a significant impact on the evolution of the inner binary

What are the common evolutionary pathways that triple systems go through? Are there any evolutionary pathways open to triples, which are not open to isolated binaries?

- ❖ Triple evolution can lead to:

- ❖ Enhanced formation of compact binaries

- ❖ Enhanced occurrence rate of mass transfer

- ❖ Evolution: Three body dynamics + stellar evolution

- ❖ Rich interacting regime (Shappee+ '13, Hamers+ '13, Michaely+ '14, Toonen+ '16)

- ❖ New code **TRES** for (coeval stellar hierarchical) triple evolution (Toonen + 2016, Toonen+ in prep.)

- ❖ **Goal veni: Create comprehensive model of triple evolution**



# Code for Triple Evolution

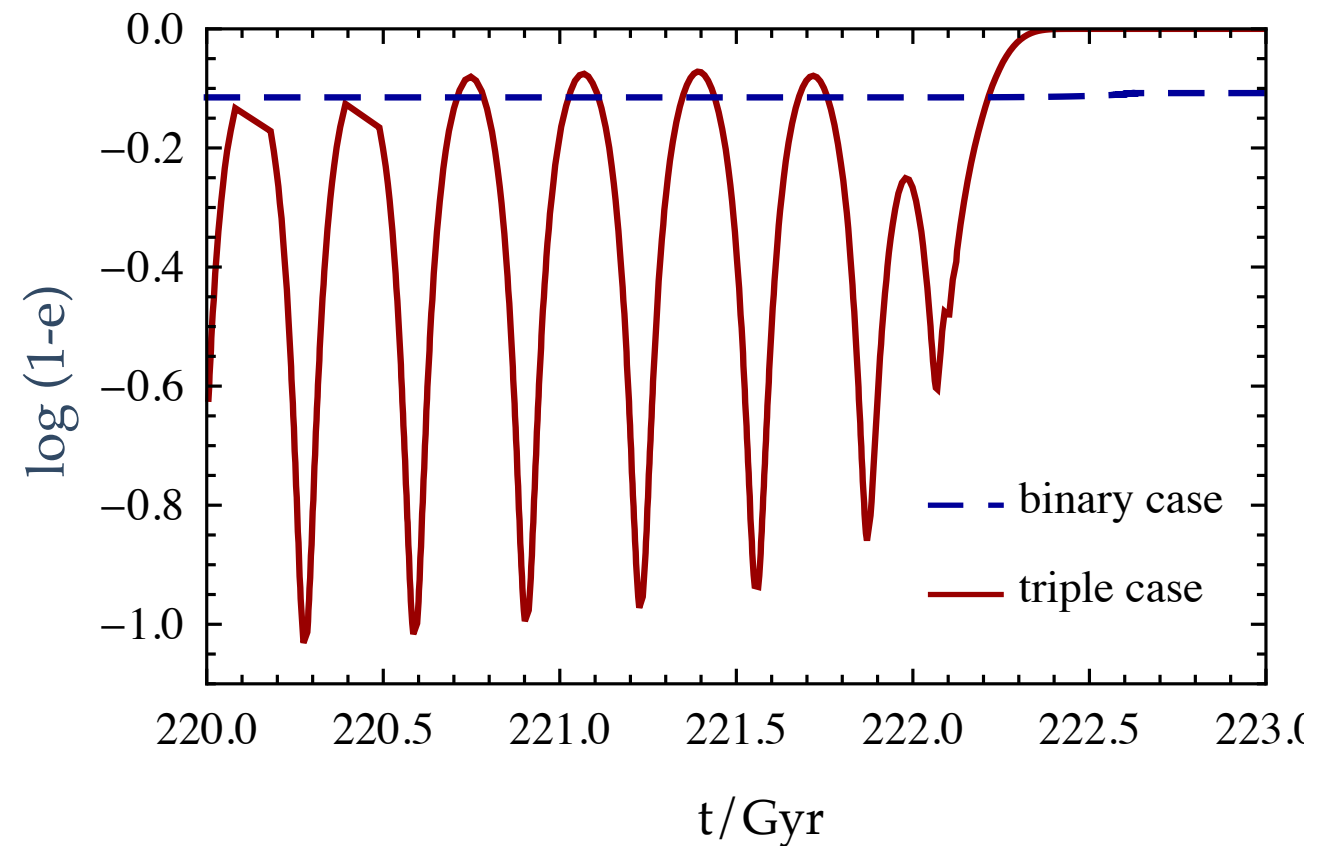
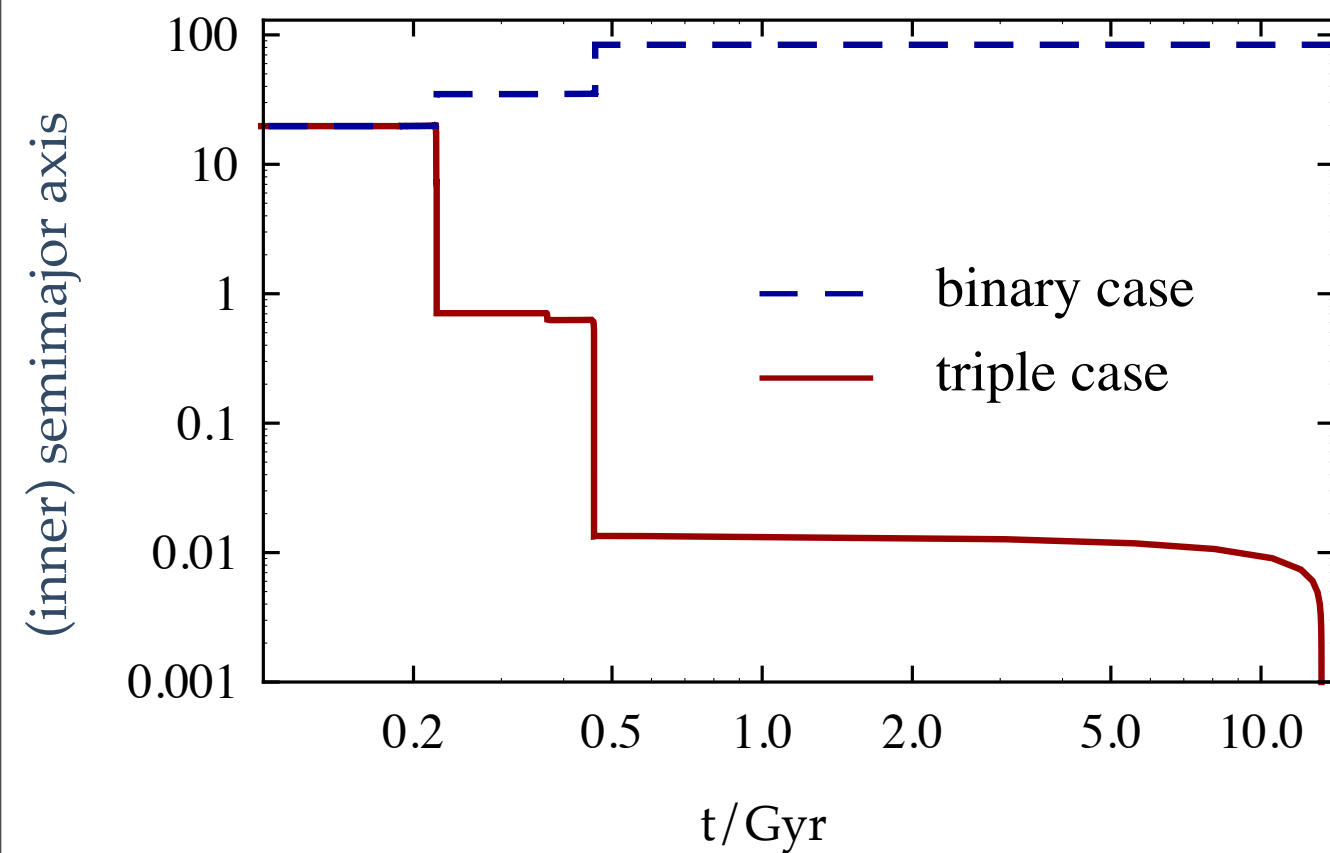
- ❖ New code **TRES** for (isolated coeval stellar hierarchical) triple evolution (Toonen+ 2016, Toonen+ in prep.)
  - ❖ Will become publicly available
- ❖ Written in Astrophysical Multipurpose Software Environment
  - ❖ software framework astrophysical simulations
  - ❖ existing codes from different domains (stellar dynamics, stellar evolution, hydrodynamics and radiative transfer)
  - ❖ easy coupling between the codes
  - ❖ easy coupling to N-body code or detailed stellar evolution codes



# Long-term effect

Example: (inner) binary:  $M_1 = 3.95$ ,  $M_2 = 3.03M_{\odot}$ ,  $a_{in} = 19.7$  AU,  $e_{in} = 0.23$

tertiary star  $M_3 = 2.73M_{\odot}$  on an orbit with  $a_{out} = 636$  AU,  $e_{out} = 0.82$ ,  $i = 116^{\circ}$



. Put another way, some systems which one might think have no hope of merging in tH, actually merge in tH with a suitably placed tertiary

# Initial population

- ❖ Monte Carlo method to generate initial systems containing zero-age main-sequence stars
- ❖ 3 sets of distributions

	uncorrelated binaries 1	uncorrelated binaries 2	Eggleton
m1			
m2			
m3			
a			
e	thermal	thermal	thermal
i	circular uniform [0,pi]	circular uniform [0,pi]	circular uniform [0,pi]
g	uniform [-pi,pi]	uniform [-pi,pi]	uniform [-pi,pi]
$\Omega$	fit of Hurley ea '00 to Lang '92	fit of Hurley ea '00 to Lang '92	fit of Hurley ea '00 to Lang '92

# Initial population

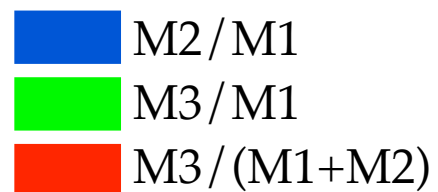
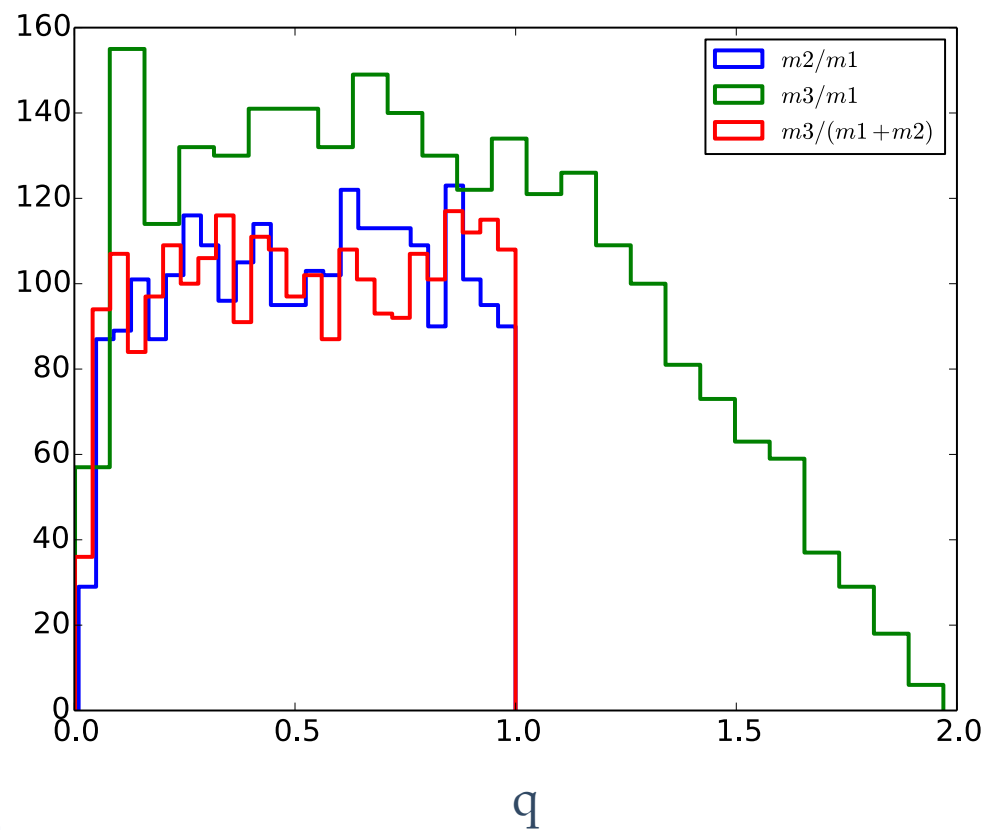
- ❖ Monte Carlo method to generate initial systems containing zero-age main-sequence stars
- ❖ 3 sets of distributions

	uncorrelated binaries 1	uncorrelated binaries 2	Eggleton
$m_1$	Kroupa IMF (Kroupa ea '93)	Kroupa IMF (Kroupa ea '93)	Eggleton '09
$m_2$	flat in $m_2/m_1$	flat in $m_2/m_1$	Eggleton '09
$m_3$	flat in $m_3/(m_1+m_2)$	flat in $m_3/(m_1+m_2)$	Eggleton '09
$a$	flat in $\log a$ (Abt '83)	Tokovinin '14 (lognormal, $\mu = 1e5d$ , $\sigma=2.3$ )	Eggleton '09
$e$	thermal	thermal	thermal
$i$	circular uniform $[0,\pi]$	circular uniform $[0,\pi]$	circular uniform $[0,\pi]$
$g$	uniform $[-\pi,\pi]$	uniform $[-\pi,\pi]$	uniform $[-\pi,\pi]$
$\Omega$	fit of Hurley ea '00 to Lang '92	fit of Hurley ea '00 to Lang '92	fit of Hurley ea '00 to Lang '92

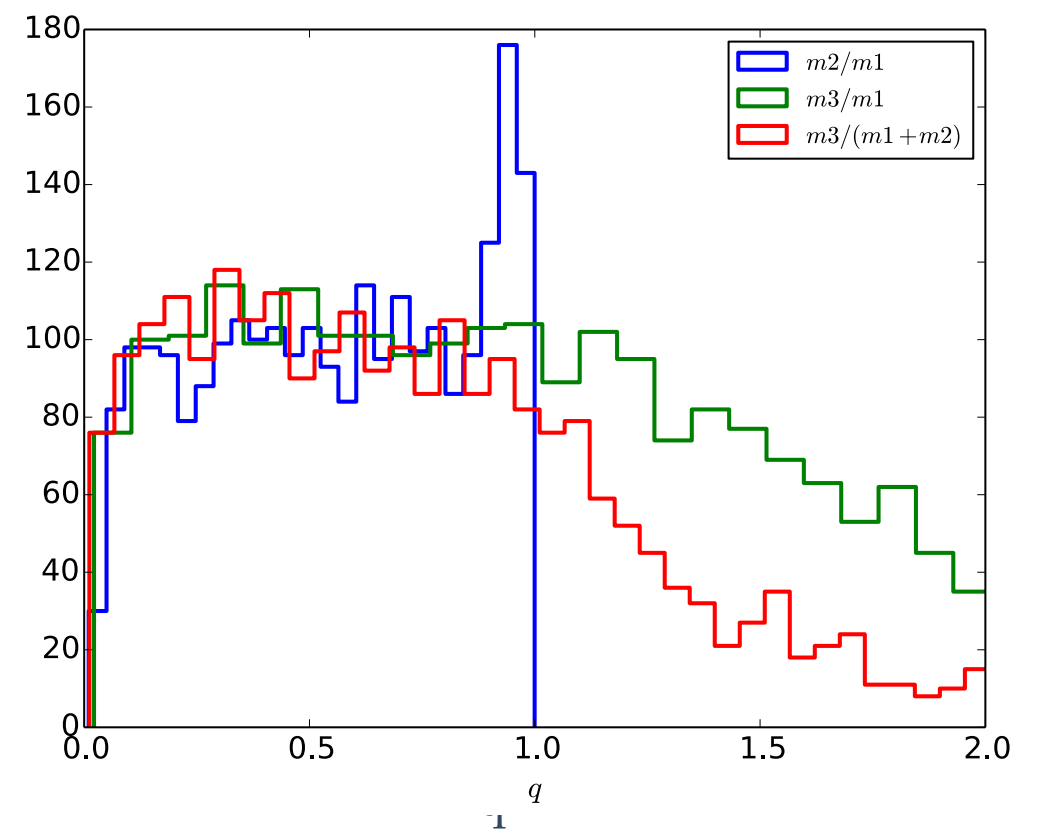
# Initial population

- ❖ Mass ratio distribution

- ❖ Uncorrelated binaries



- ❖ Eggleton '09

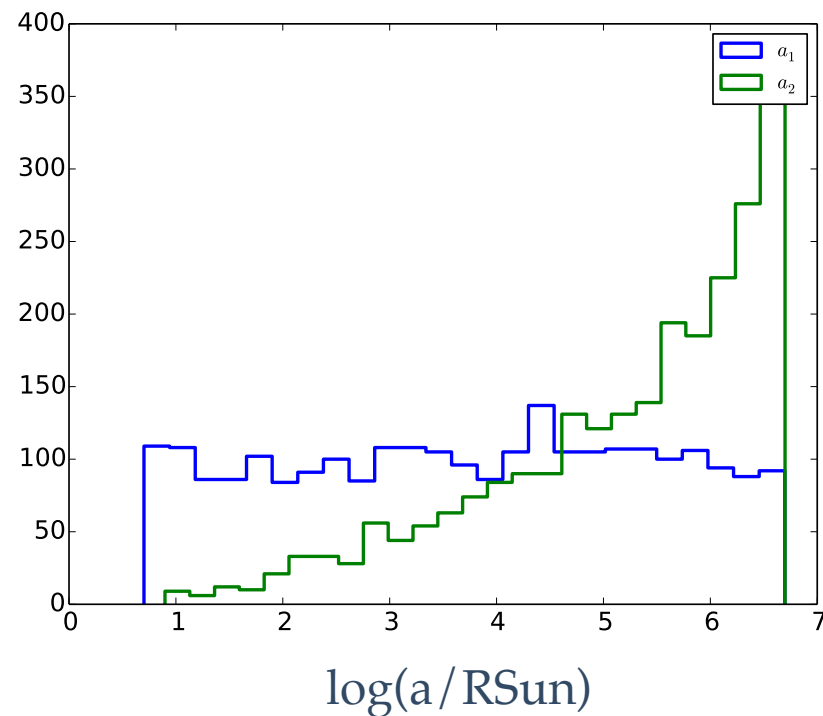


- ❖ Note: per definition  $m_1 > m_2$
- ❖  $m_1 \Rightarrow$  primary,  $m_2 \Rightarrow$  secondary

# Initial population

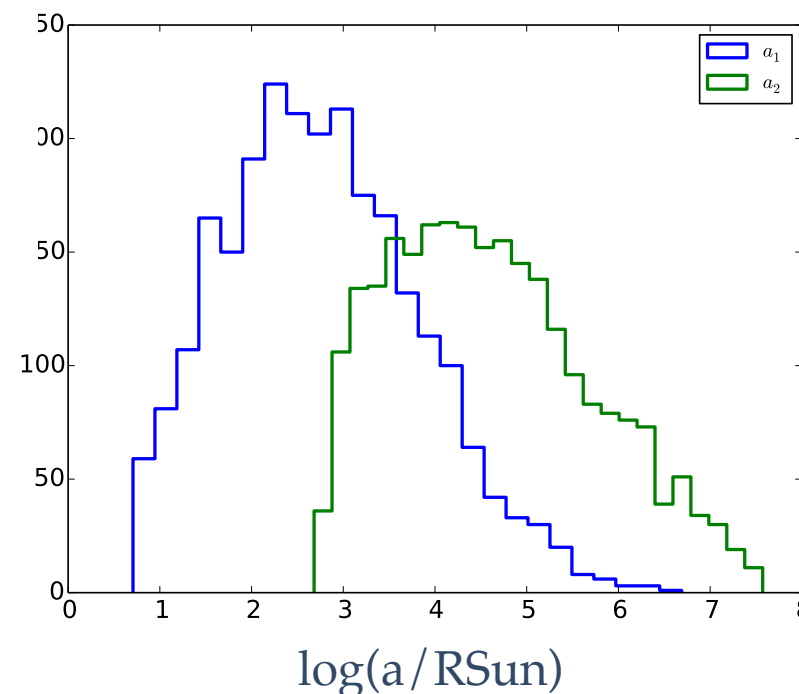
- ✧ Distribution of orbital separation

- ✧ Uncorrelated binaries 1:  
Abt '83 - flat in log a

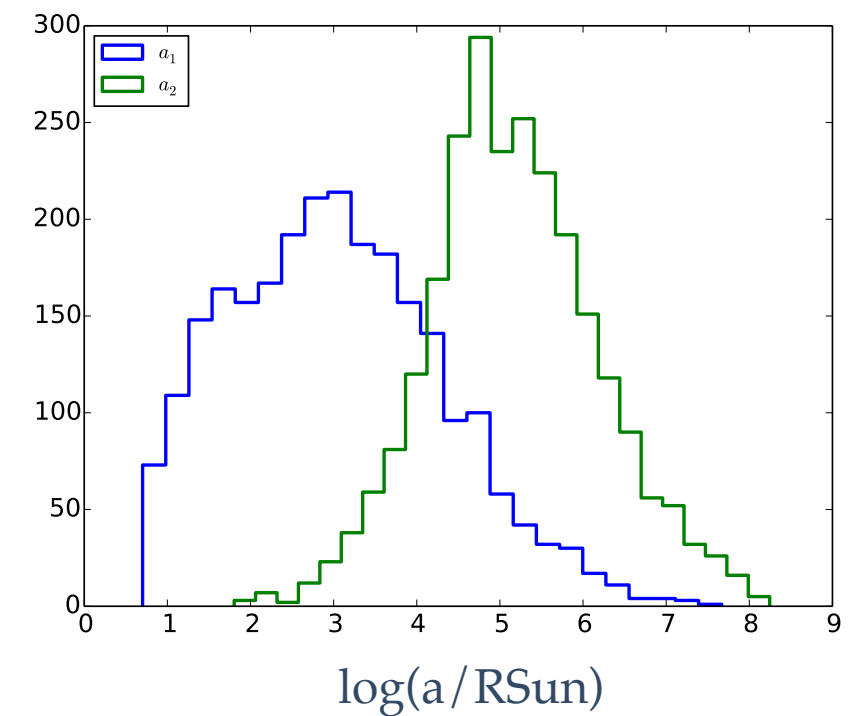


■ a1  
■ a2

- ✧ Uncorrelated binaries 2:  
Tokovinin '14 - log normal periods

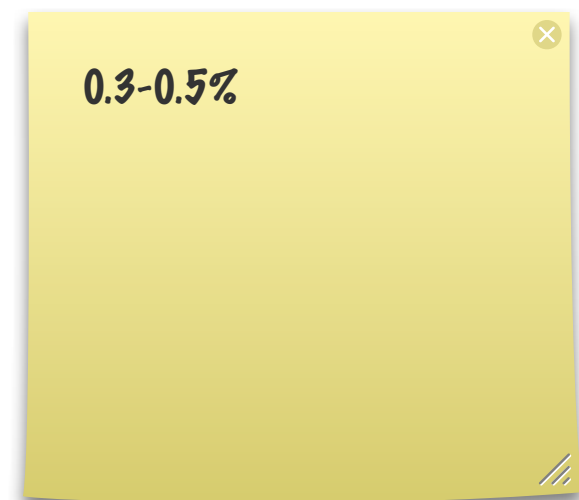


- ✧ Eggleton '09



# Roche Lobe Overflow

- ❖ In the inner binary by the secondary
  - ❖ after the primary has become a compact object
  - ❖ special evolutionary channel
    - ❖ to form compact binaries without mass transfer (see also: Shappee & Thompson '13, Michaely & Perets '14)
- ❖ How often does this happen in triples?
  - ❖ a few in a 1000 systems for all models
- ❖ How eccentric is the orbit?
  - ❖ Roughly half of systems:  $e_{in} \sim 0$
  - ❖ Other half:  $e_{in} > 0.8$
- ❖ Donor stars can be evolved or non-evolved stars



# MIEK

Shappee & Thompson (2013) studied the case of mass-loss from a component in the inner binary, which leads to a transition from a more regular Kozai-Lidov secular behavior to the regime where octupole level perturbations become significant, and the amplitude of eccentricity changes become significant; a behavior

## MIEK - Mass-loss induced eccentric Kozai (Shappee & Thompson 2013)

- ❖ Mass-loss from the inner binary causes a transition from regular quadrupole Kozai behaviour to where the octupole becomes significant
- ❖ Standard example:  
 $m_1=7M_{\text{Sun}}, m_2=6.5M_{\text{Sun}}, m_3=6M_{\text{sun}}; a_1=10\text{AU}, a_2=250\text{AU}, e_1=0.1, e_2=0.7, g_1=0, g_2=180, i=60$
- ❖ Varying  $i, e_1, e_2, g_1$ :
  - ❖ 2 up to 7% of systems go through MIEK (Shappee & Thompson 2013)
- ❖ However...
  - ❖ Even if the inner binary was isolated, RLOF when  $a_1 < 15\text{AU}$
  - ❖ Slightly wider orbits affected by Kozai-Lidov induced-RLOF and wind-induced dynamical instabilities



# Secular evolution

Solve set of first-order ordinary differential equations

- As a function of semimajor-axis  $a$ , eccentricity  $e$ , inclination  $i$ , argument of pericenter  $\omega$ , longitude of ascending node  $h$ , spin angular frequency  $\Omega$

$$\left\{ \begin{array}{l} \dot{a}_1 = \dot{a}_{1,GR} + \dot{a}_{1,TF} + \dot{a}_{1,wind} + \dot{a}_{1,MT}, \\ \dot{a}_2 = \dot{a}_{2,GR} + \dot{a}_{2,TF} + \dot{a}_{2,wind} + \dot{a}_{2,MT}, \\ \dot{e}_1 = \dot{e}_{1,STD} + \dot{e}_{1,GR} + \dot{e}_{1,TF}, \\ \dot{e}_2 = \dot{e}_{2,STD} + \dot{e}_{2,GR} + \dot{e}_{2,TF}, \\ \dot{\theta} = \frac{-1}{G_1 G_2} [\dot{G}_1 (G_1 + G_2 \theta) + \dot{G}_2 (G_2 + G_1 \theta)], \\ \dot{g}_1 = \dot{g}_{1,STD} + \dot{g}_{1,GR} + \dot{g}_{1,tides} + \dot{g}_{1,rotate}, \\ \dot{g}_2 = \dot{g}_{2,STD} + \dot{g}_{2,GR} + \dot{g}_{2,tides} + \dot{g}_{2,rotate}, \\ \dot{h}_1 = \dot{h}_{1,STD}, \\ \dot{\Omega}_{\star 1} = \dot{\Omega}_{\star 1,TF} + \dot{\Omega}_{\star 1,I}, \\ \dot{\Omega}_{\star 2} = \dot{\Omega}_{\star 2,TF} + \dot{\Omega}_{\star 2,I}, \\ \dot{\Omega}_{\star 3} = \dot{\Omega}_{\star 3,TF} + \dot{\Omega}_{\star 3,I}, \end{array} \right.$$

processes that are described are independent such that the in time derivative terms can be added linearly. In addition, in the expressions for  $\dot{g}_{1,tide}$ ,  $\dot{g}_{1,rotate}$ ,  $\dot{e}_{1,TF}$ ,  $\dot{a}_{1,TF}$ ,  $\dot{i}_{1,TF}$  and  $\dot{\theta}$  we assume coplanarity of spin and orbit at all times, even though Kozai cycles in principle affect the relative orientations between the spin and orbit angular momentum vectors and in turn a misalignment of these vectors affects the Kozai cycles themselves (e.g. Correia et al. 2011). We justify this assumption by noting that for the majority of systems that we study, the orbital angular momenta of both inner and outer orbits greatly exceed the spin angular momenta in magnitude (Harrington '68; Ford, Kozinsky & Rasio '00; Naoz et al. '11).

With: **STD = secular three-body dynamics** (Harrington '68; Ford, Kozinsky & Rasio '00; Naoz et al. '11)  
**GR include 1 & 2.5 PN** (Peters '64, Blaes et al. '02)

**TF = tidal friction** (Hut '81)

**tides** (Smeyers & Willems '01)

**rotate** (Fabrycky & Tremaine 2002)

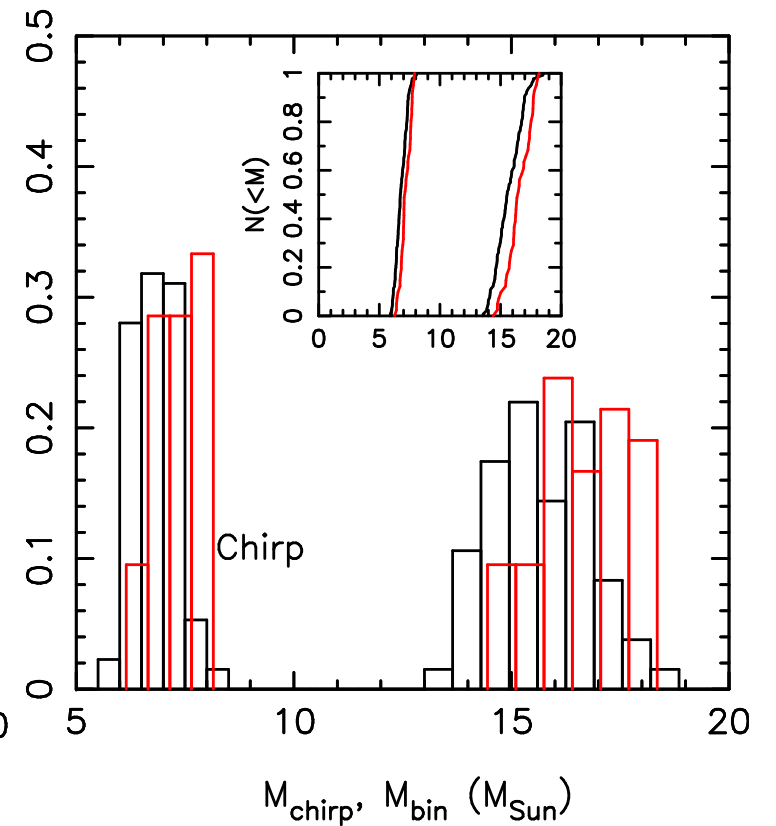
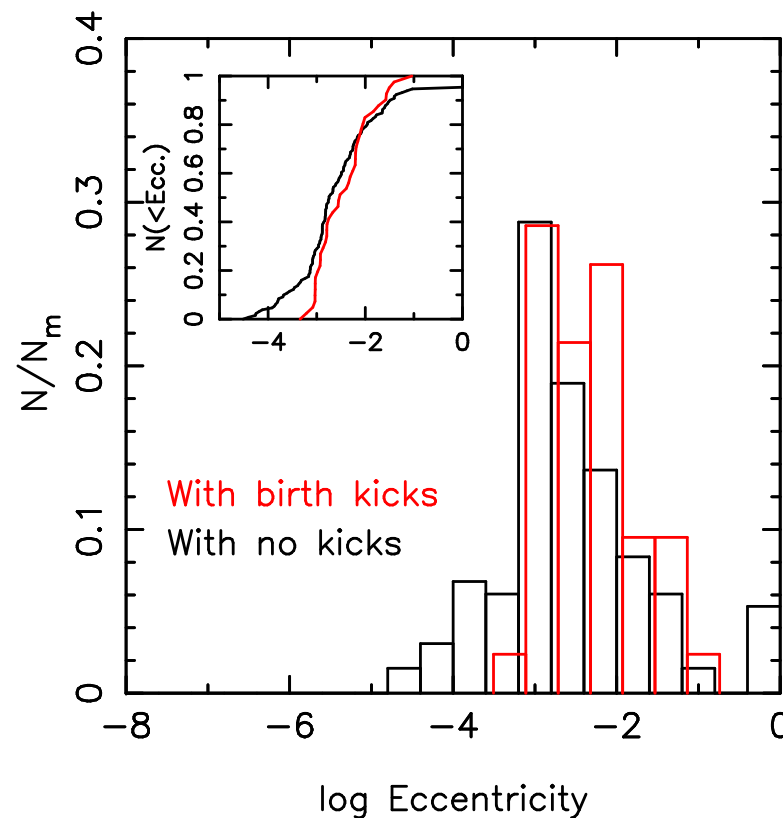
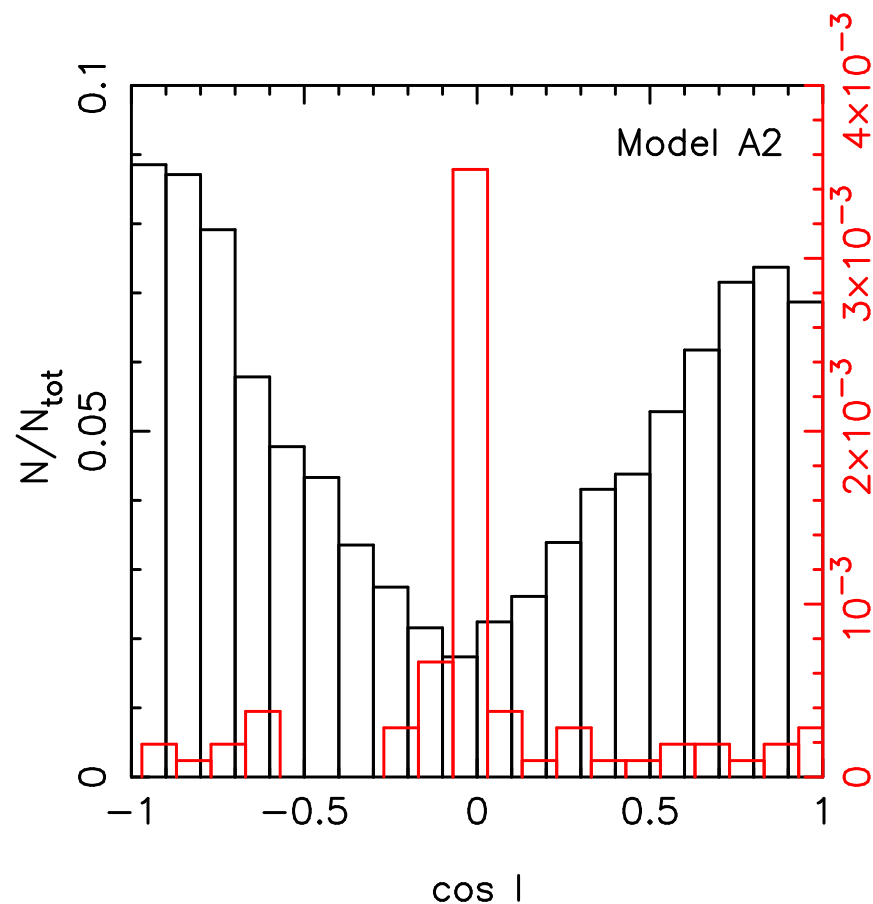
**= Moment of inertia**

**G orbital angular momentum**

$\theta \equiv \cos(i)$

# BH-BH mergers

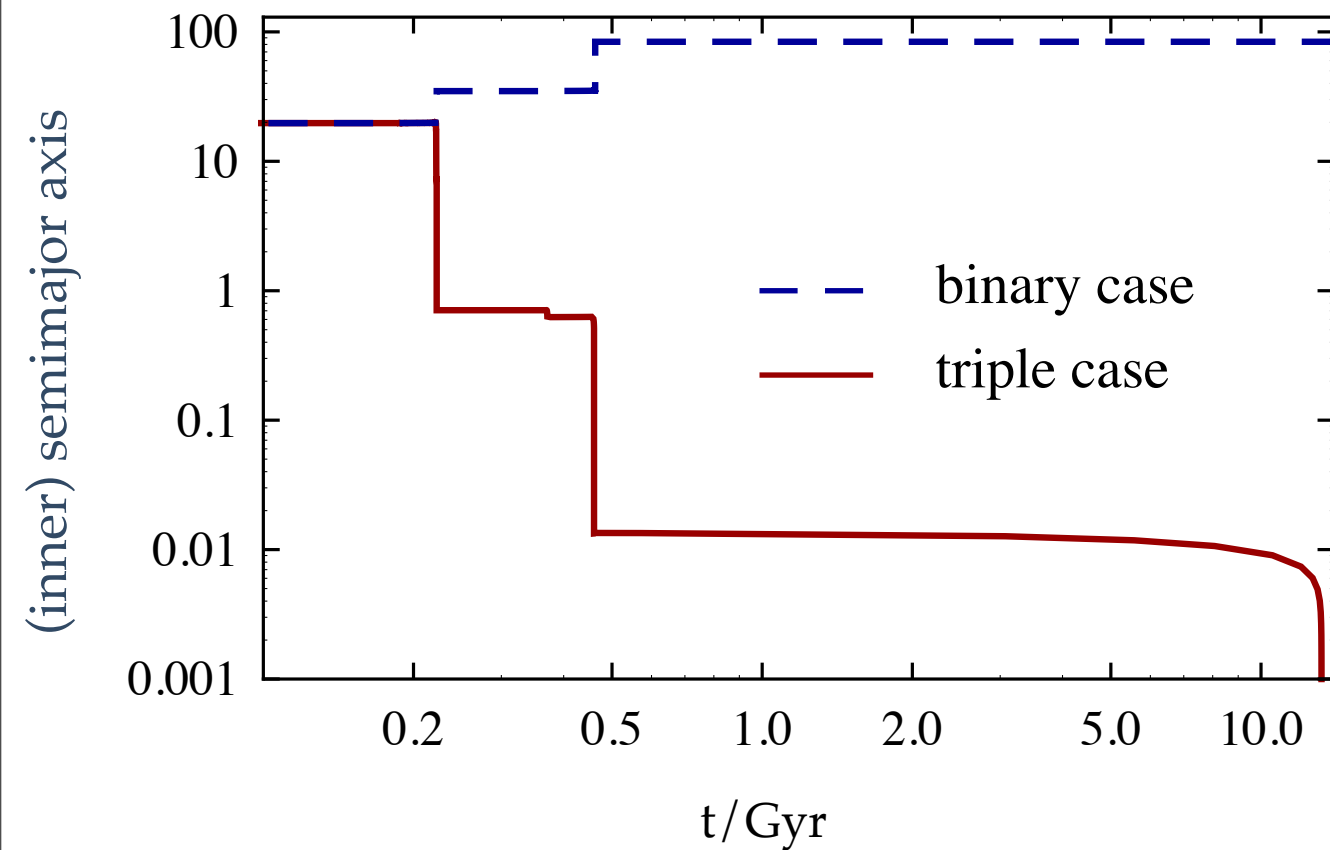
- ➔ Enhanced occurrence rate of mass transfer
- ➔ Enhanced formation rate of compact binaries
- ➔ Enhanced merger rate of compact objects
  - ❖ Antonini, Toonen & Hamers in prep.
  - ❖ Preliminary rate: 0.3-1.2 per year per  $\text{Gpc}^3$



# Interacting regime

Example: (inner) binary:  $M_1 = 3.95$ ,  $M_2 = 3.03M_{\odot}$ ,  $a_{in} = 19.7$  AU,  $e_{in} = 0.23$

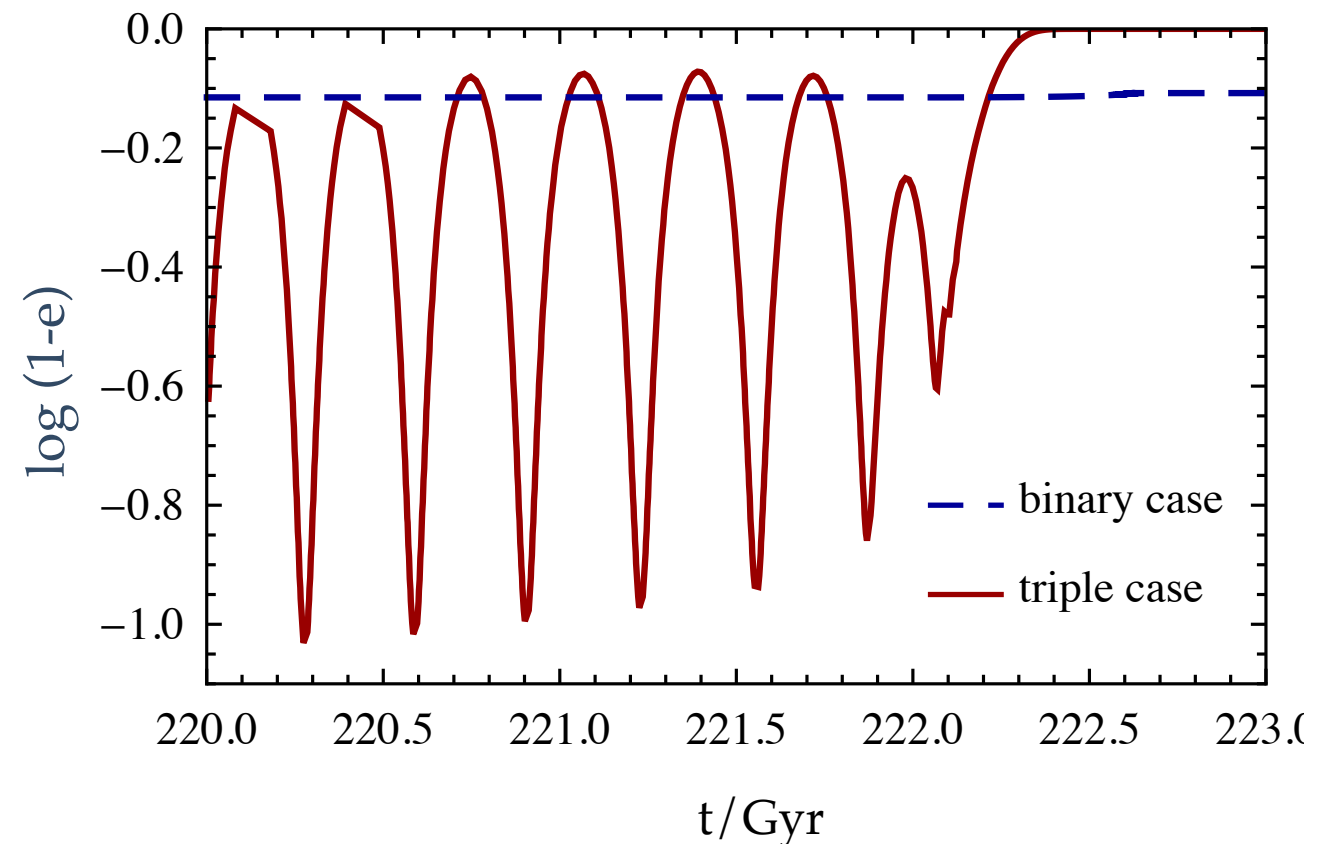
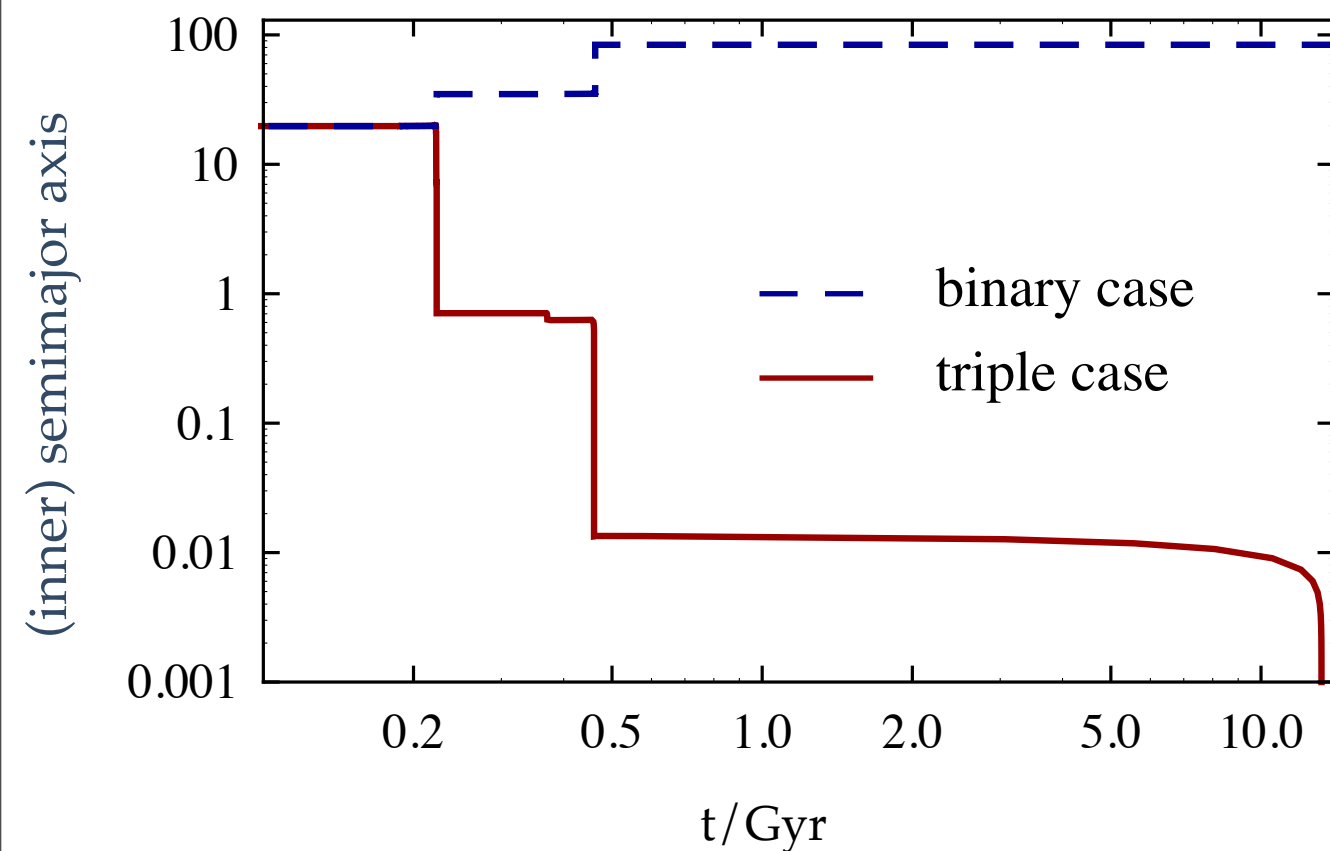
tertiary star  $M_3 = 2.73M_{\odot}$  on an orbit with  $a_{out} = 636$  AU,  $e_{out} = 0.82$ ,  $i = 116^{\circ}$



# Interacting regime

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tertiary star  $M_3 = 2.73M_{\odot}$  on an orbit with  $a_{out} = 636$  AU,  $e_{out} = 0.82$ ,  $i = 116^{\circ}$



- \* Kozai-Lidov cycles relevant for 90% of triples
- \* 1.5x more often Roche lobe overflow compared to binaries
- \* 40% of Roche lobe overflow in an eccentric orbit

ref: Toonen, Hamers, Portegies Zwart in prep.

# Triples

- ❖ Fairly common

	Triple fraction	Binary fraction
Low-mass stars	10-15%	40-50%
High-mass stars	~50	>70%
Refs	Tokovinin '08, '14, Ramage Evans '11	Raghavan ea '08, Duchene & Kraus '13)

- ❖ Evolution: Three body dynamics + stellar evolution
  - ❖ Rich interacting regime (Shappee ea 2013, Hamers ea 2013, Michaely ea 2014)
- ❖ **TRES**: self-consistent treatment of triple evolution (Toonen et al. in prep.)
  - ❖ judge the importance of this interacting regime
  - ❖ curious evolutionary products from triples

# Initial population

- ❖ Monte Carlo method to generate initial systems containing zero-age main-sequence stars
- ❖ 3 sets of distributions

	uncorrelated binaries 1	uncorrelated binaries 2	Eggleton
m1			
m2			
m3			
a			
e	thermal	thermal	thermal
i	circular uniform [0,pi]	circular uniform [0,pi]	circular uniform [0,pi]
g	uniform [-pi,pi]	uniform [-pi,pi]	uniform [-pi,pi]
$\Omega$	fit of Hurley ea '00 to Lang '92	fit of Hurley ea '00 to Lang '92	fit of Hurley ea '00 to Lang '92

# Initial population

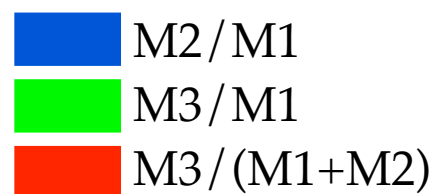
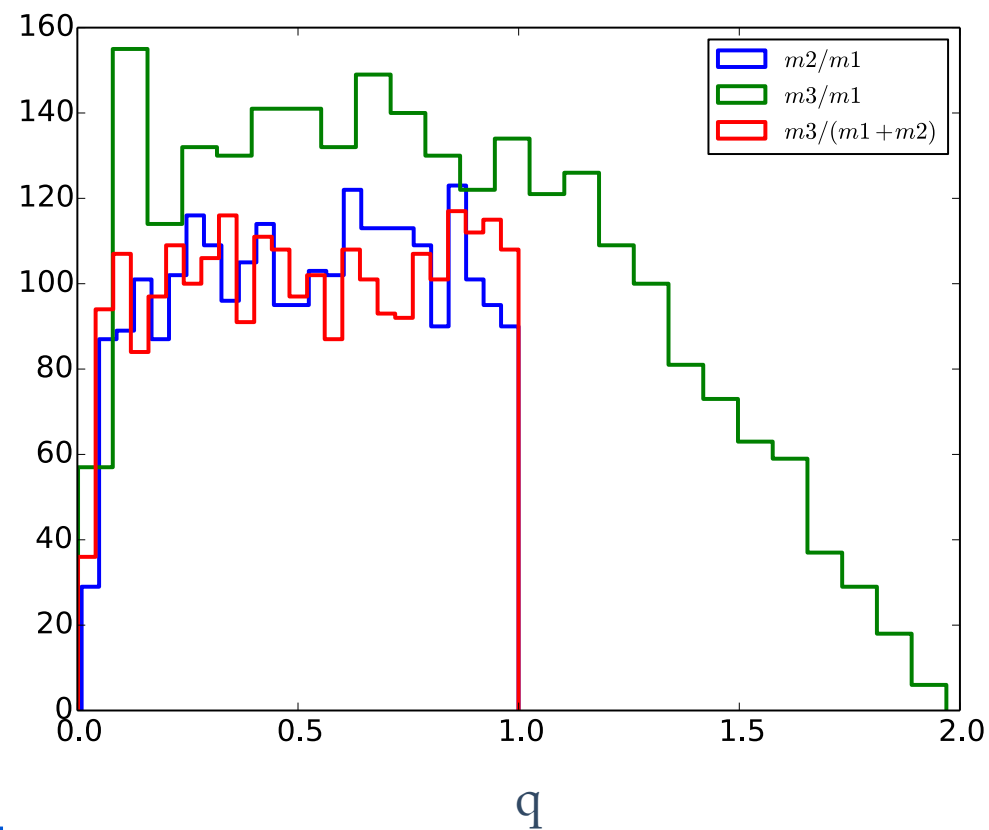
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$m_3$	flat in $m_3/(m_1+m_2)$	flat in $m_3/(m_1+m_2)$	Eggleton '09
$a$	flat in $\log a$ (Abt '83)	Tokovinin '14 (lognormal, $\mu = 1e5d$ , $\sigma=2.3$ )	Eggleton '09
$e$	thermal	thermal	thermal
$i$	circular uniform $[0,\pi]$	circular uniform $[0,\pi]$	circular uniform $[0,\pi]$
$g$	uniform $[-\pi,\pi]$	uniform $[-\pi,\pi]$	uniform $[-\pi,\pi]$
$\Omega$	fit of Hurley ea '00 to Lang '92	fit of Hurley ea '00 to Lang '92	fit of Hurley ea '00 to Lang '92

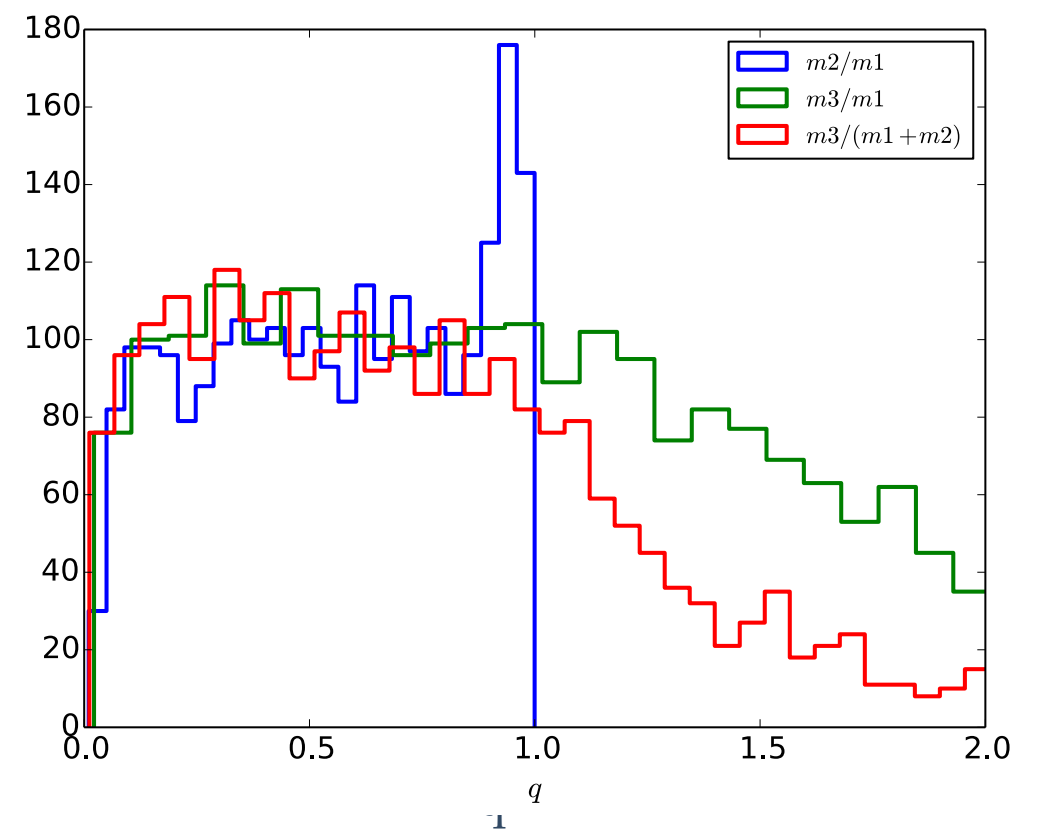
# Initial population

- ❖ Mass ratio distribution

- ❖ Uncorrelated binaries



- ❖ Eggleton '09



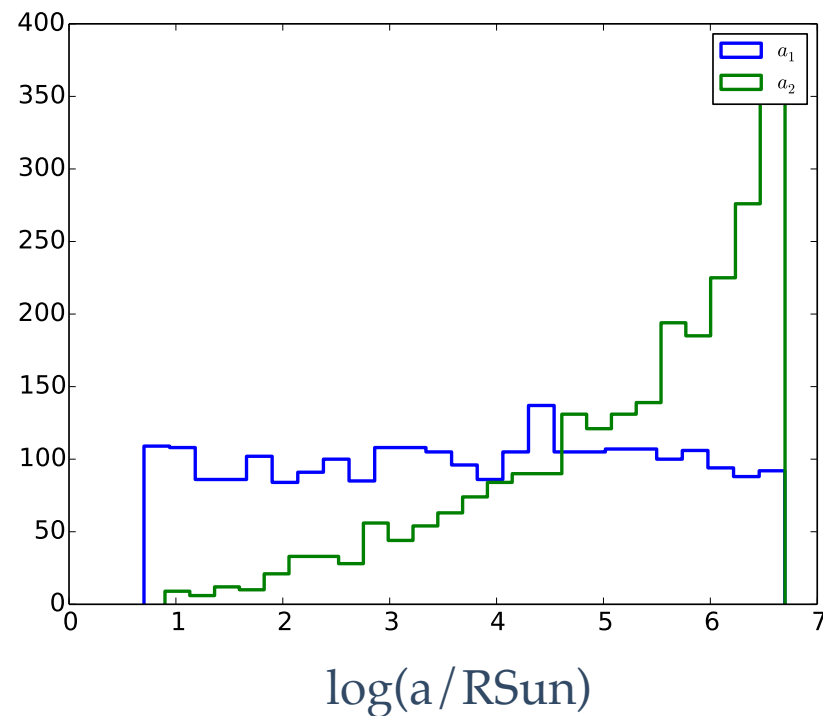
- ❖ Note: per definition  $m_1 > m_2$
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# Initial population

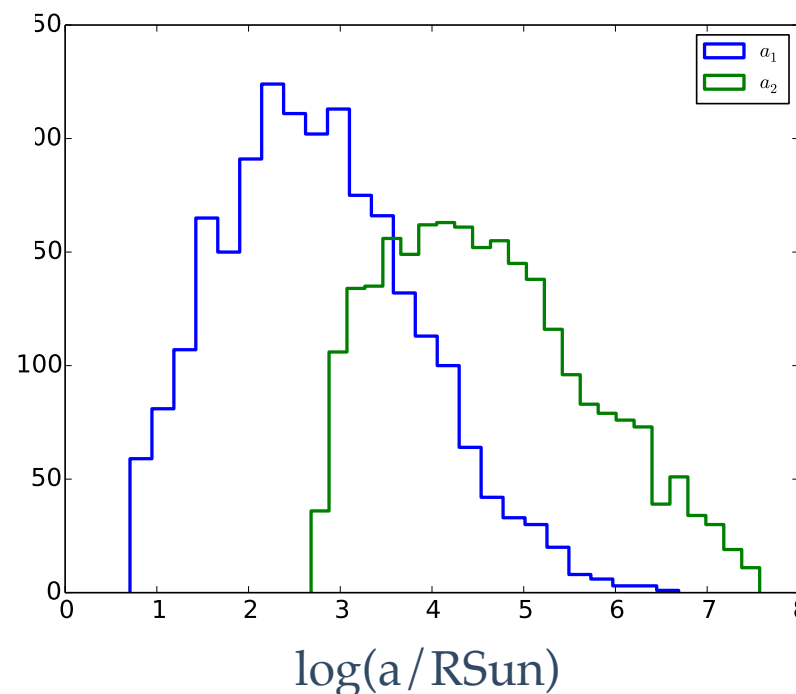
- ✦ Distribution of orbital separation

- ✦ Uncorrelated binaries 1:  
Abt '83 - flat in log a

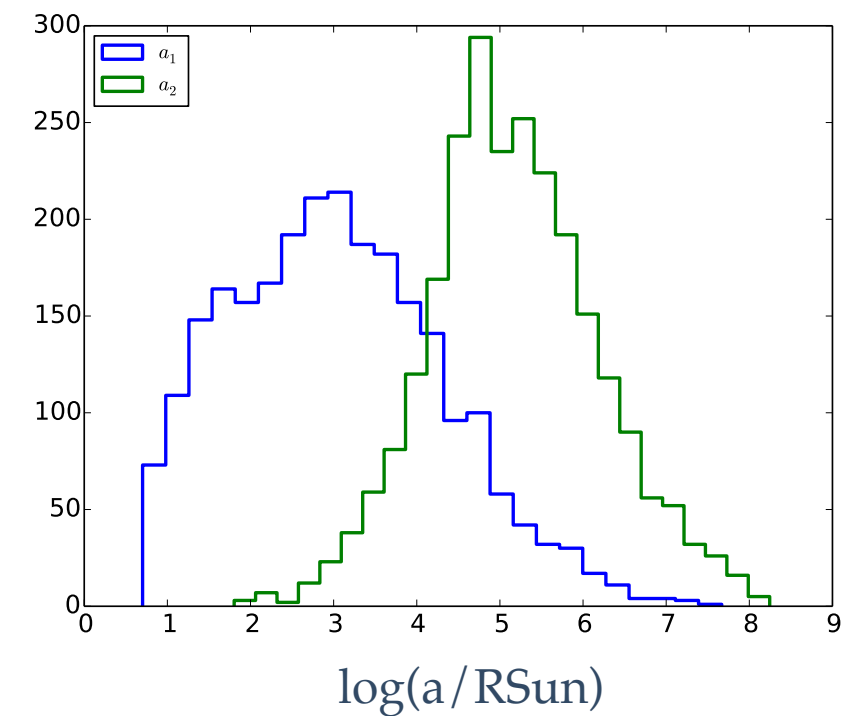


 a1  
 a2

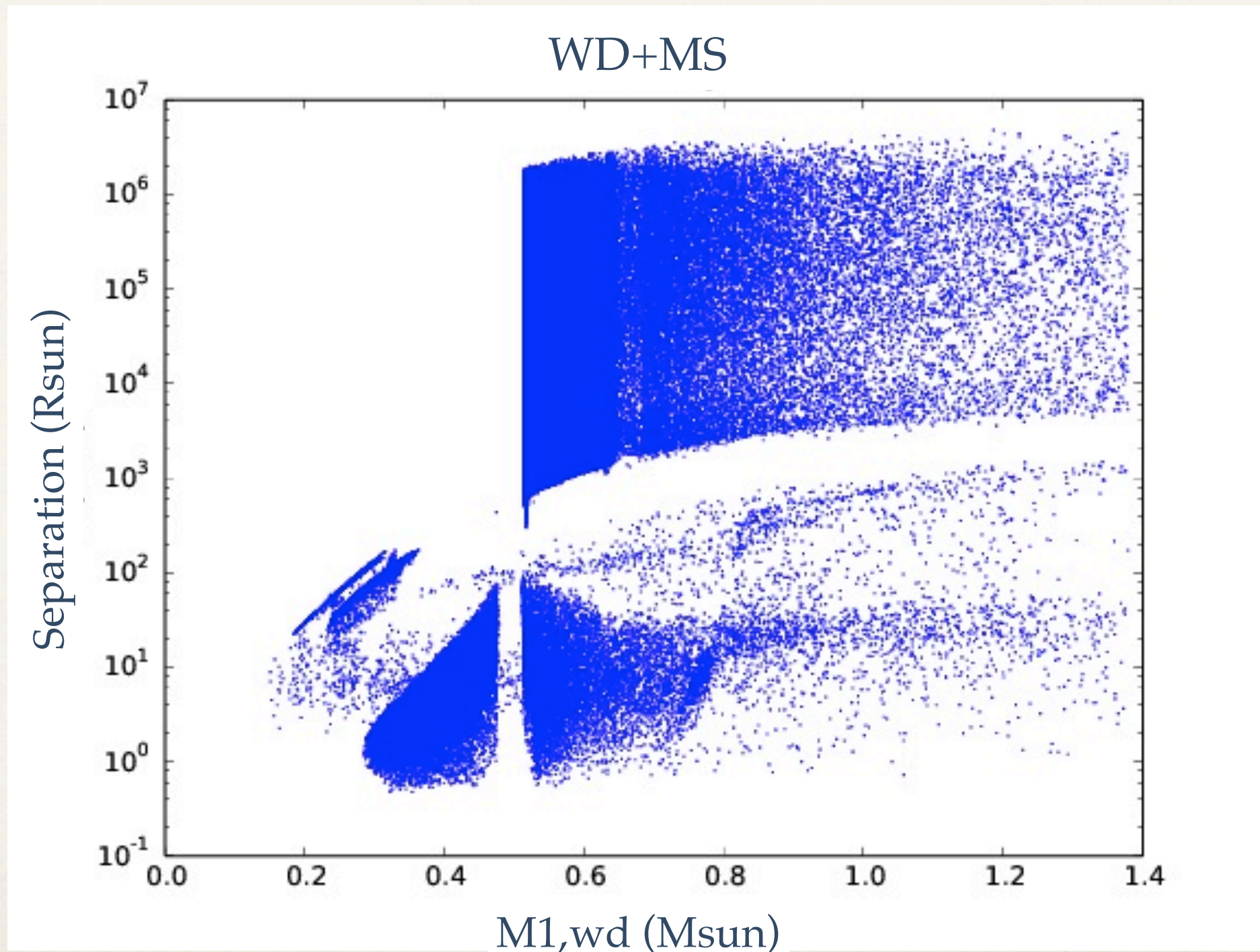
- ✦ Uncorrelated binaries 2:  
Tokovinin '14 - log normal periods



- ✦ Eggleton '09

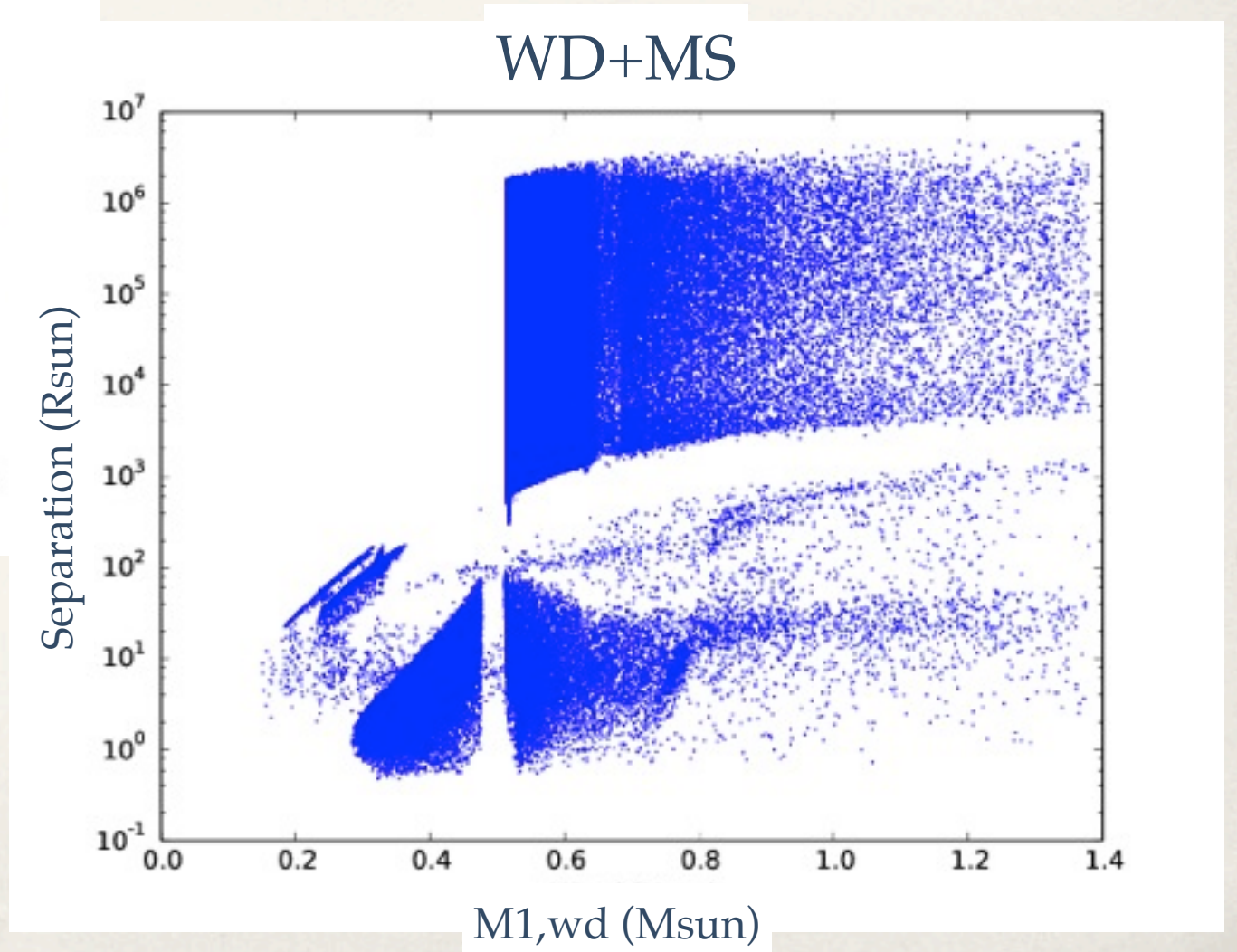
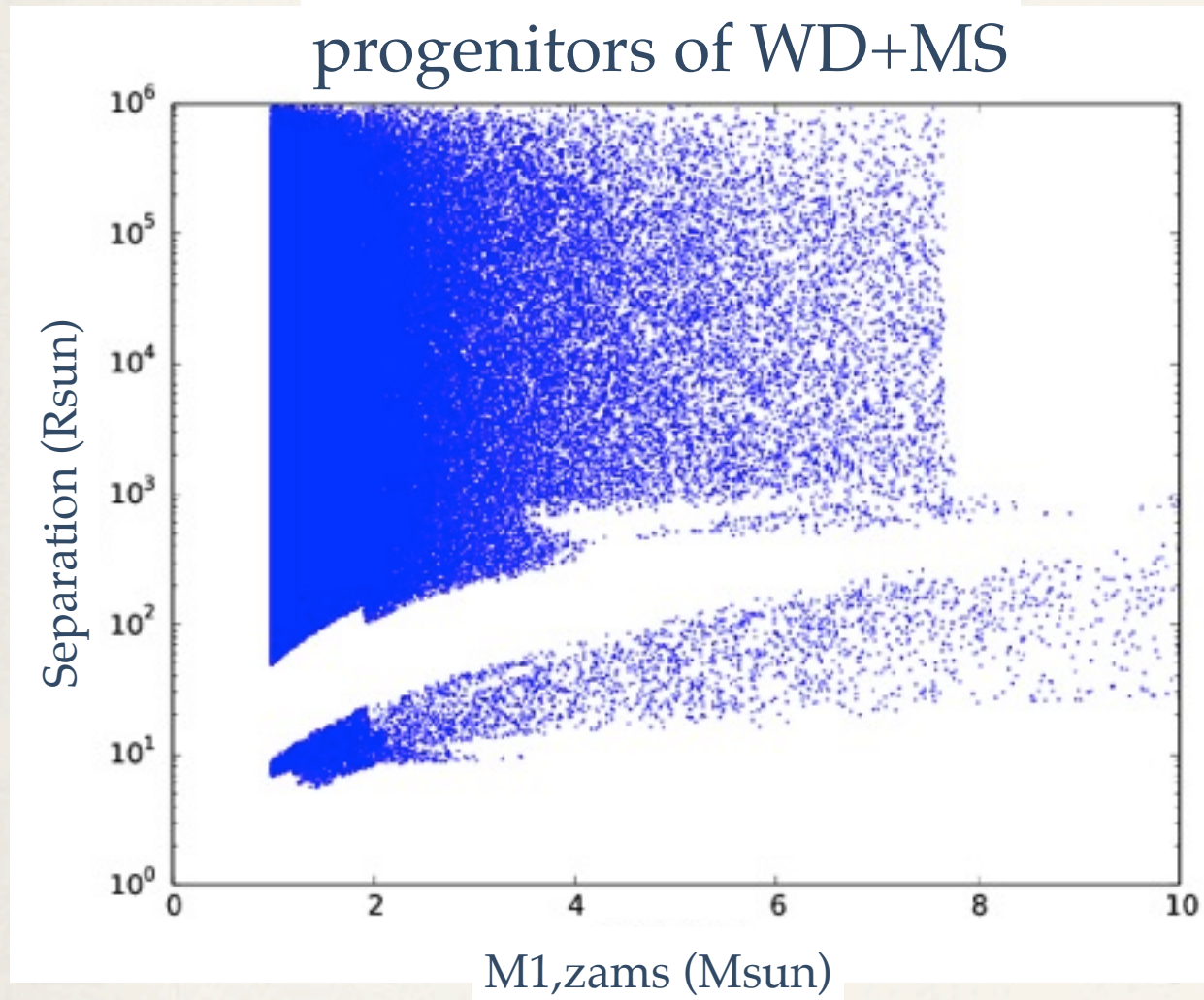


# Binary population synthesis



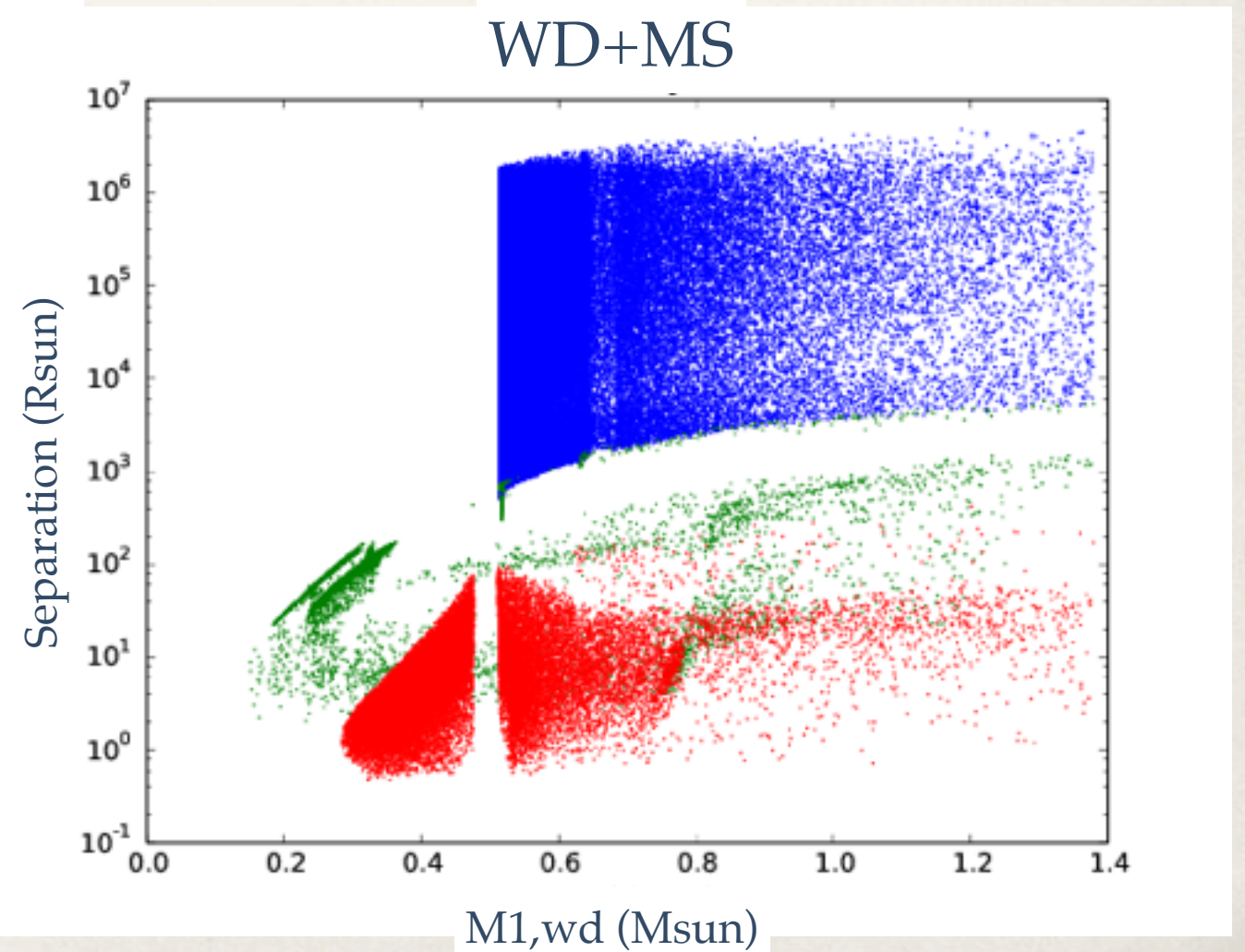
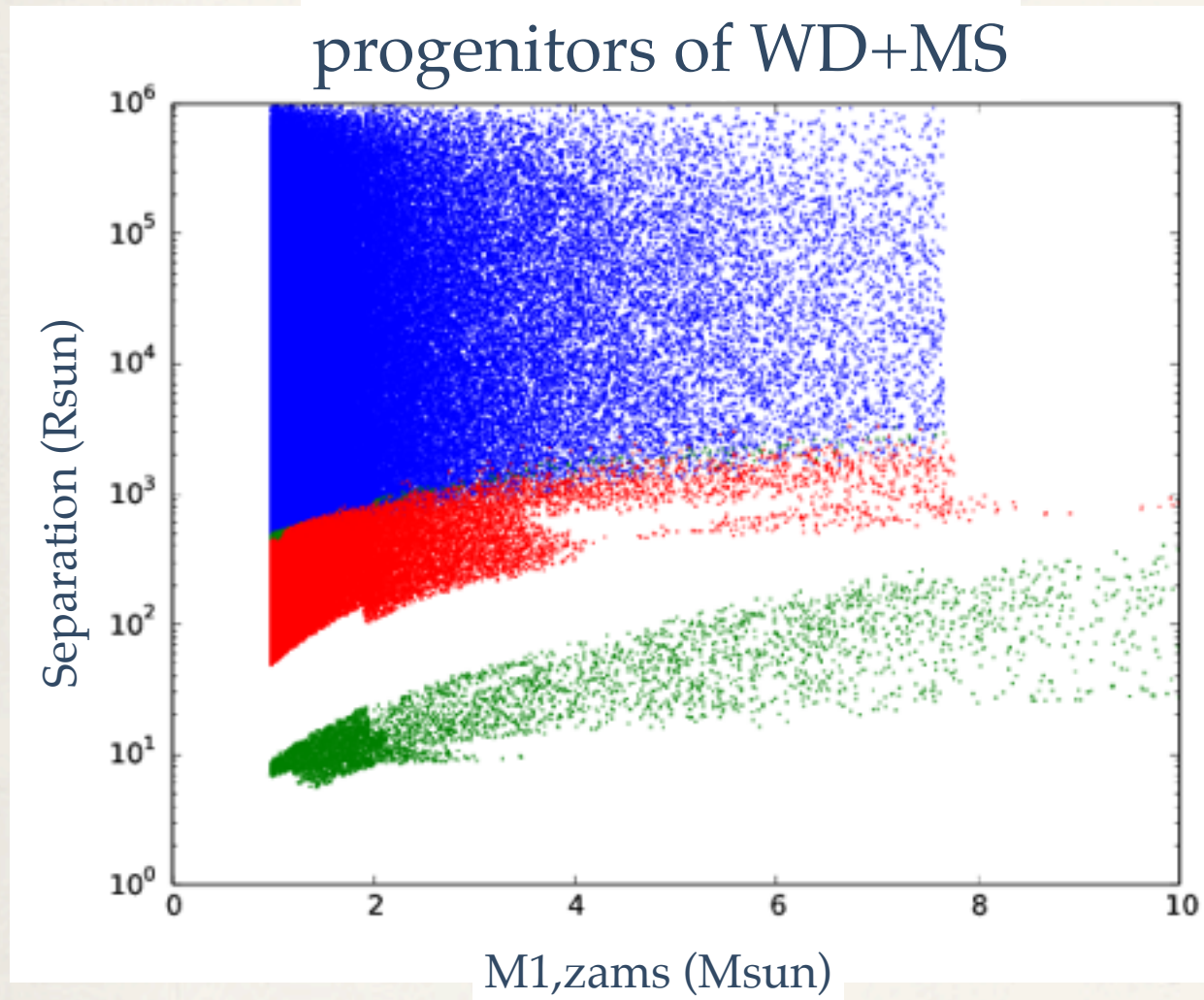


# Binary population synthesis





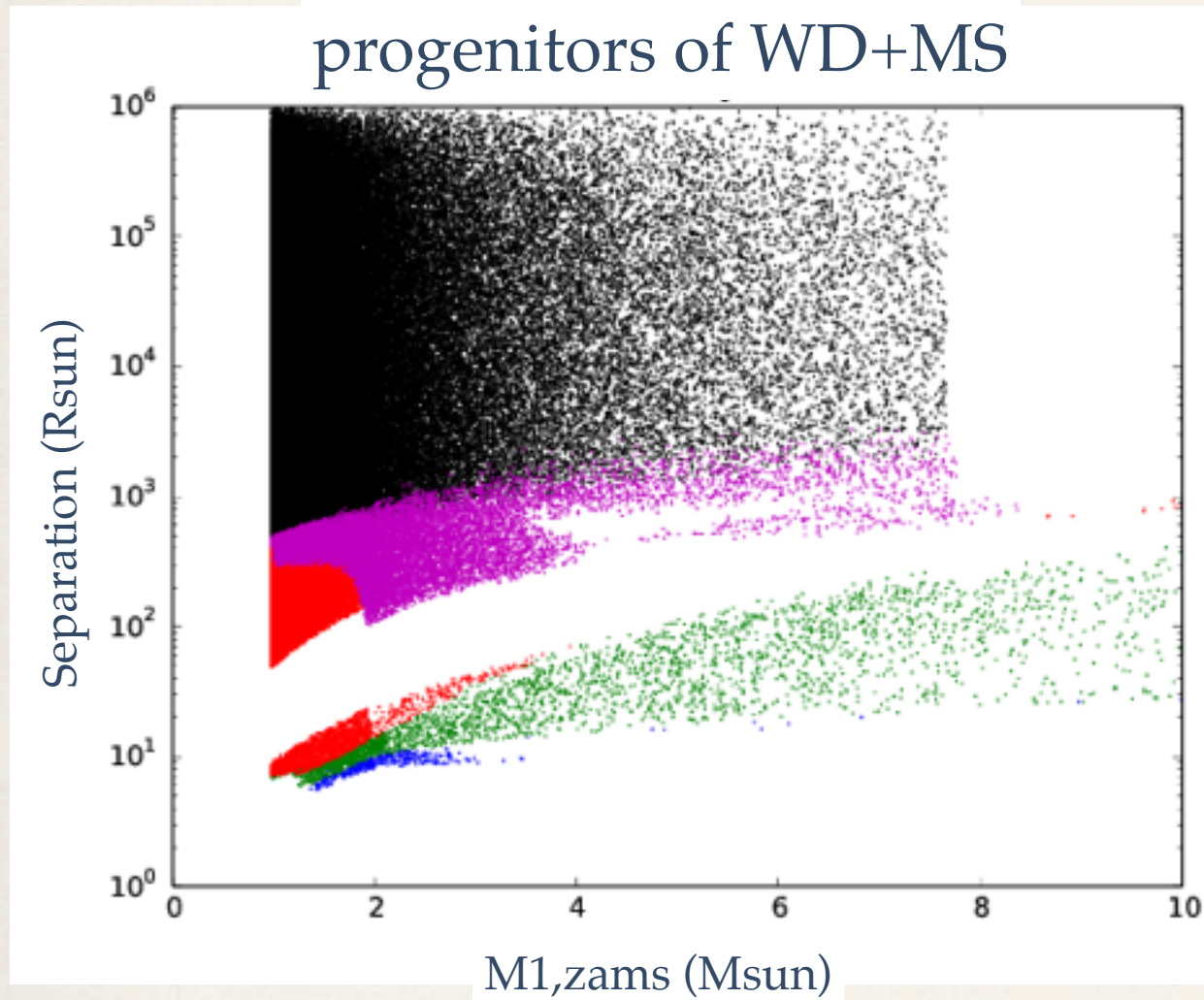
# Binary population synthesis



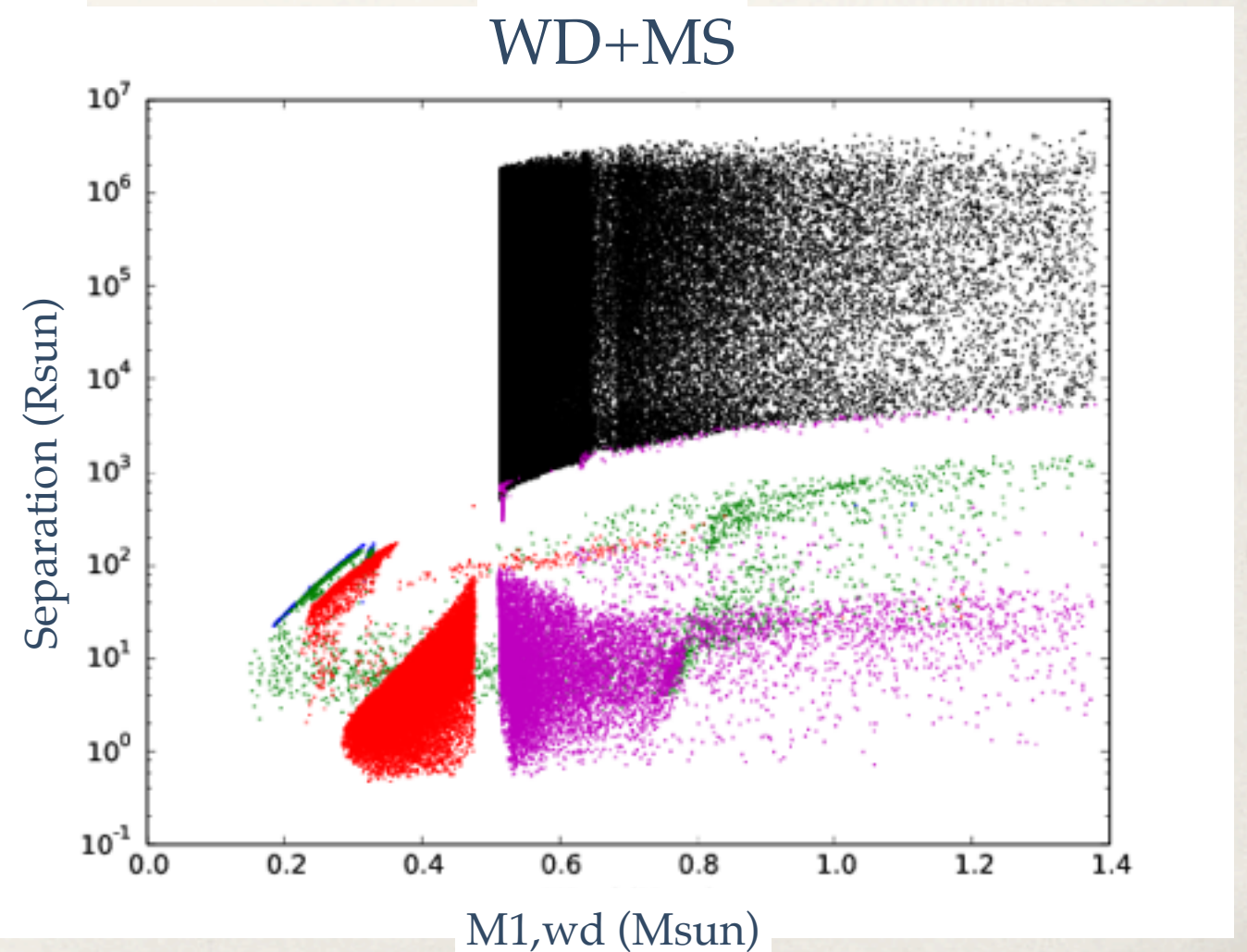
- No interaction
- Stable mass transfer
- Common-Envelope



# Binary population synthesis



- No contact
- MS donor
- HG donor
- GB donor
- AGB donor



# PopCORN



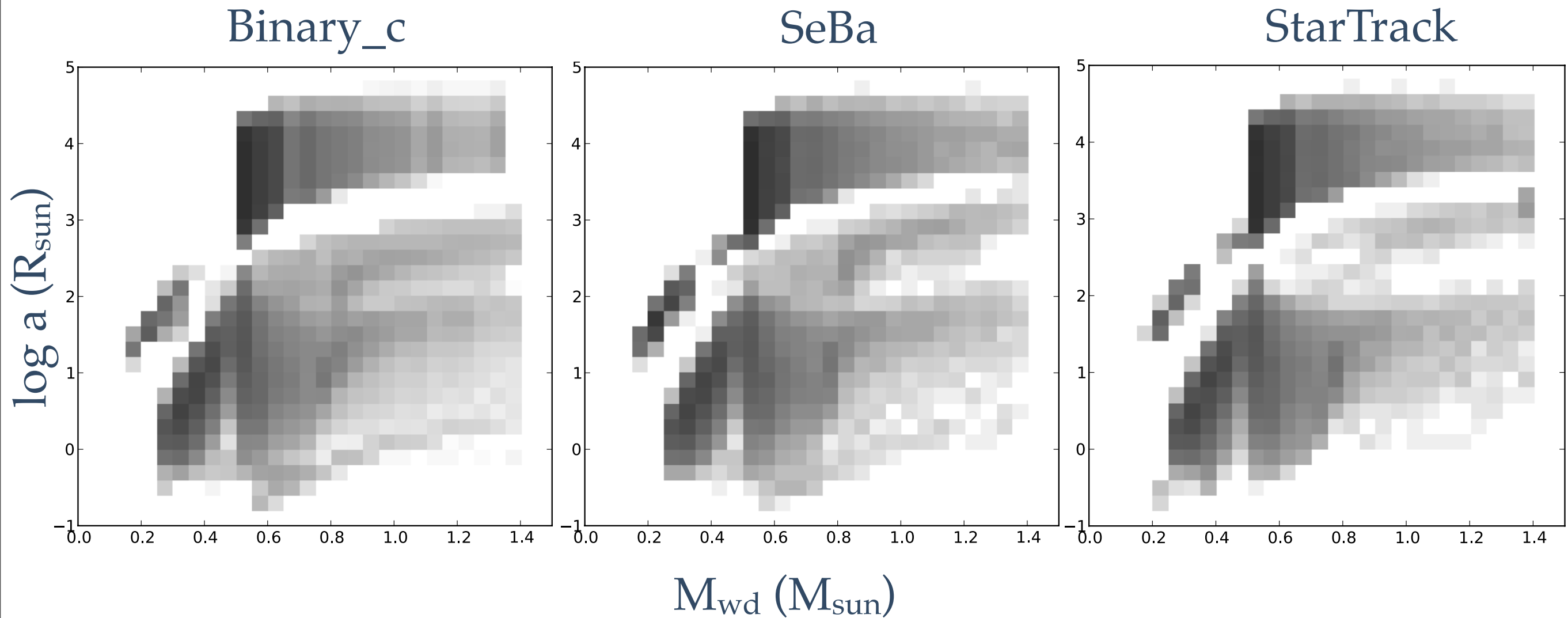
- Population synthesis used extensively for **binaries** (e.g. Eggleton '89, de Kool ea '92, Willems & Kolb '94, Nelemans ea '01, Han ea '02, Belczynski ea '08, Ruiter ea '12, Mennekens ea '13, Claeys ea '14, Toonen ea '12,13,14)
  - ❖ Comparison of codes for binary population synthesis
- 
- ❖ When input assumptions are equalized: different binary population synthesis codes give **similar** populations.
  - ❖ Differences are **not** caused by **numerical** differences, **but** can be explained by differences in the **input physics**

ref: Toonen, Claeys, Mennekens, Ruiter 2014

see also: [www.astro.ru.nl/~silviato/popcorn](http://www.astro.ru.nl/~silviato/popcorn)

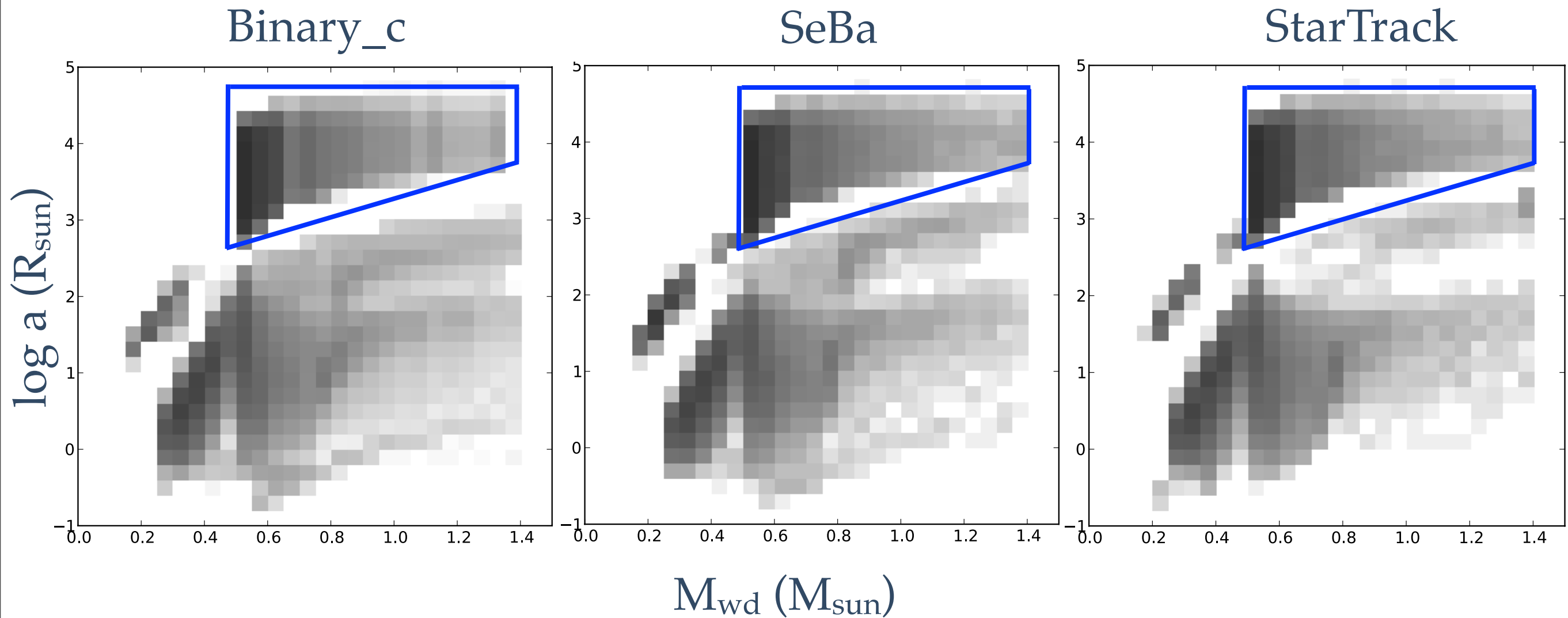


# WD + non-degenerate companion

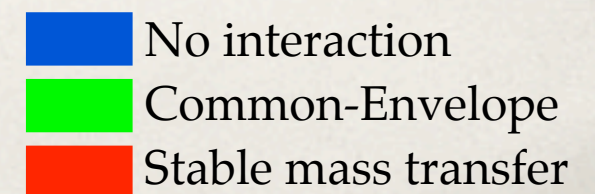


Similar simulated populations

# WD + non-degenerate companion

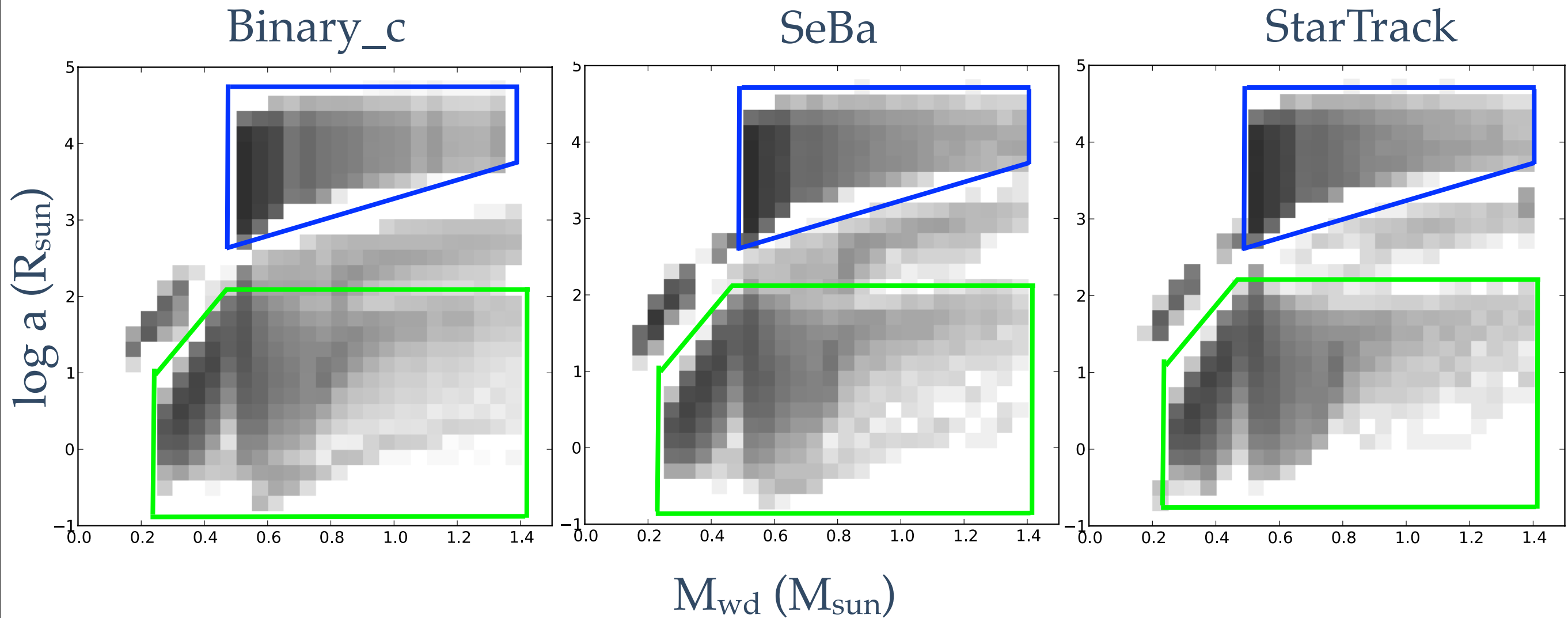


Similar simulated populations





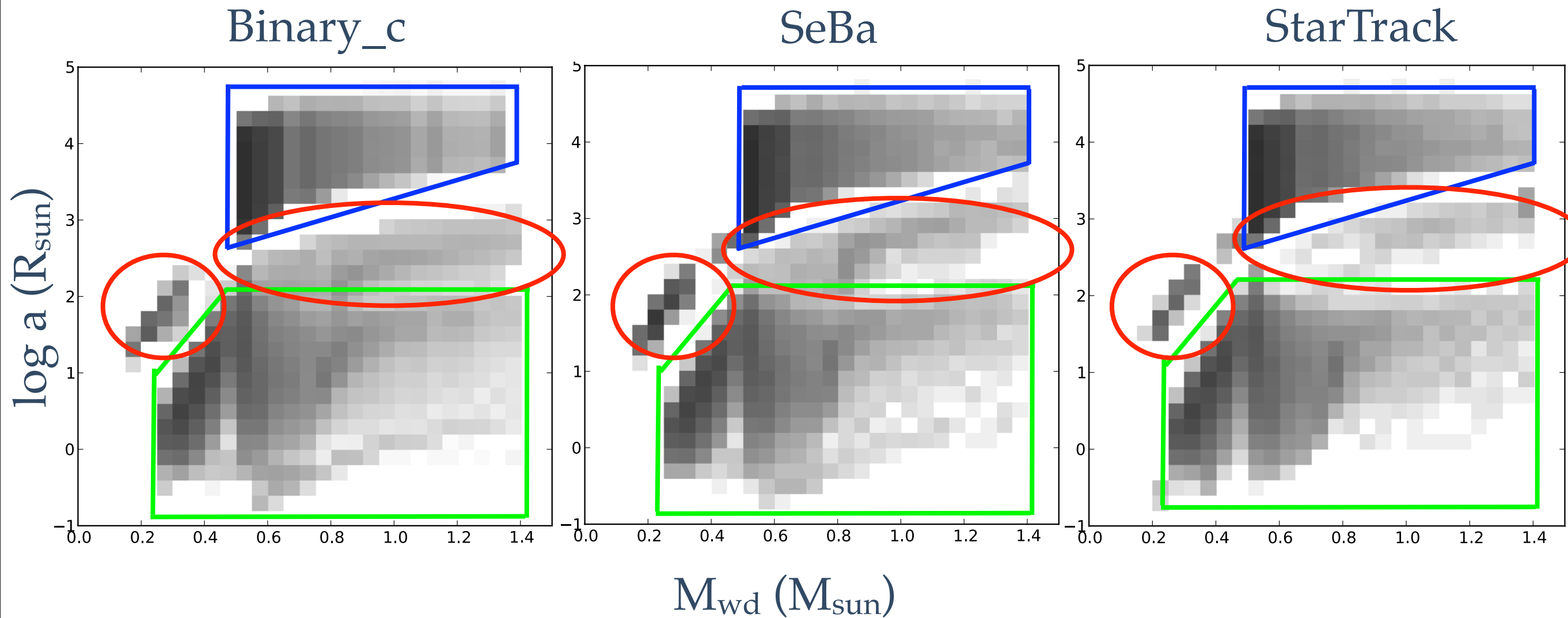
# WD + non-degenerate companion



Similar simulated populations

- No interaction
- Common-Envelope
- Stable mass transfer

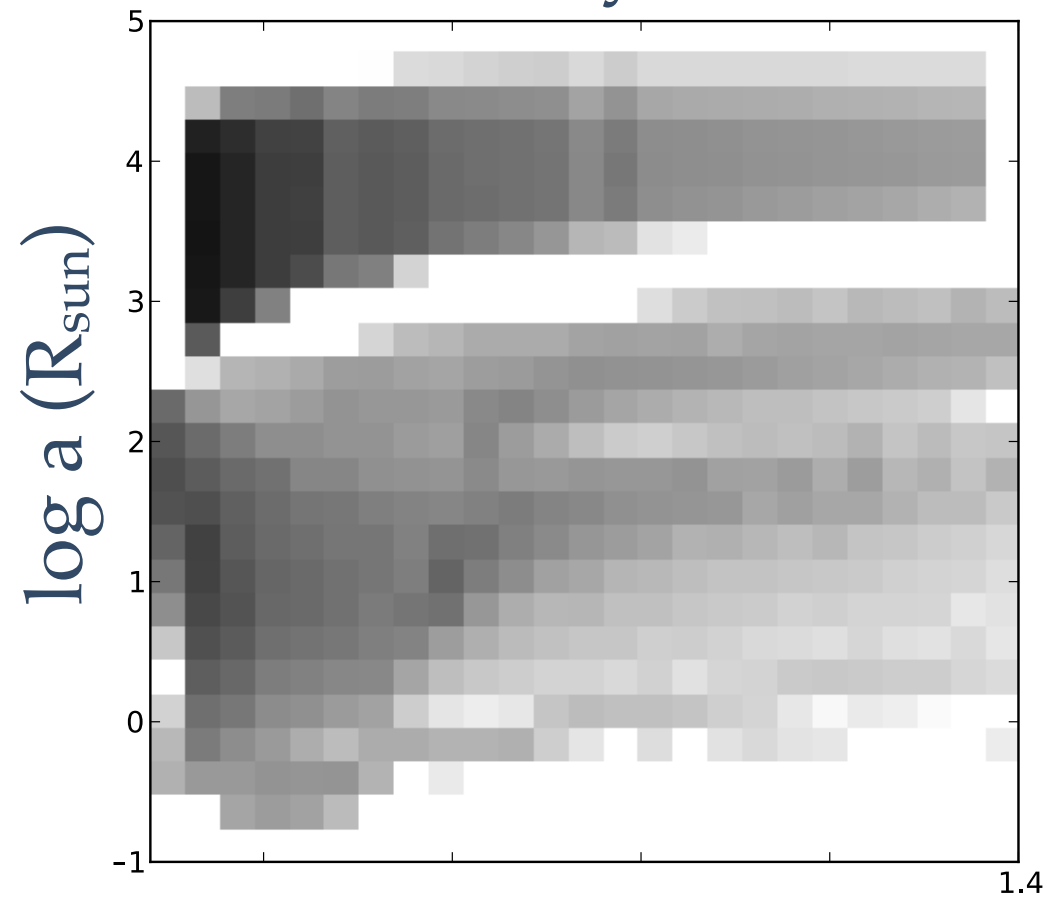
# WD + non-degenerate companion



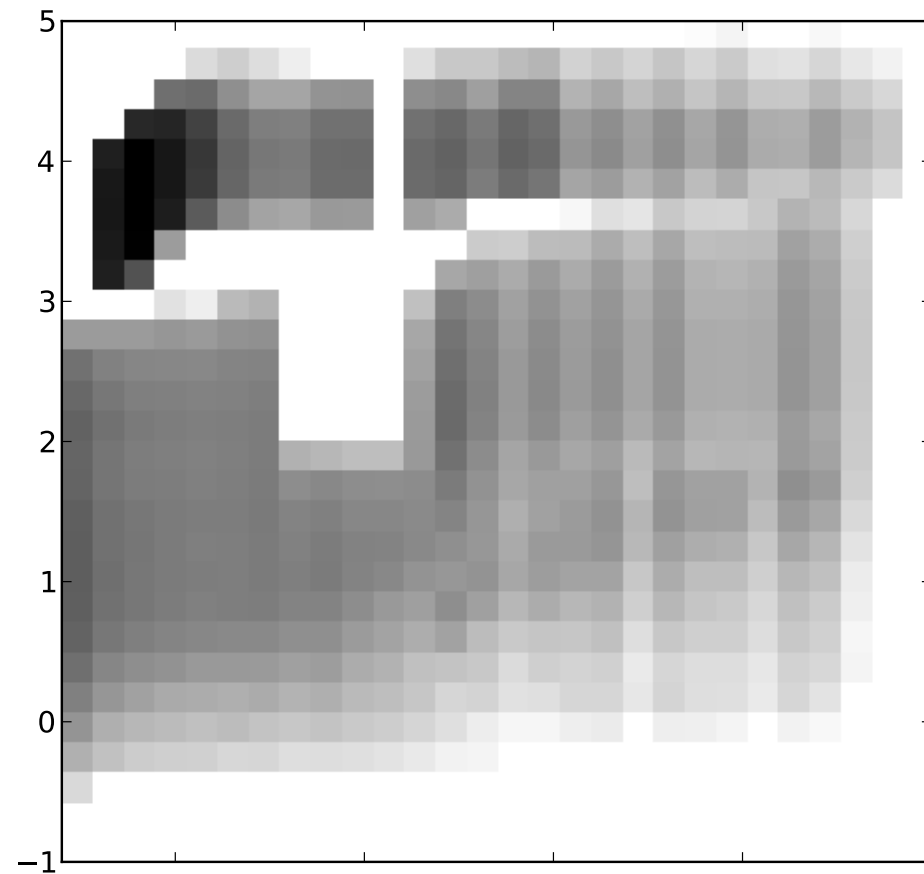
Similar simulated populations

- No interaction
- Common-Envelope
- Stable mass transfer

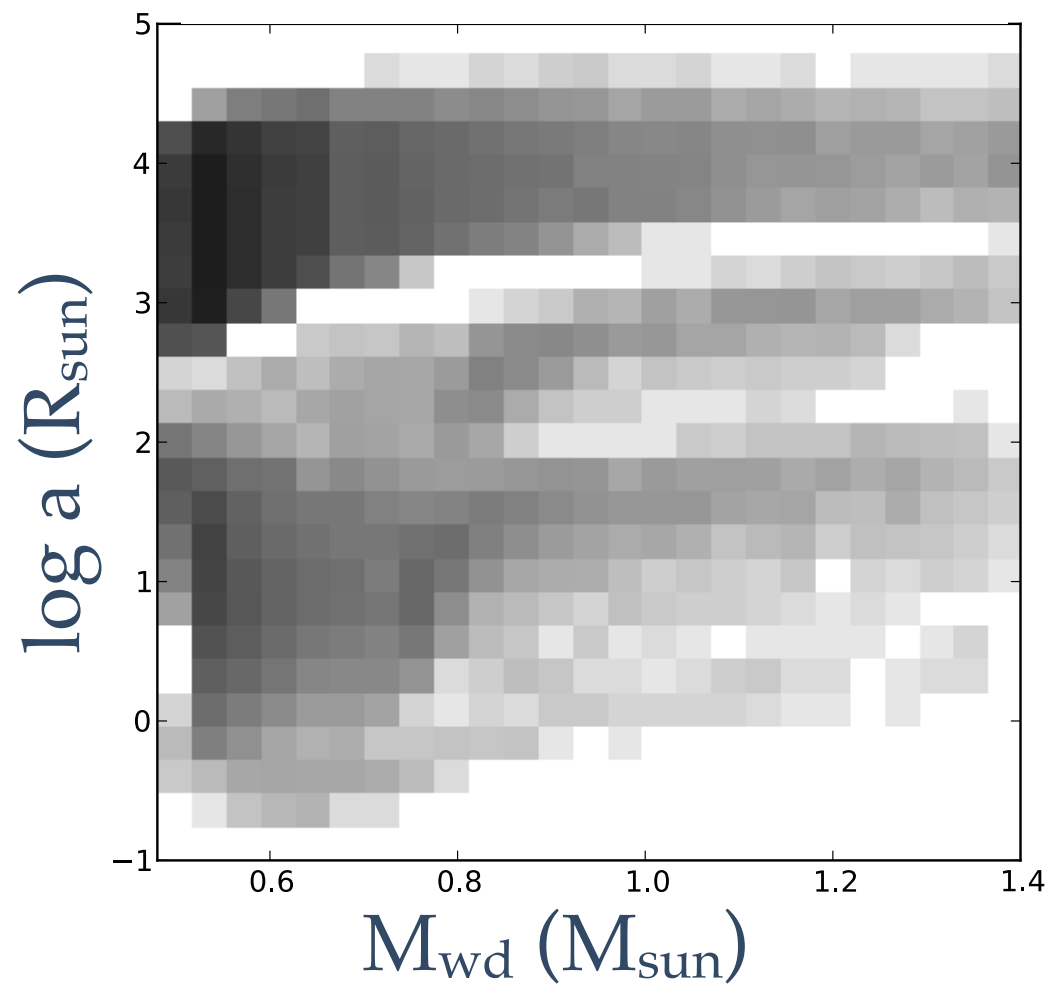
Binary\_c



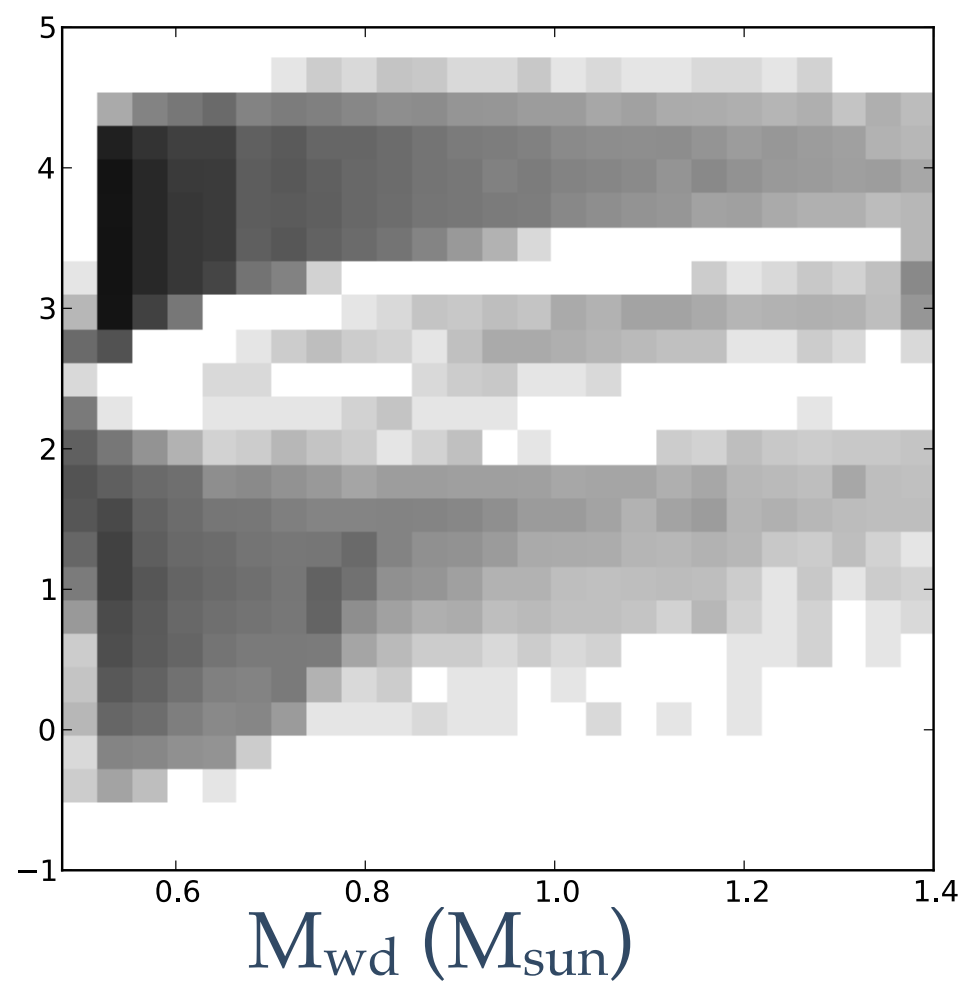
Brussels code



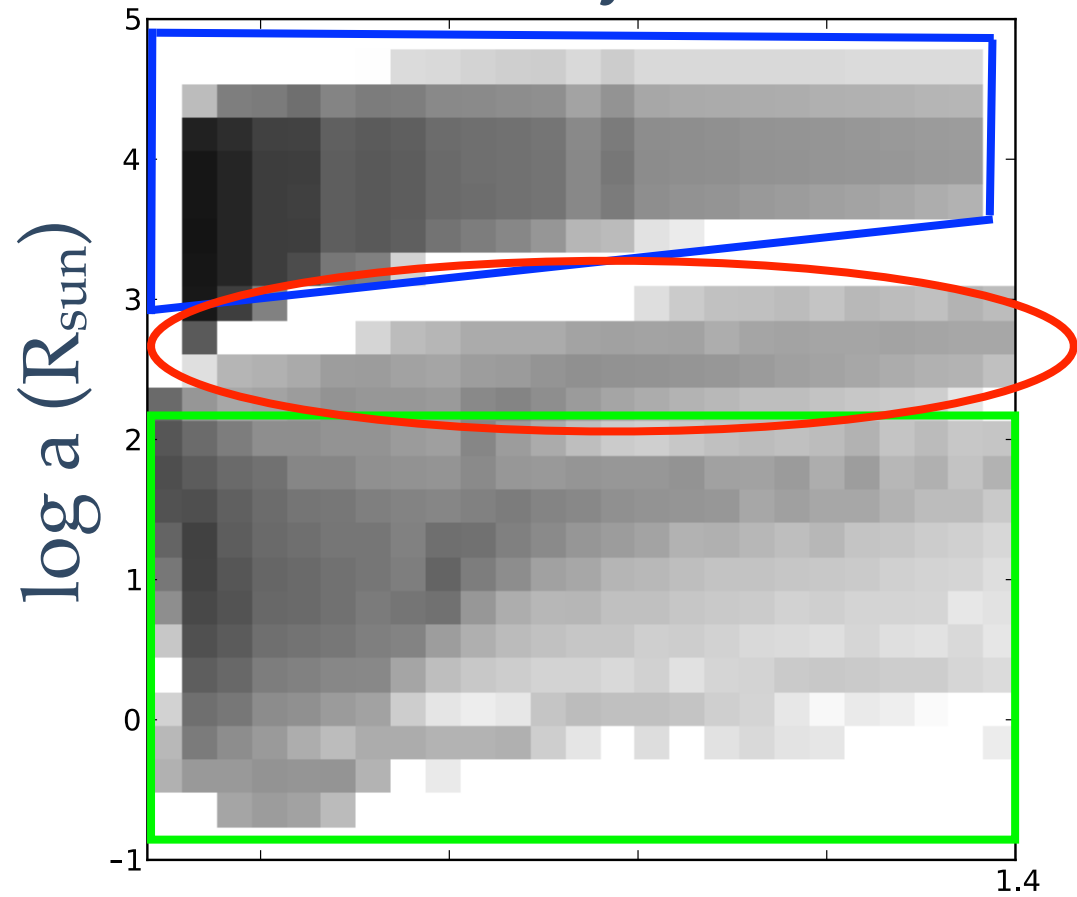
SeBa



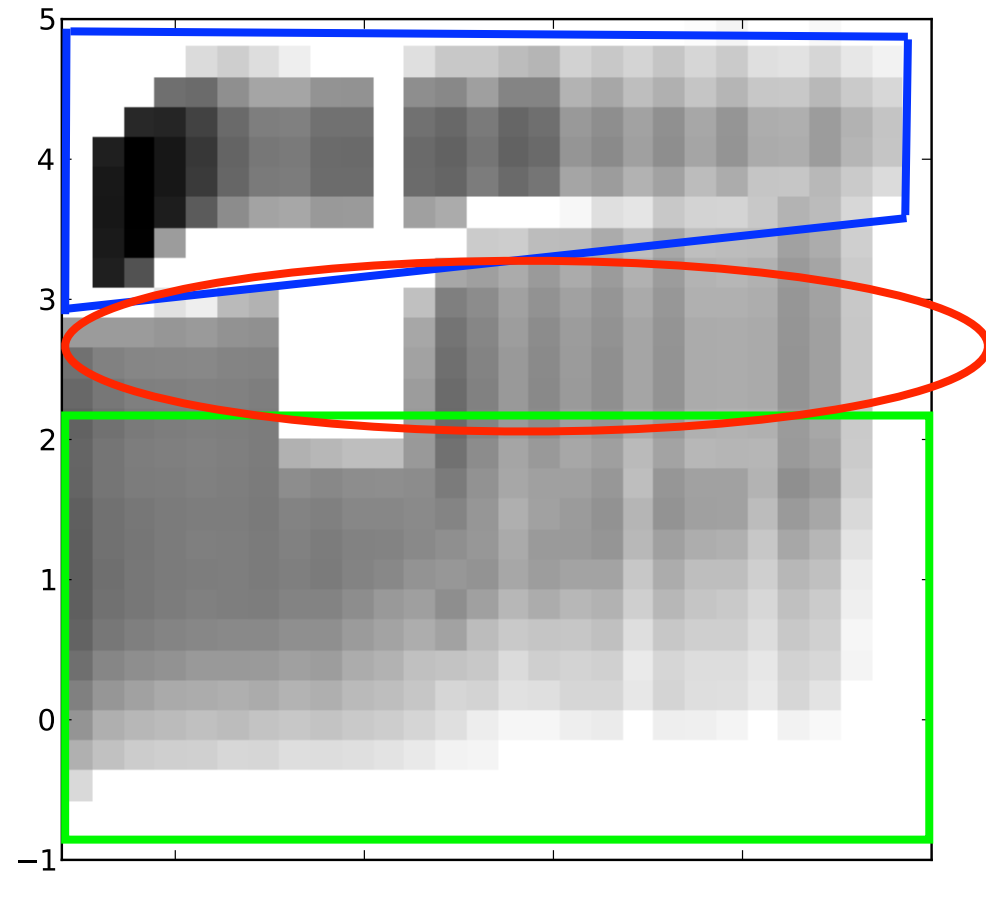
StarTrack

(for  $M_{1,i} > 3 M_{\text{sun}}$ )

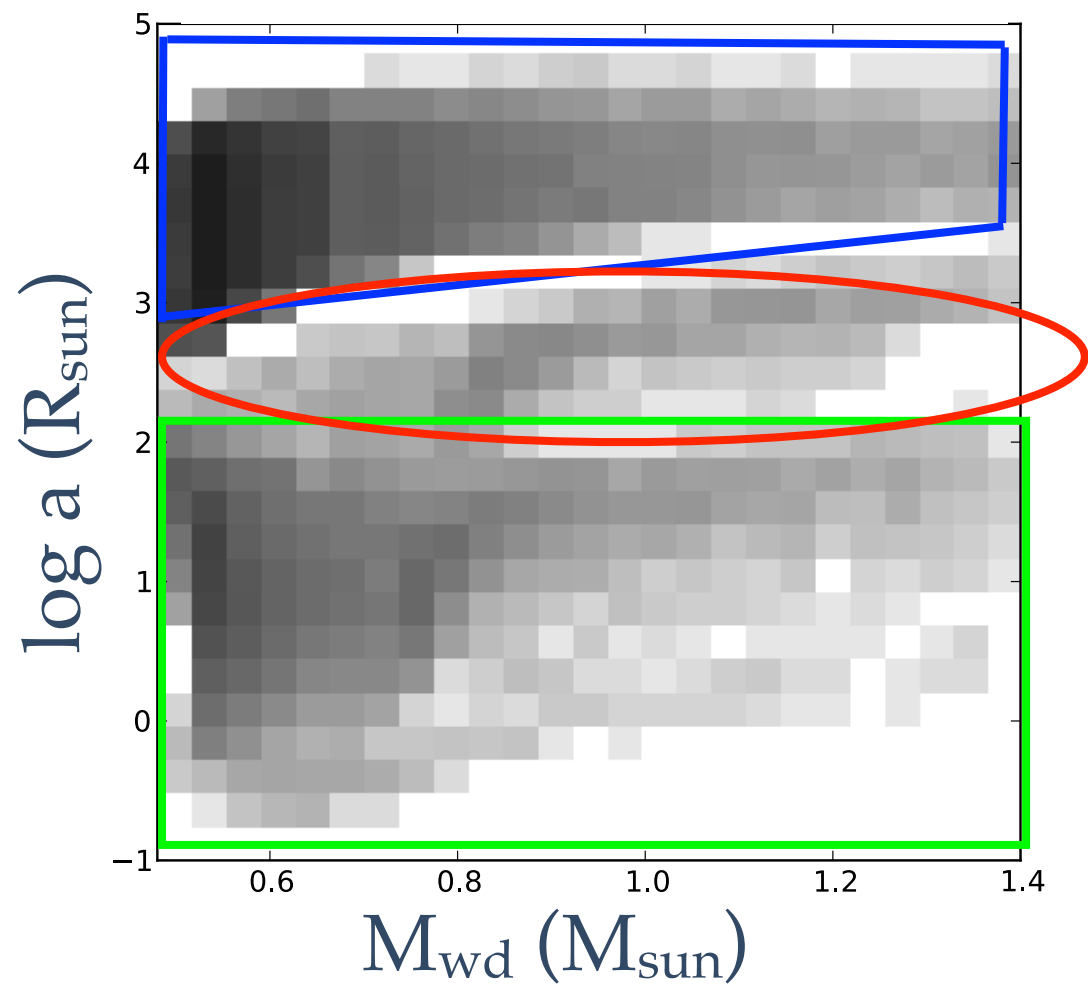
Binary\_c



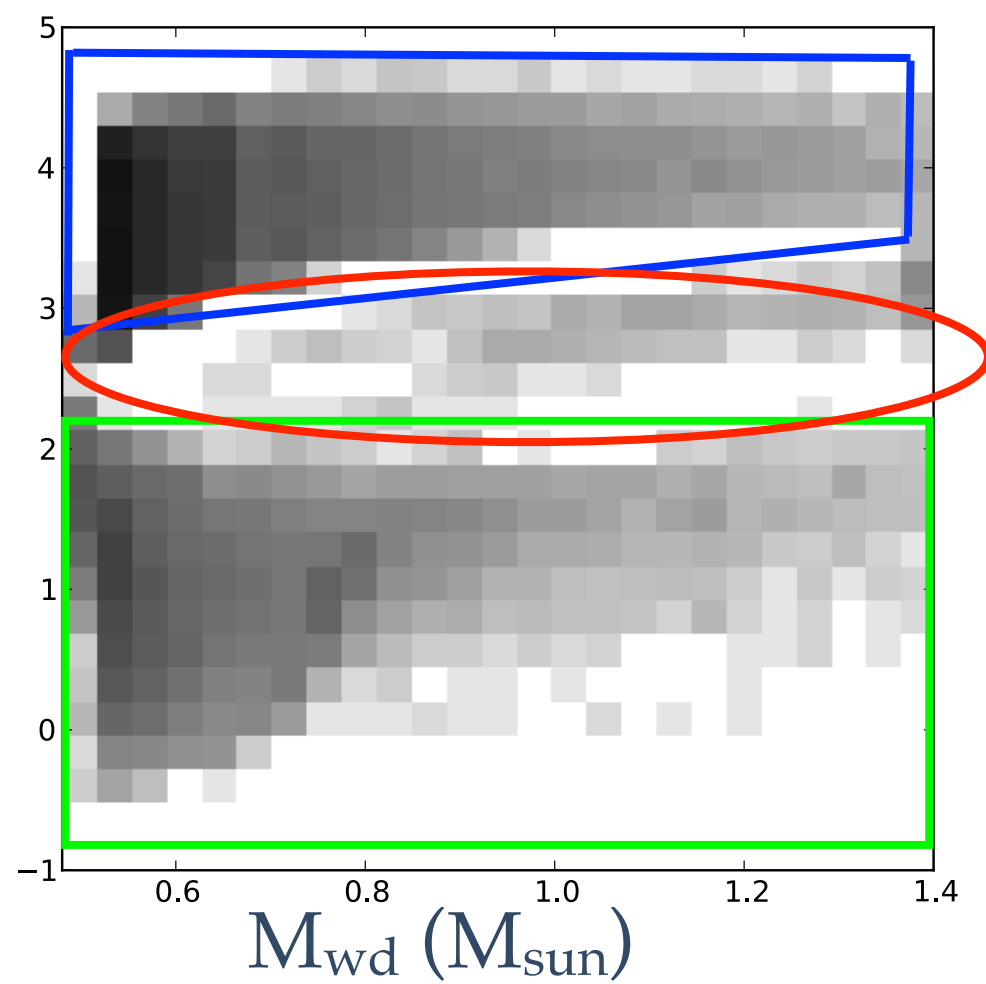
Brussels code



SeBa



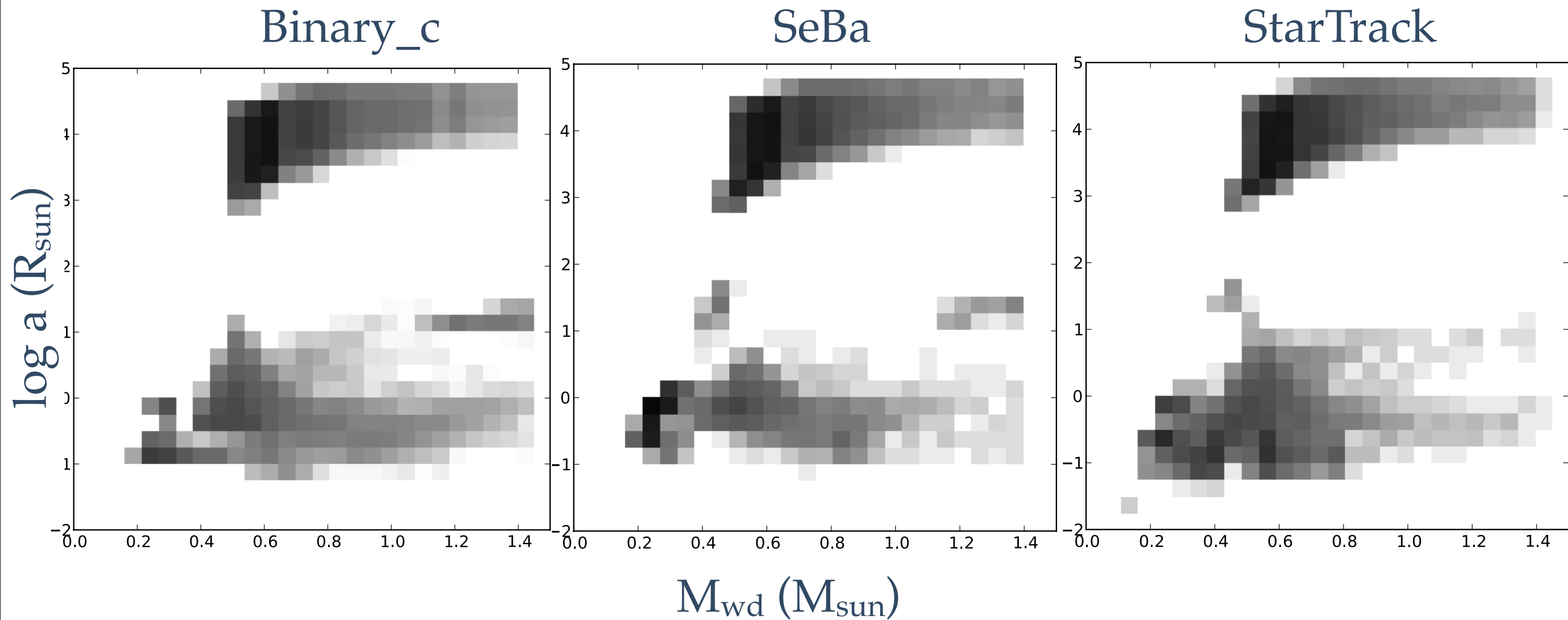
StarTrack

(for  $M_{1,i} > 3 M_{\text{sun}}$ )

- No contact
- Common-Envelope
- Stable mass transfer

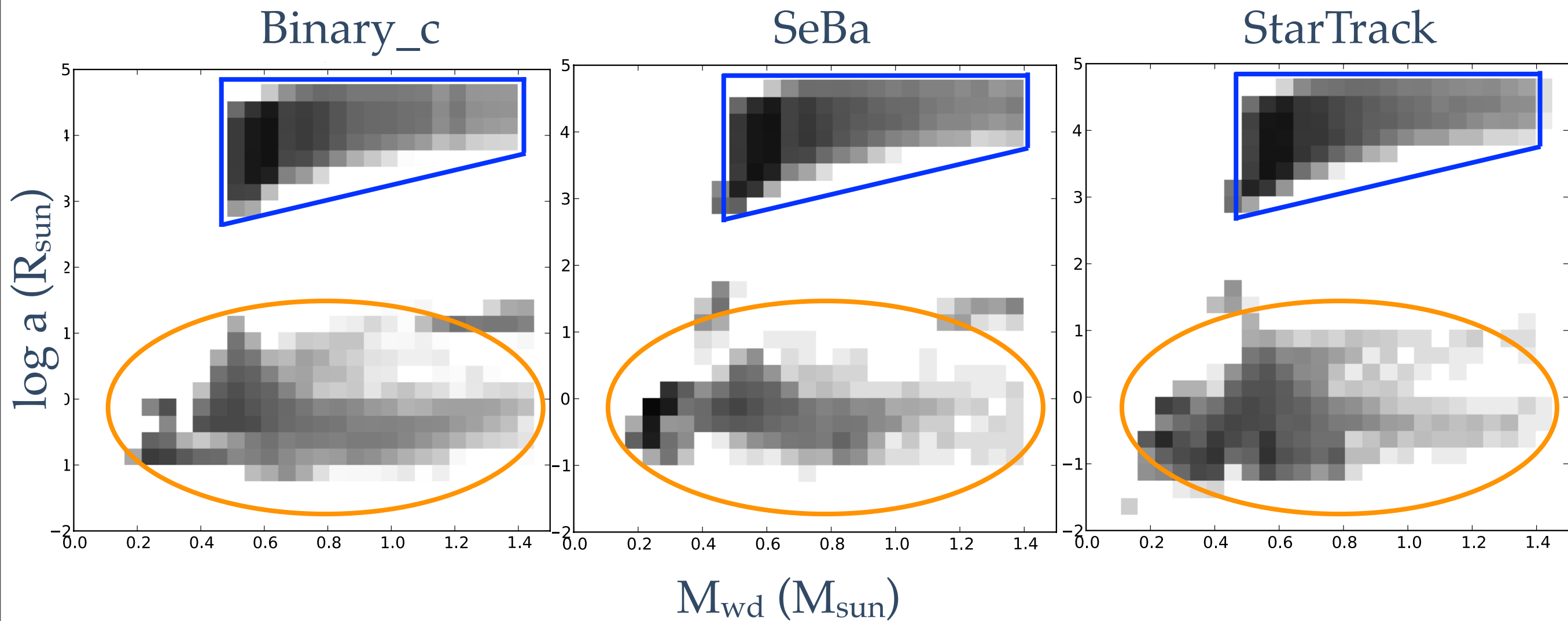


# Double white dwarfs

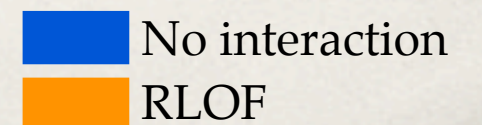


Similar simulated populations

# Double white dwarfs



Similar simulated populations

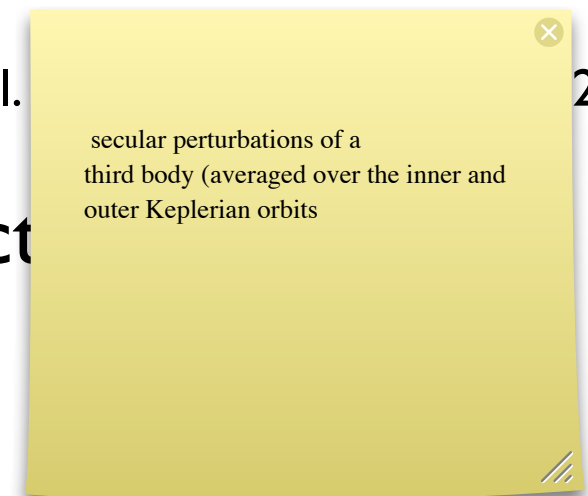


# Triples

New code for the evolution of coeval hierarchical stellar triples

Combines:

- ❖ Stellar evolution
  - ❖ Using SeBa (Portegies Zwart & Verbunt 1996, Nelemans et al. 2013)
  - ❖ Tracking realistic mass and radius as a function of time
- ❖ Secular hierarchical triple dynamics
  - ❖ Based on Hamers et al. 2013
    - ❖ in which the orbit-averaged equations of motion are solved numerically
- ❖ Simulate consistently stellar evolution + tides + Kozai-Lidov
  - ❖ including mass transfer, supernova kicks



# Secular evolution

Solve set of first-order ordinary differential equations

- As a function of semimajor axis  $a$ , inclination  $i$ , argument of periastron  $\omega$ , longitude of ascending node  $h$ , spin angular frequency  $\dot{\theta}$

$$\left\{ \begin{array}{l} \dot{a}_1 = \dot{a}_{1,GR} + \dot{a}_{1,TF} + \dot{a}_{1,wind} \\ \dot{a}_2 = \dot{a}_{2,GR} + \dot{a}_{2,TF} + \dot{a}_{2,wind} \\ \dot{e}_1 = \dot{e}_{1,STD} + \dot{e}_{1,GR} + \dot{e}_{1,TF}, \\ \dot{e}_2 = \dot{e}_{2,STD} + \dot{e}_{2,GR} + \dot{e}_{2,TF}, \\ \dot{\theta} = \frac{-1}{G_1 G_2} [\dot{G}_1 (G_1 + G_2 \theta) + \dot{G}_2 (G_2 + G_1 \theta)], \\ \dot{g}_1 = \dot{g}_{1,STD} + \dot{g}_{1,GR} + \dot{g}_{1,tides} + \dot{g}_{1,rotate}, \\ \dot{g}_2 = \dot{g}_{2,STD} + \dot{g}_{2,GR} + \dot{g}_{2,tides} + \dot{g}_{2,rotate}, \\ \dot{h}_1 = \dot{h}_{1,STD}, \\ \dot{\Omega}_{\star 1} = \dot{\Omega}_{\star 1,TF} + \dot{\Omega}_{\star 1,I}, \\ \dot{\Omega}_{\star 2} = \dot{\Omega}_{\star 2,TF} + \dot{\Omega}_{\star 2,I}, \\ \dot{\Omega}_{\star 3} = \dot{\Omega}_{\star 3,TF} + \dot{\Omega}_{\star 3,I}, \end{array} \right.$$

processes that are described are independent such that the individual time derivative terms can be added linearly. In addition, in the expressions for  $\dot{g}_{1,tide}$ ,  $\dot{g}_{1,rotate}$ ,  $\dot{e}_{1,TF}$ ,  $\dot{a}_{1,TF}$ ,  $\dot{e}_{2,TF}$  and  $\dot{\theta}$  we assume coplanarity of spin and orbit at all times, even though Kozai cycles in principle affect the relative orientations between the spin and orbit angular momentum vectors and in turn a misalignment of these vectors affects the Kozai cycles themselves (e.g. Correia et al. 2011). We justify this assumption by noting that for the majority of systems that we study the orbital angular momenta of both inner and outer orbits greatly exceed the spin angular momenta in magnitude, therefore, the stellar spins cannot greatly affect the exchange of angular momentum between both orbits.

$\dot{g}_{1,tides}$  (Smeyers & VVillems '01)

$\dot{g}_{1,rotate}$  (Fabrycky & Tremaine 2002)

$\dot{I}$  = Moment of inertia

$G$  orbital angular momentum

$\theta \equiv \cos(i)$



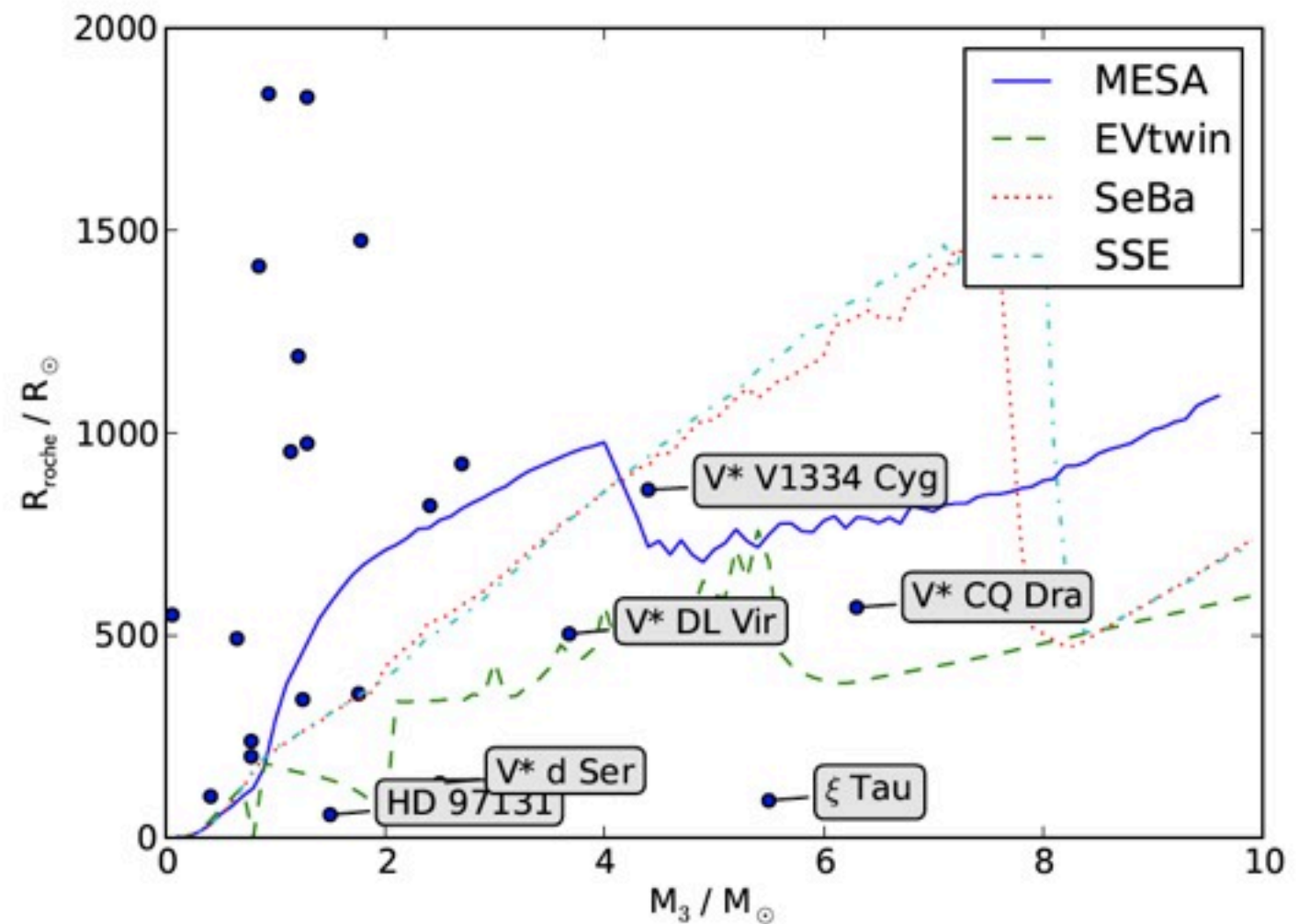
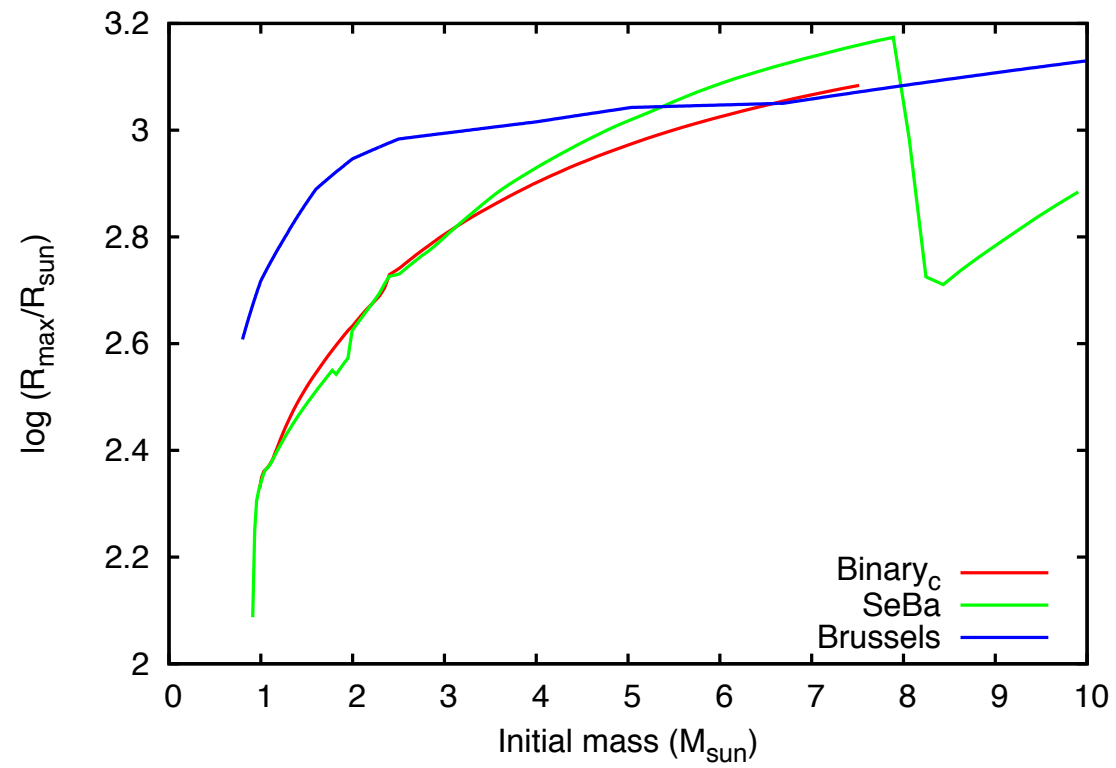
# Coupling of the codes:



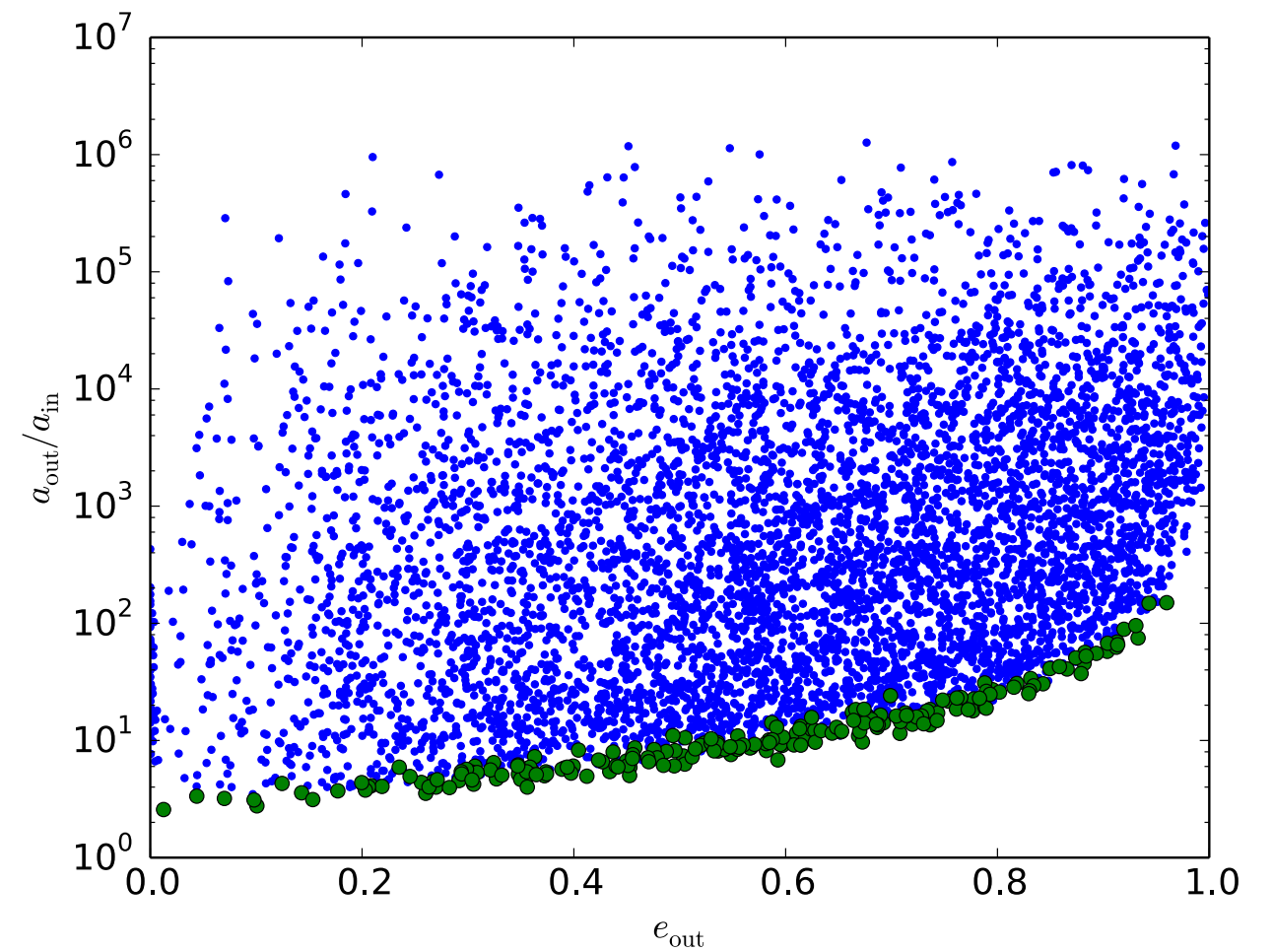
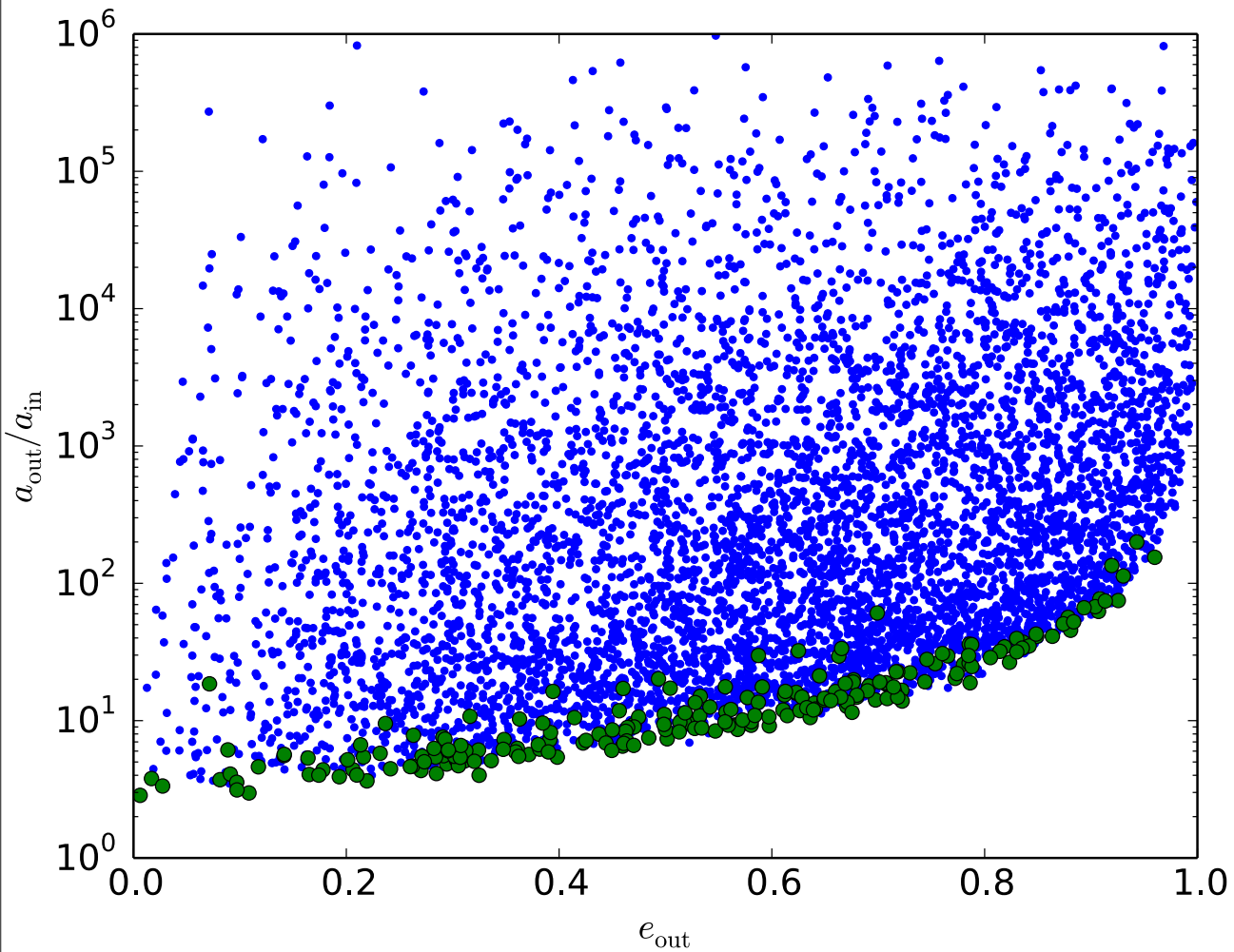
## Astrophysical Multipurpose Software Environment

- ❖ software framework astrophysical simulations,
- ❖ existing codes from different domains (stellar dynamics, stellar evolution, hydrodynamics and radiative transfer)
- ❖ easy coupling between the codes
- ❖ easy coupling to N-body code

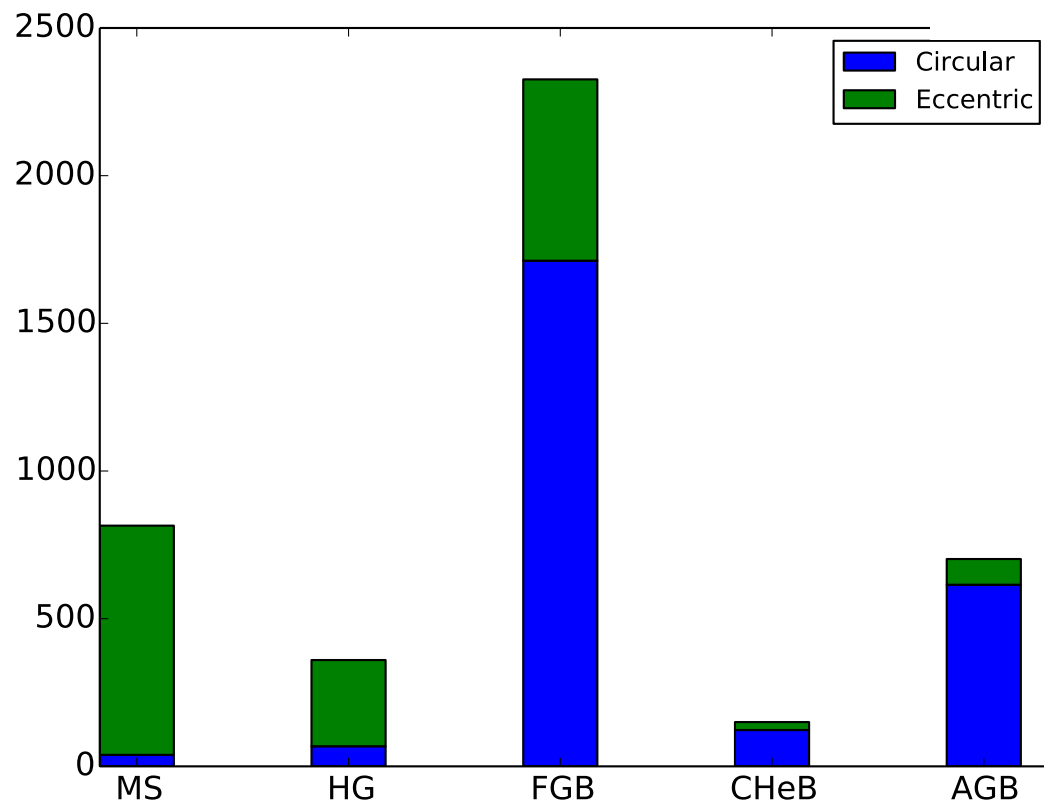
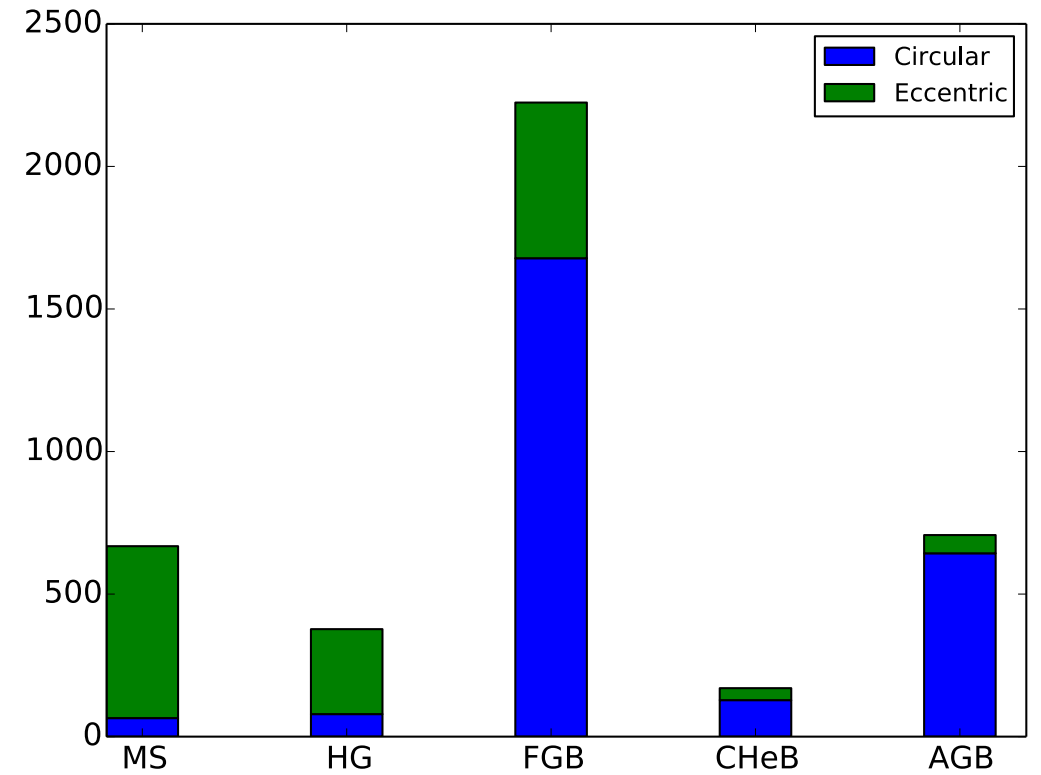
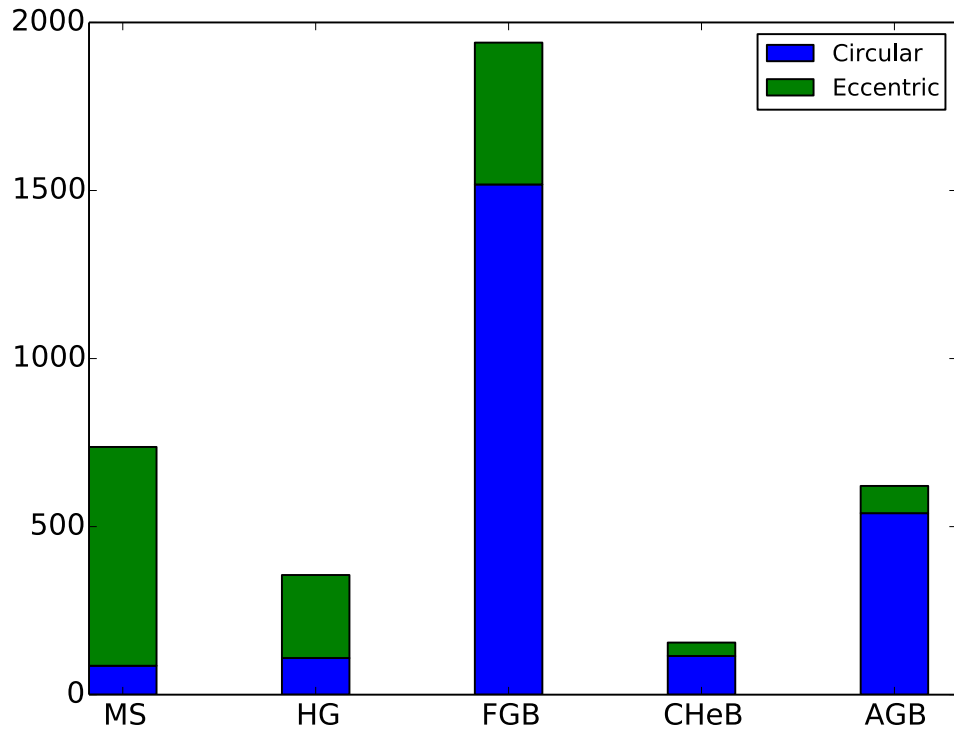
# Maximum radius



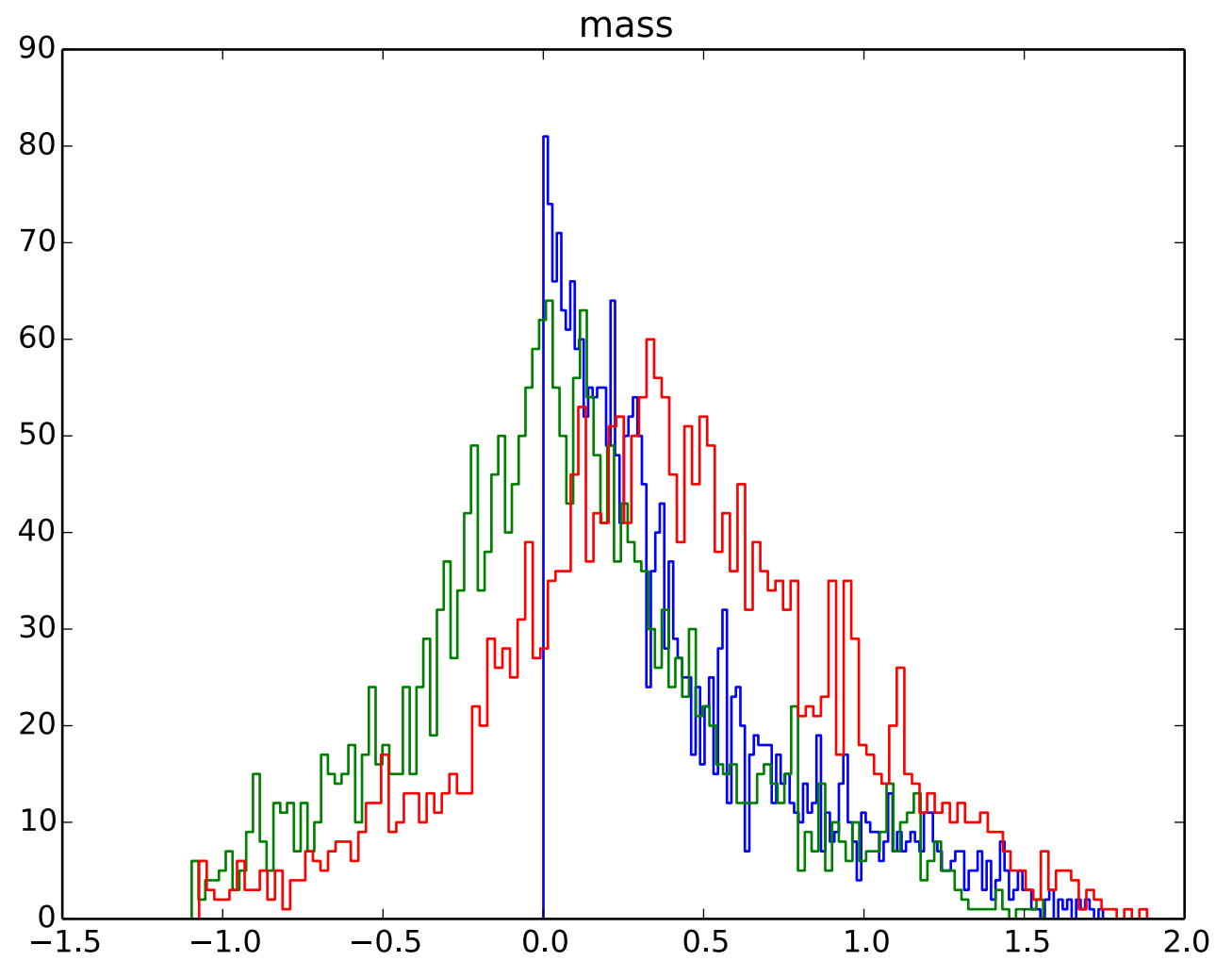
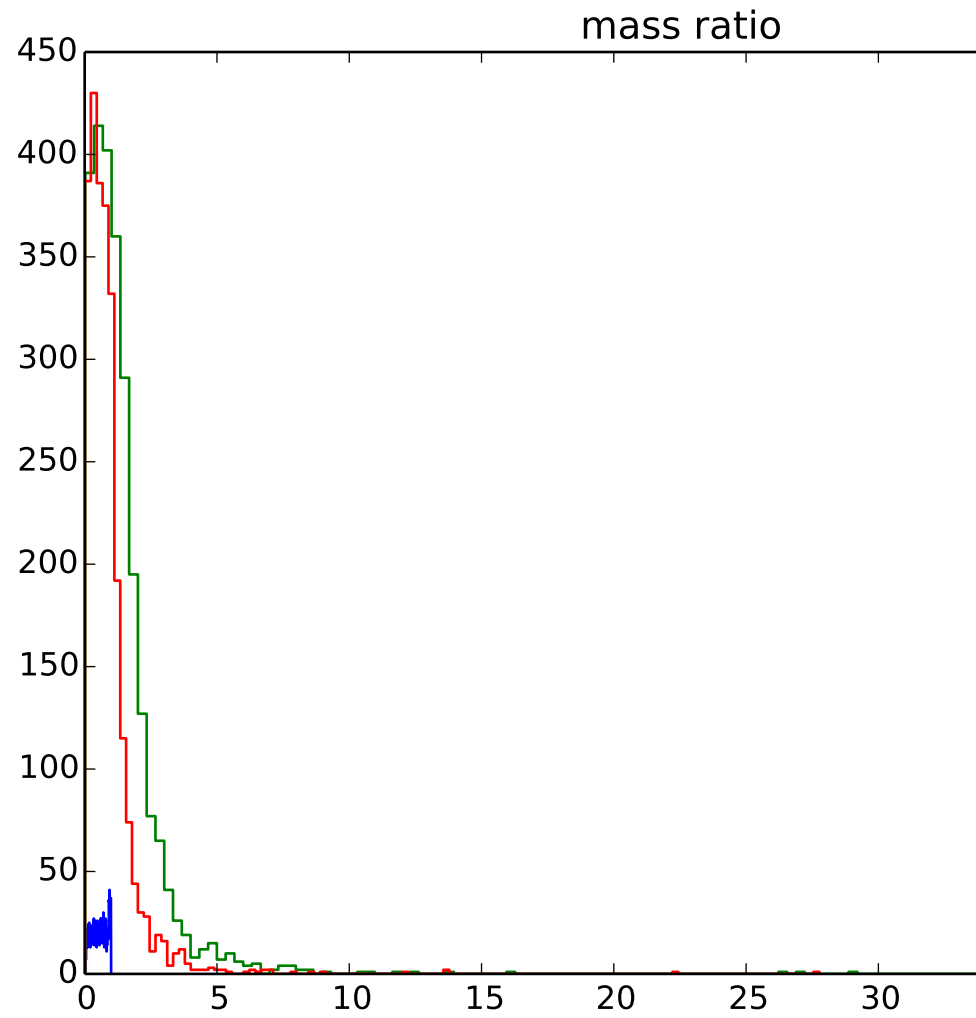
# Dynamical instability



# Inner RLOF donor type

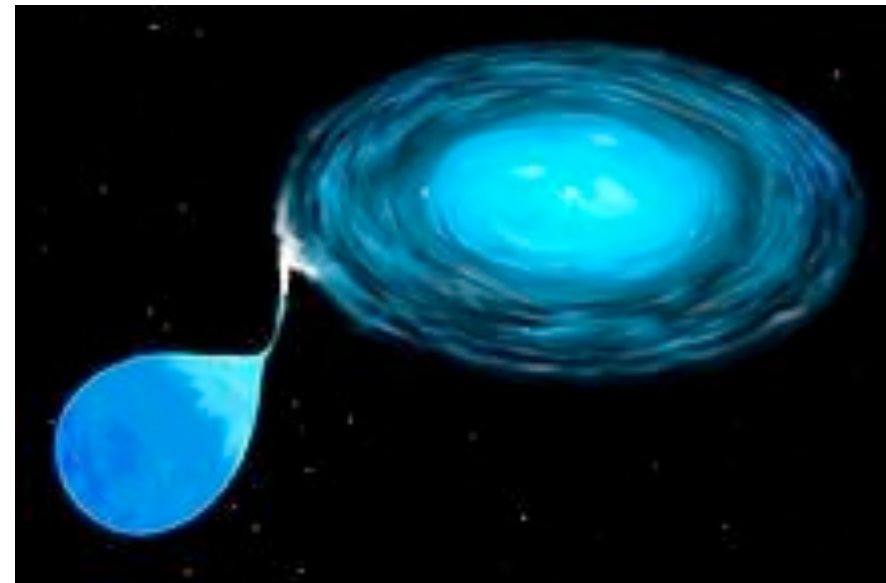


# Distribution of Eggleton '09



# Binary population synthesis

- Simulate the evolution of a large number of binaries
- From ZAMS to remnant formation (or any desired evolutionary phase)
- At each timestep for each binary take into account relevant physics  
=> Combination of stellar evolution & dynamics

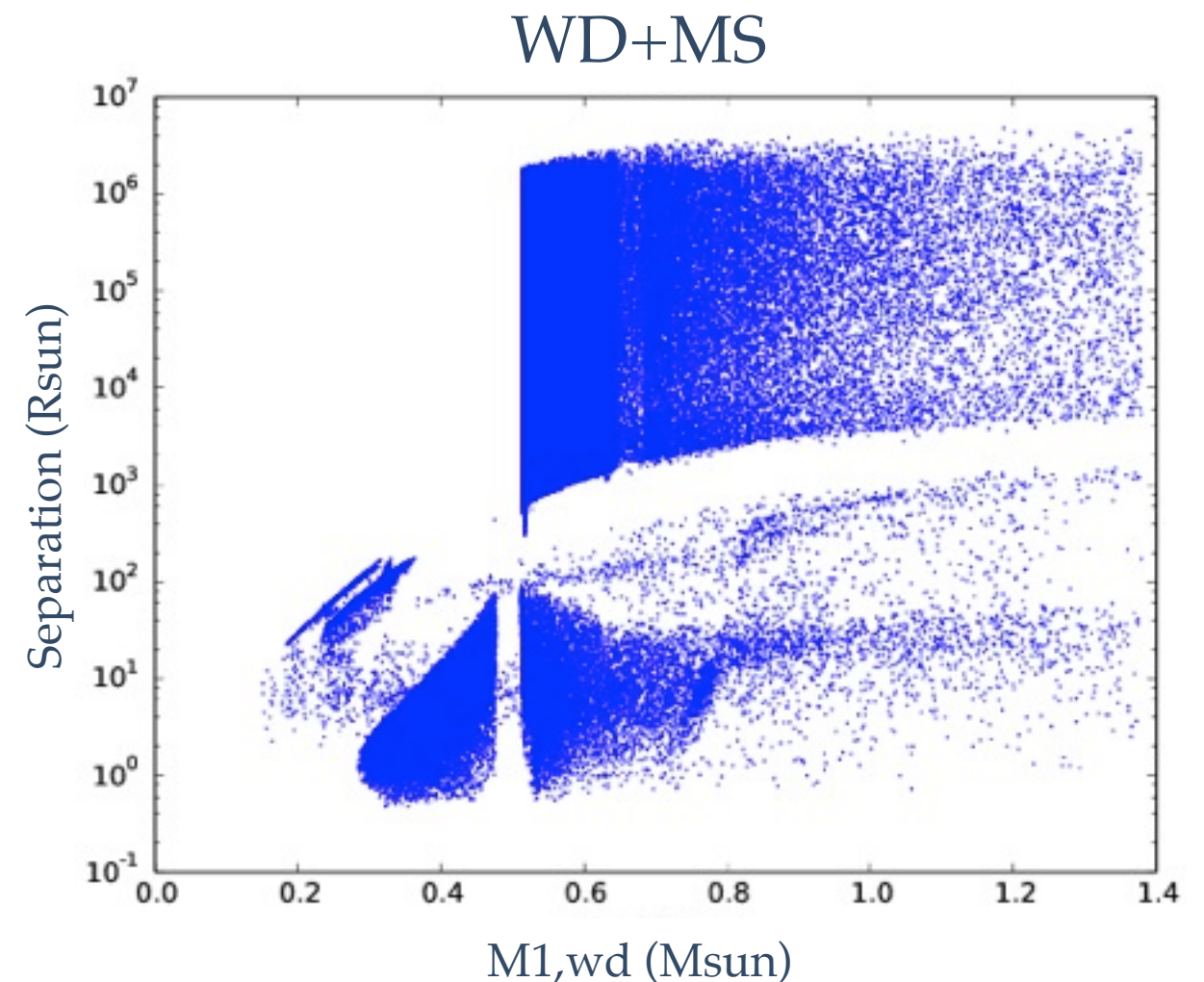
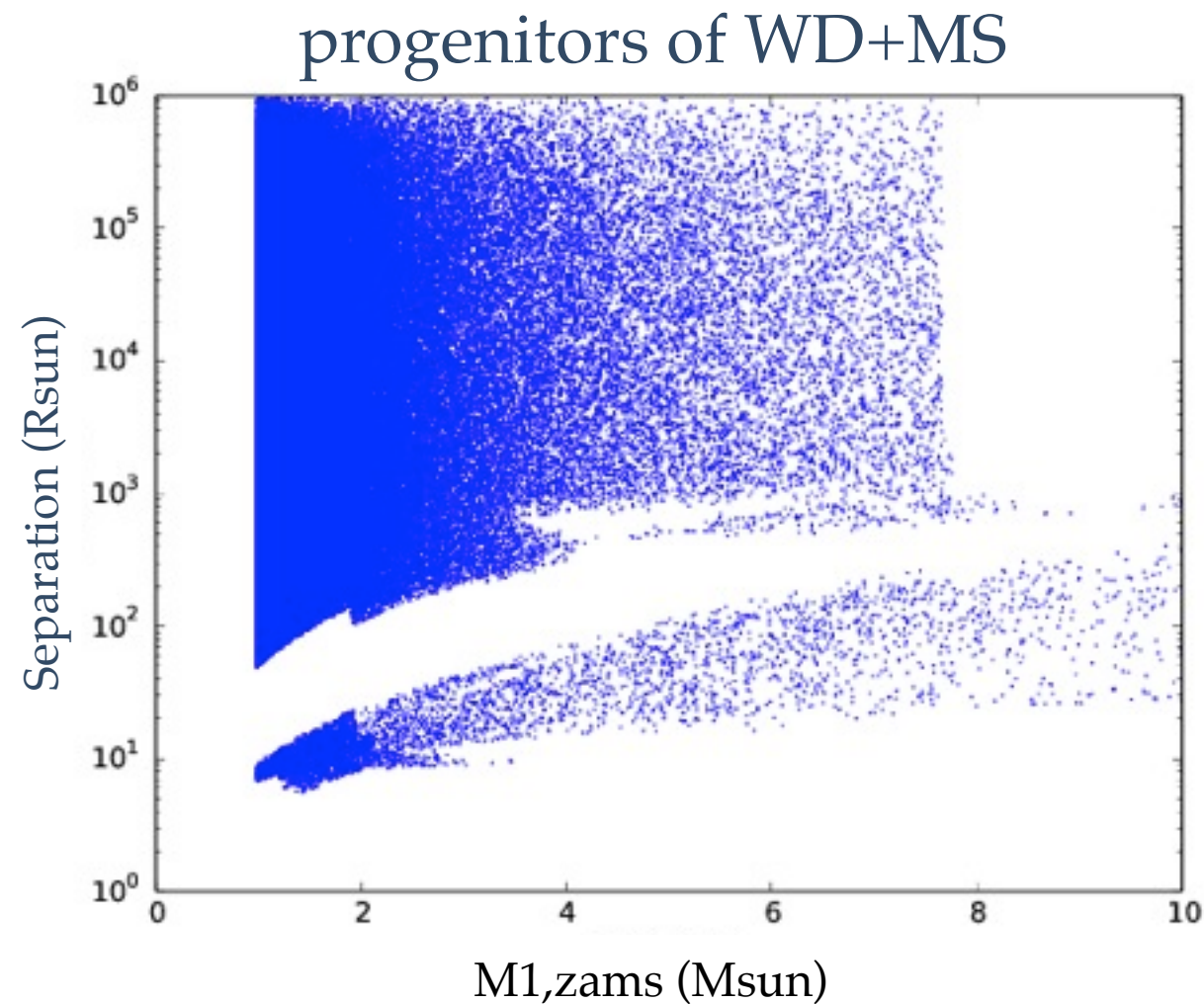


- Consensus on the principles of binary evolution

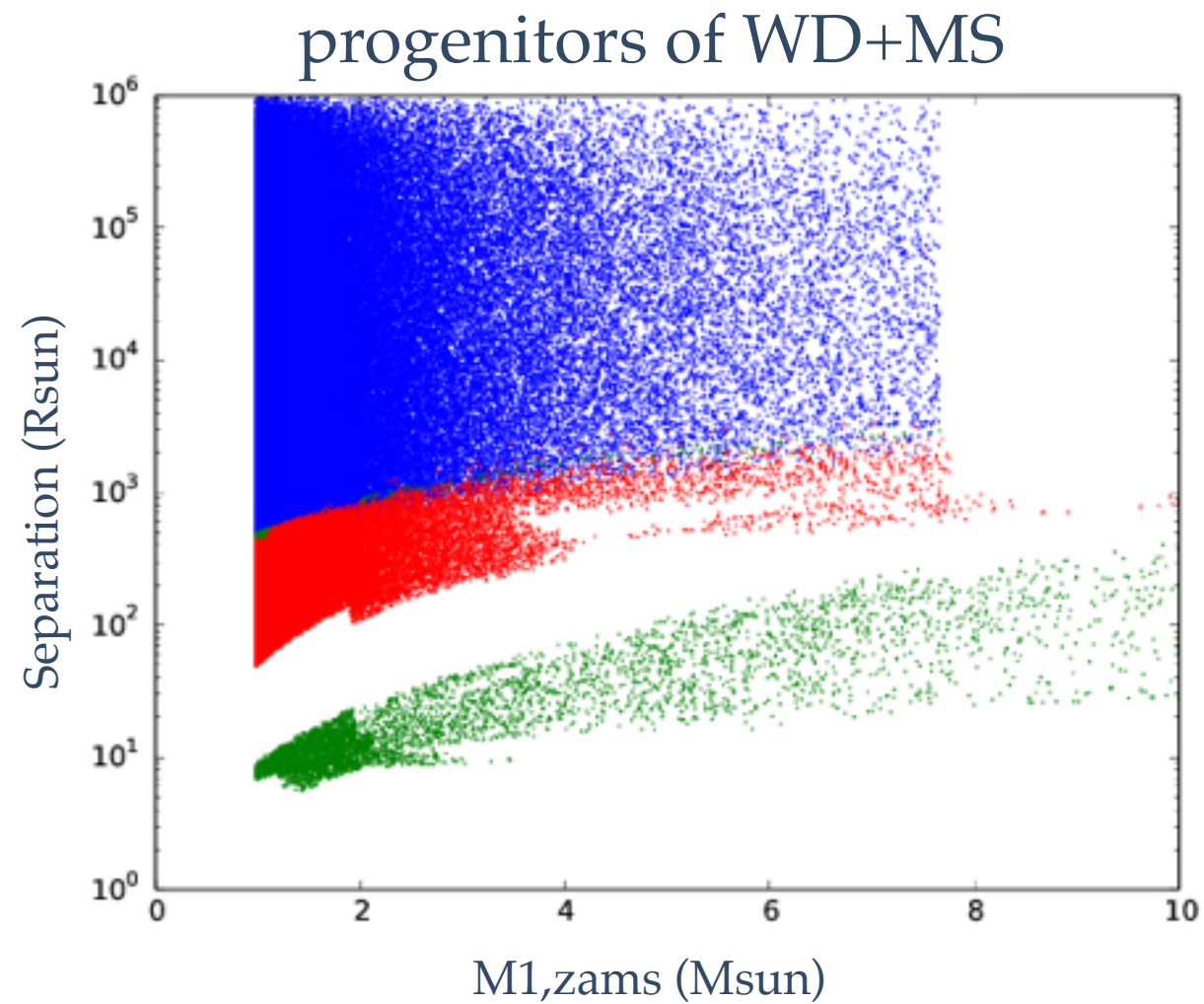


# Binary population synthesis

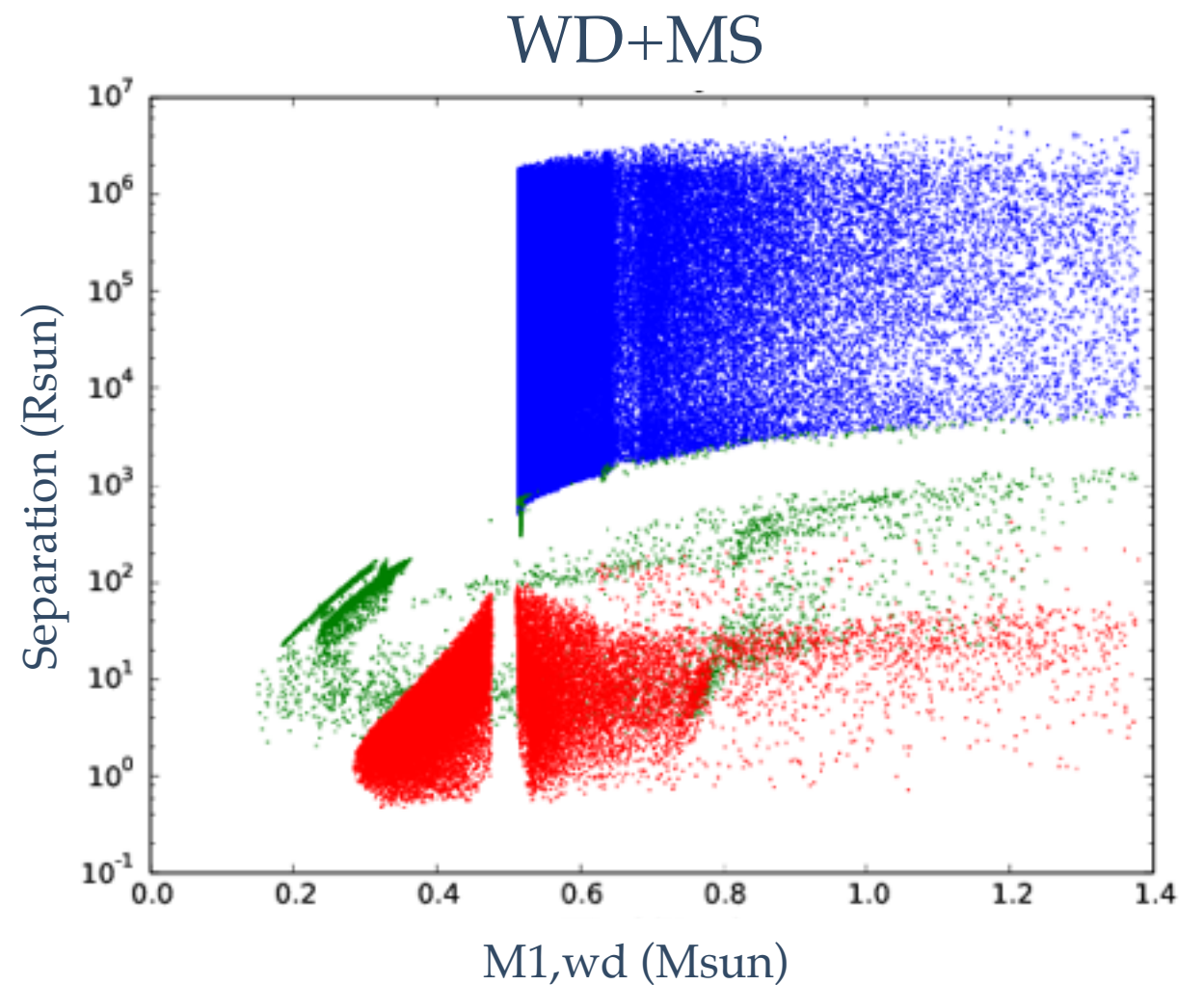
- Consensus on the principles of binary evolution



# Evolutionary channels



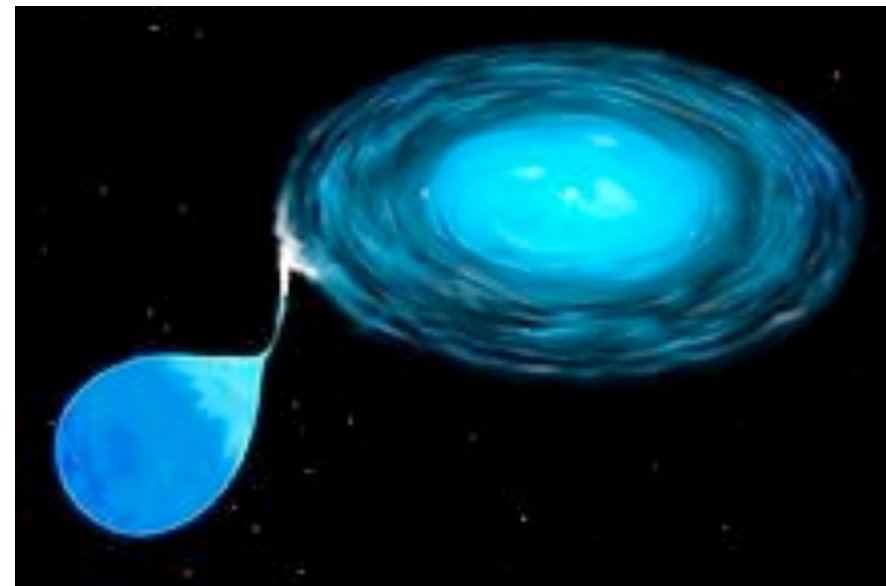
- No interaction
- Stable mass transfer
- Common-Envelope





# Population synthesis

- **Population synthesis used extensively for binaries** (e.g. Eggleton '89, de Kool ea '92, Willems & Kolb '94, Nelemans ea '01, Han ea '02, Belczynski ea '08, Ruiters ea '12, Mennekens ea '13, Claeys ea '14, Toonen ea '12,13,14)
- Studying e.g. supernova type Ia progenitors, WD-MS stars, cataclysmic variables, double white dwarf, X-ray binaries, SdB stars, gravitational wave sources



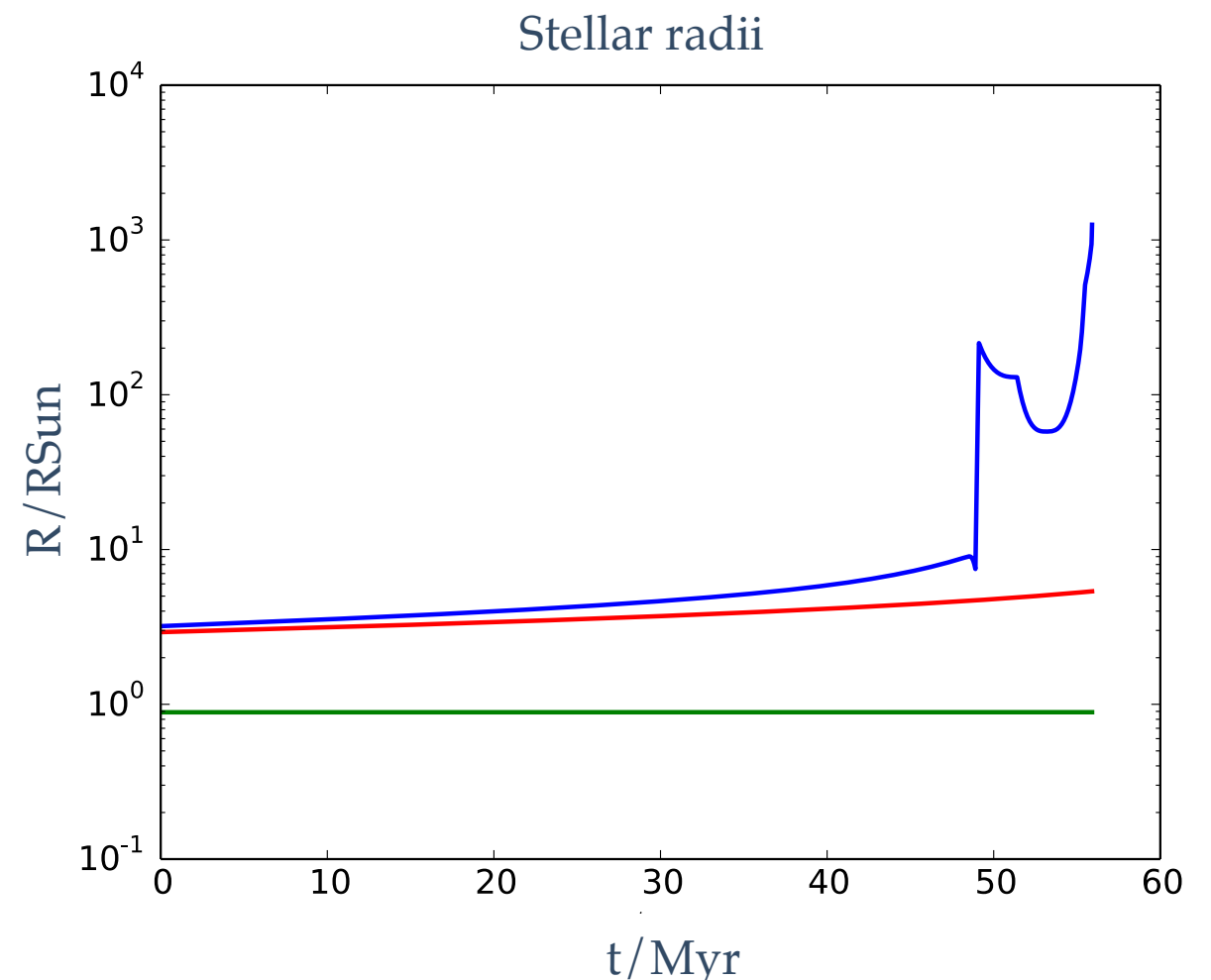
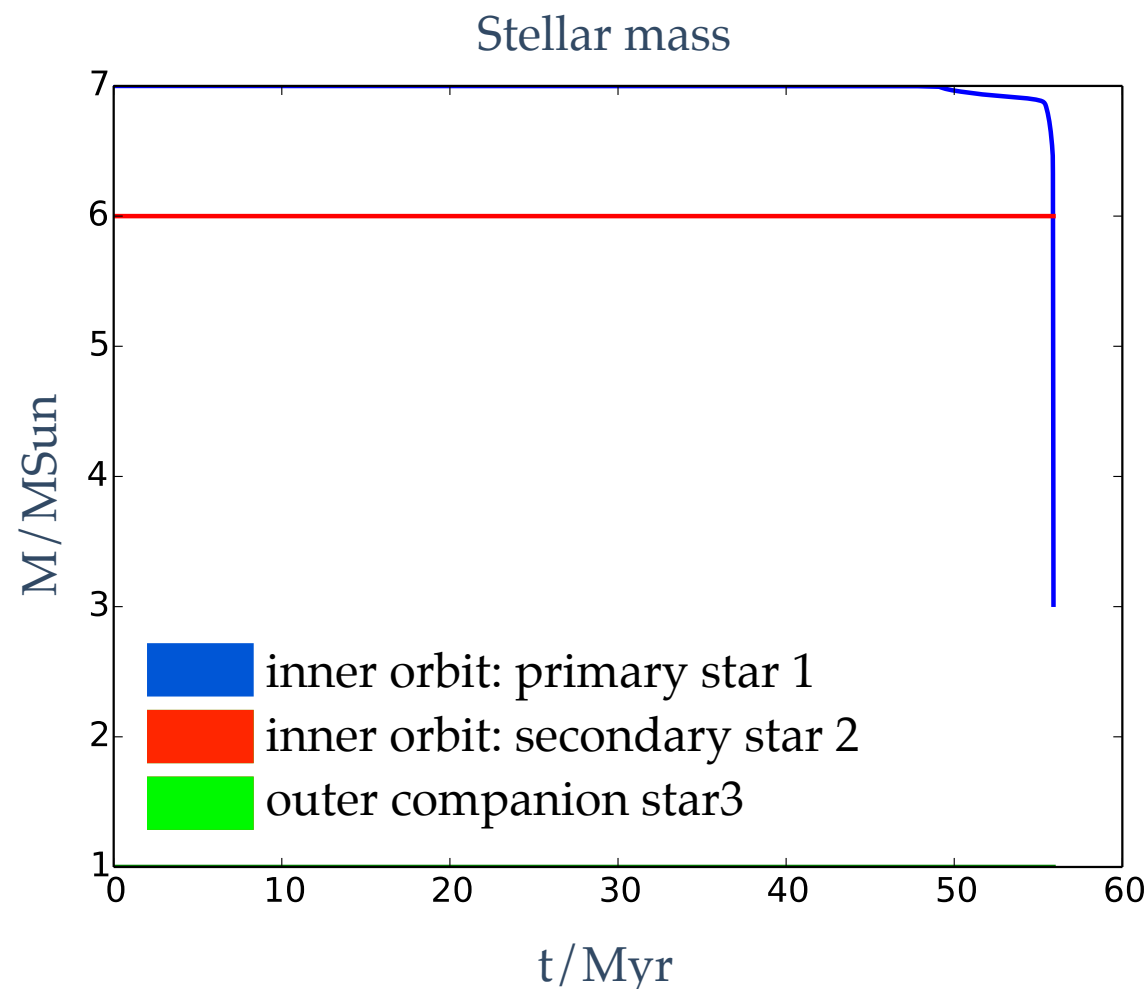
- Consensus on the principles of binary evolution
- Many questions remain, e.g. regarding mass transfer stability, unstable mass transfer (common-envelope phase), accretion

# Dynamical instability

Triple evolution dynamical instability (e.g. Kiseleva et al. '94, Iben & Tutukov '99, Perets & Kratter '11)

Through wind mass loss:

Example:  $M_1 = 7$ ,  $M_2 = 1$ ,  $M_3 = 6 M_{\text{Sun}}$ ,  $a_1 = 1e4$ ,  $a_2 = 5e5 R_{\text{Sun}}$ ,  $e_1 = 0.1$ ,  $e_2 = 0.8$ ,  $i = 0$ ,  $g_1 = 0.1$ ,  $g_2 = 0.5$

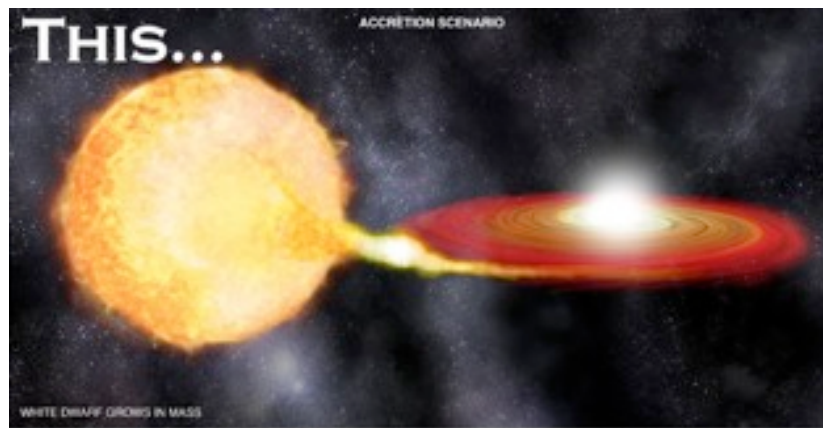


# Supernova Type Ia progenitors

Triples as SNIa progenitors (Katz & Dong 2012, Hamers ea 2013)

Classical progenitor systems:

- ❖ Single degenerate (Whelan & Iben '73)



- ❖ Double degenerate (Iben & Tutukov '84)



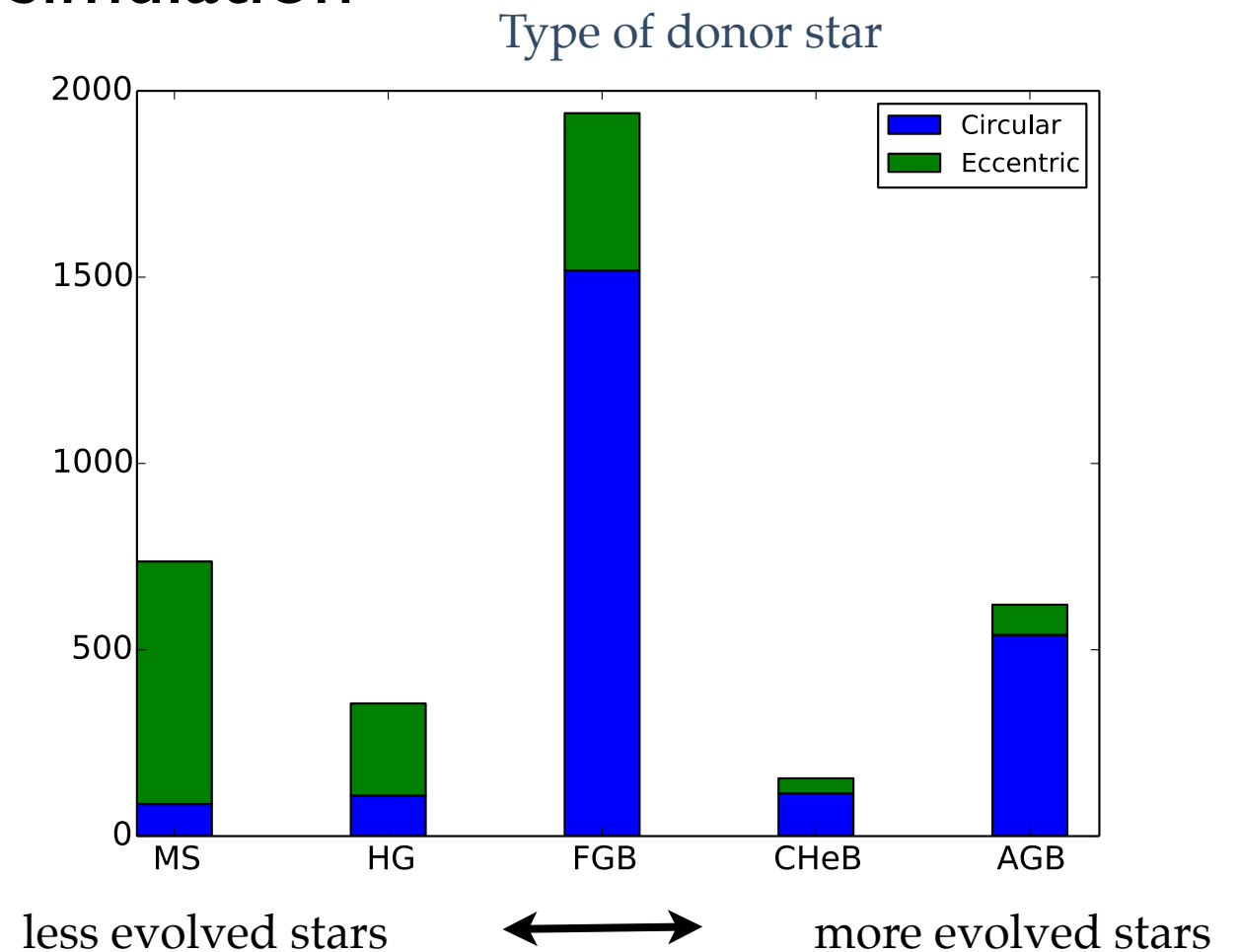
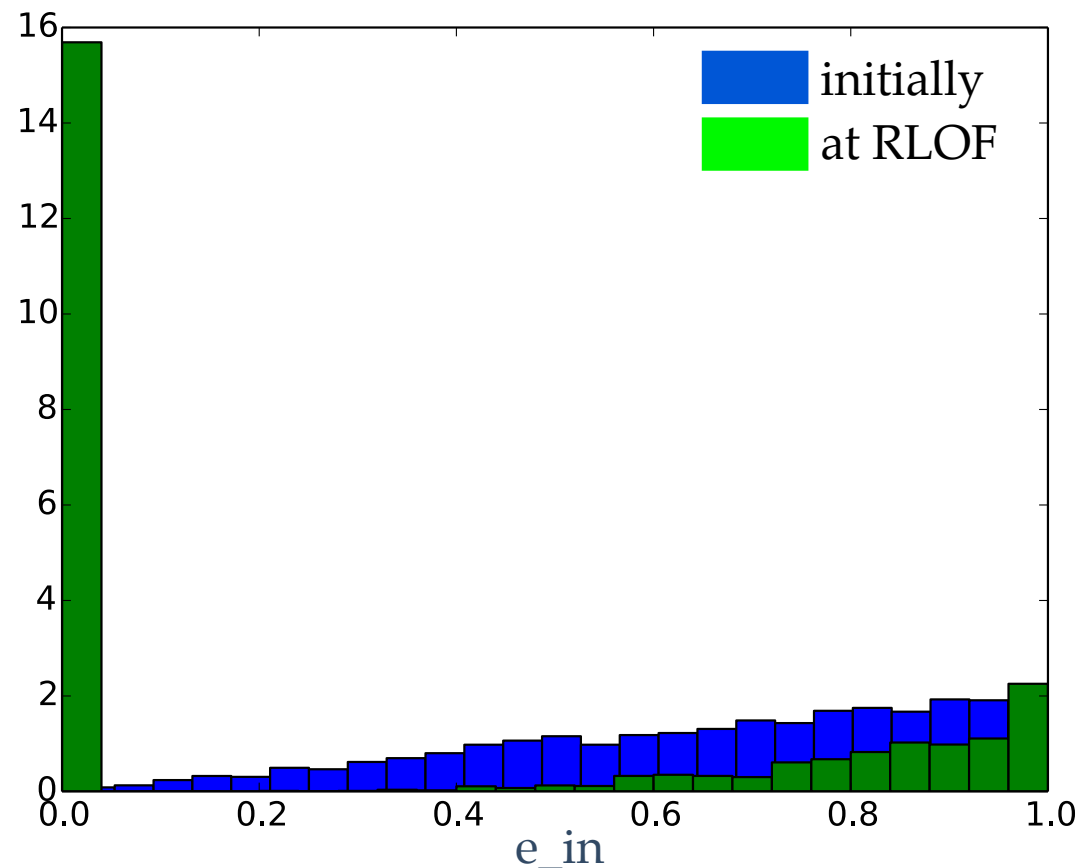
Rate (per $10^4 M_{\odot}$ )	
Observed (Maoz ea 2011, 2012, Perrett ea 2012, Graur ea 2012)	4-23
Triples	>0.02
Single degenerate	<0.001-1.3
Double degenerate	2-3.3

# Roche Lobe Overflow

- ❖ In the inner binary by the primary
- ❖ How often does this happen in triples?
  - ❖ 63% for model uncorrelated binaries I
  - ❖ 72% for model uncorrelated binaries II (Tokovinin)
  - ❖ 69% for model Eggleton
- ❖ How does this compare to isolated binary evolution?
  - ❖ Educated guess: ~40%
    - ❖ Assuming RLOF occurs when  $a(1-e^2) < 1e3 | \text{units. } R_{\text{Sun}}$  (see Toonen et al. 2014) & uncorrelated binaries I (Abt)
  - ❖ Detailed comparison: to be continued....

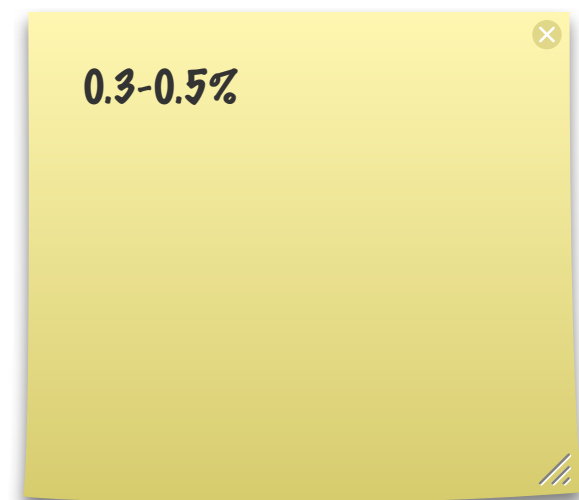
# Roche Lobe Overflow

- ❖ Mass transfer in eccentric orbit
- ❖ How often does this happen in triples?
  - ❖ 38% for model uncorrelated binaries I
  - ❖ 42% for model uncorrelated binaries II (Tokovinin)
  - ❖ 38% for model Eggleton
    - ❖ % of total number RLOF in simulation



# Roche Lobe Overflow

- ❖ In the inner binary by the secondary
  - ❖ after the primary has become a compact object
  - ❖ special evolutionary channel
    - ❖ to form compact binaries without mass transfer (see also: Shappee & Thompson '13, Michaely & Perets '14)
- ❖ How often does this happen in triples?
  - ❖ a few in a 1000 systems for all models
- ❖ How eccentric is the orbit?
  - ❖ Roughly half of systems:  $e_{in} \sim 0$
  - ❖ Other half:  $e_{in} > 0.8$
- ❖ Donor stars can be evolved or non-evolved stars



# MIEK

Shappee & Thompson (2013) studied the case of mass-loss from a component in the inner binary, which leads to a transition from a more regular Kozai-Lidov secular behavior to the regime where octupole level perturbations become significant, and the amplitude of eccentricity changes become significant; a behavior

## MIEK - Mass-loss induced eccentric Kozai (Shappee & Thompson 2013)

- ❖ Mass-loss from the inner binary causes a transition from regular quadrupole Kozai behaviour to where the octupole becomes significant
- ❖ Standard example:  
 $m_1=7M_{\text{Sun}}, m_2=6.5M_{\text{Sun}}, m_3=6M_{\text{sun}}; a_1=10\text{AU}, a_2=250\text{AU}, e_1=0.1, e_2=0.7, g_1=0, g_2=180, i=60$
- ❖ Varying  $i, e_1, e_2, g_1$ :
  - ❖ 2 up to 7% of systems go through MIEK (Shappee & Thompson 2013)
- ❖ However...
  - ❖ Even if the inner binary was isolated, RLOF when  $a_1 < 15\text{AU}$
  - ❖ Slightly wider orbits affected by Kozai-Lidov induced-RLOF and wind-induced dynamical instabilities



# Roche Lobe Overflow

- ❖ In the outer binary from the outer companion
- ❖ How often does this happen in triples?
  - ❖ 0.5% for model uncorrelated binaries I
  - ❖ 1% for model uncorrelated binaries II (Tokovinin)
  - ❖ 0.9% for model Eggleton
- ❖ In good agreement with de Vries et al. '13
  - ❖ For 1% of triples in the Tokovinin catalogue (full primary mass range), the outer companion initiates RLOF before any of the inner stars leave the main sequence
  - ❖ Predominantly evolved (AGB) donor stars
  - ❖ From SPH simulations for  $\xi$  Tau and HD97131

$$\frac{(\dot{a}_{in}/a_{in})}{(\dot{a}_{out}/a_{out})} \simeq 1$$

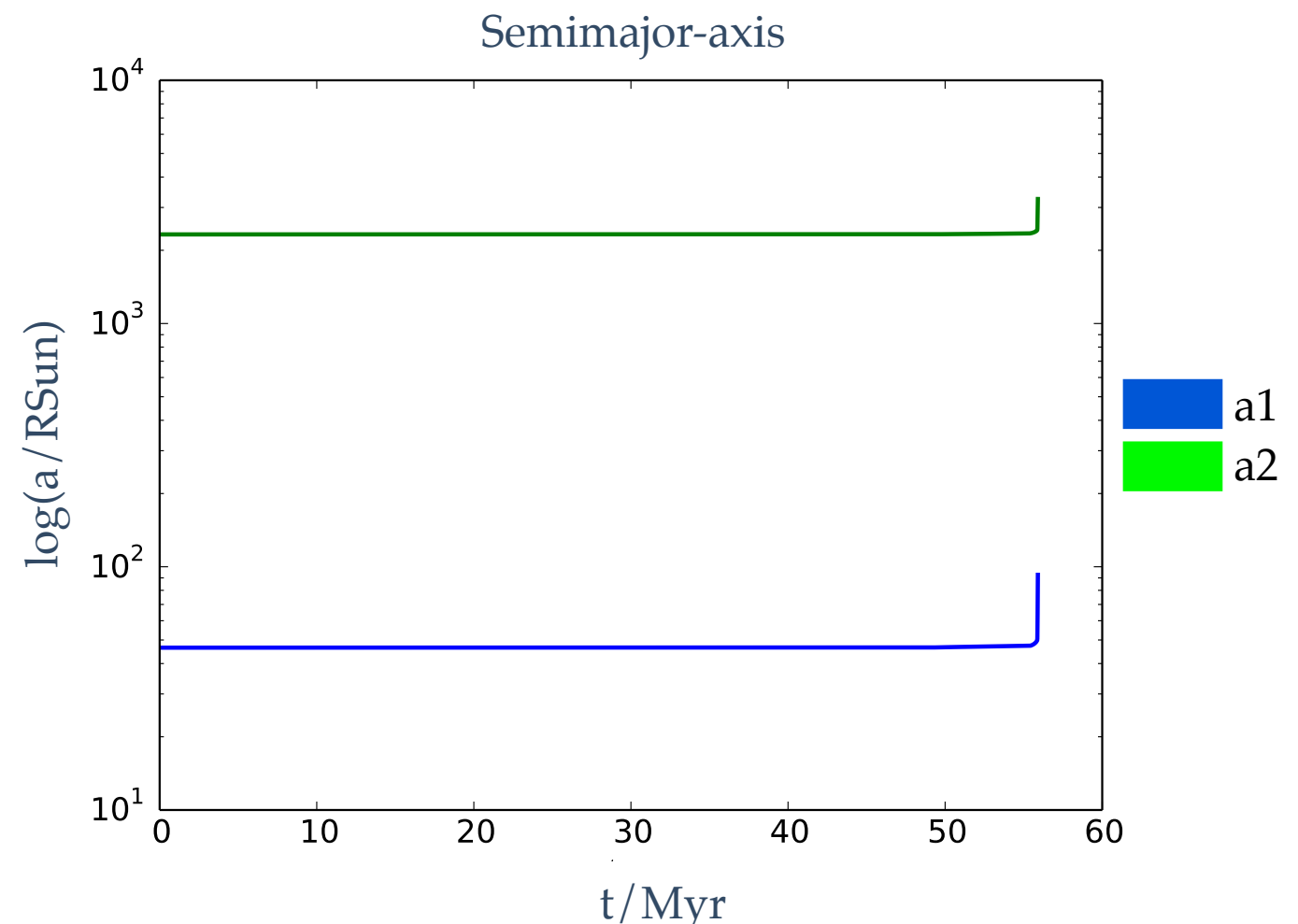
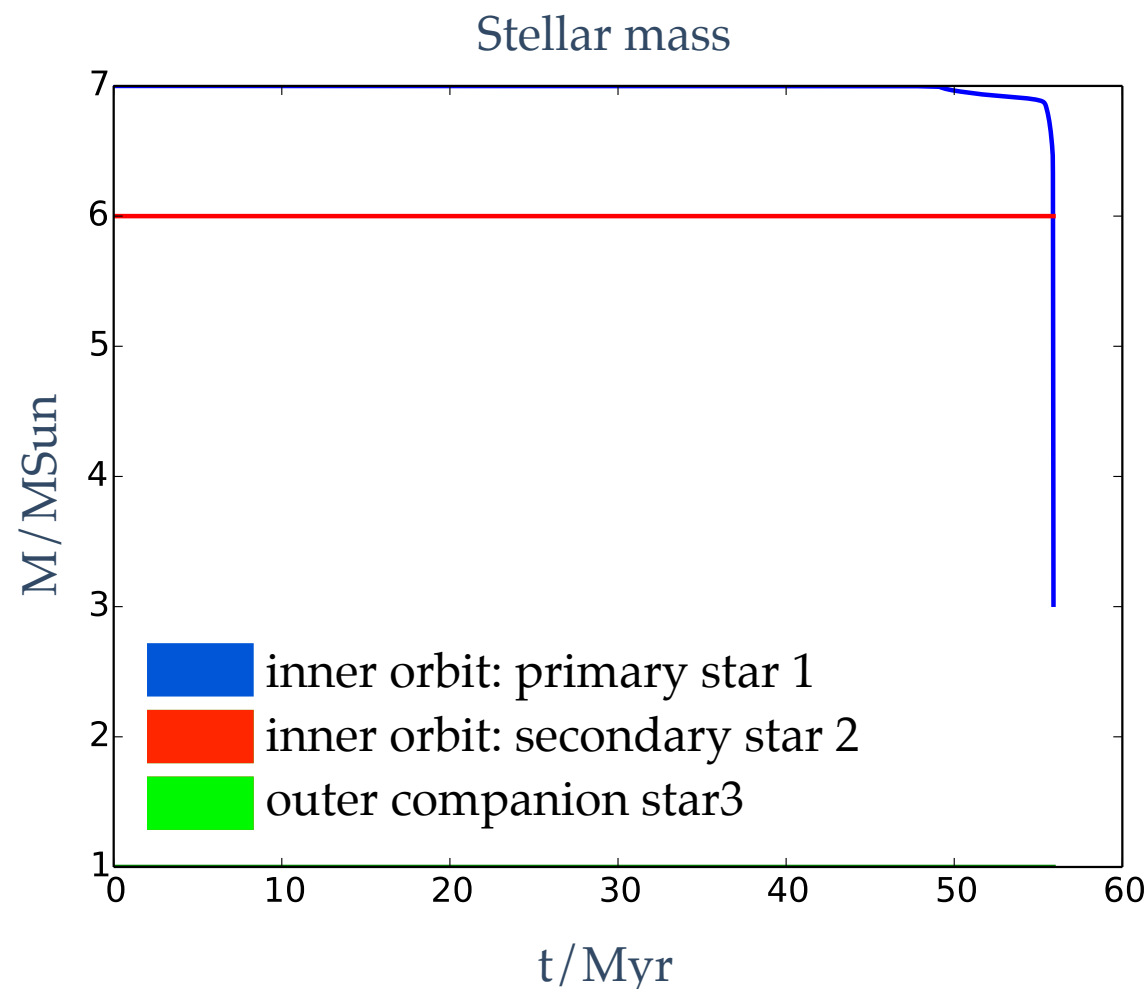


# Dynamical instability

Triple evolution dynamical instability (e.g. Kiseleva et al '94, Iben & Tutukov '99, Perets & Kratter '11)

Through wind mass loss:

Example:  $M_1 = 7$ ,  $M_2 = 1$ ,  $M_3 = 6 M_{\text{Sun}}$ ,  $a_1 = 1e4$ ,  $a_2 = 5e5 R_{\text{Sun}}$ ,  $e_1 = 0.1$ ,  $e_2 = 0.8$ ,  $i = 0$ ,  $g_1 = 0.1$ ,  $g_2 = 0.5$

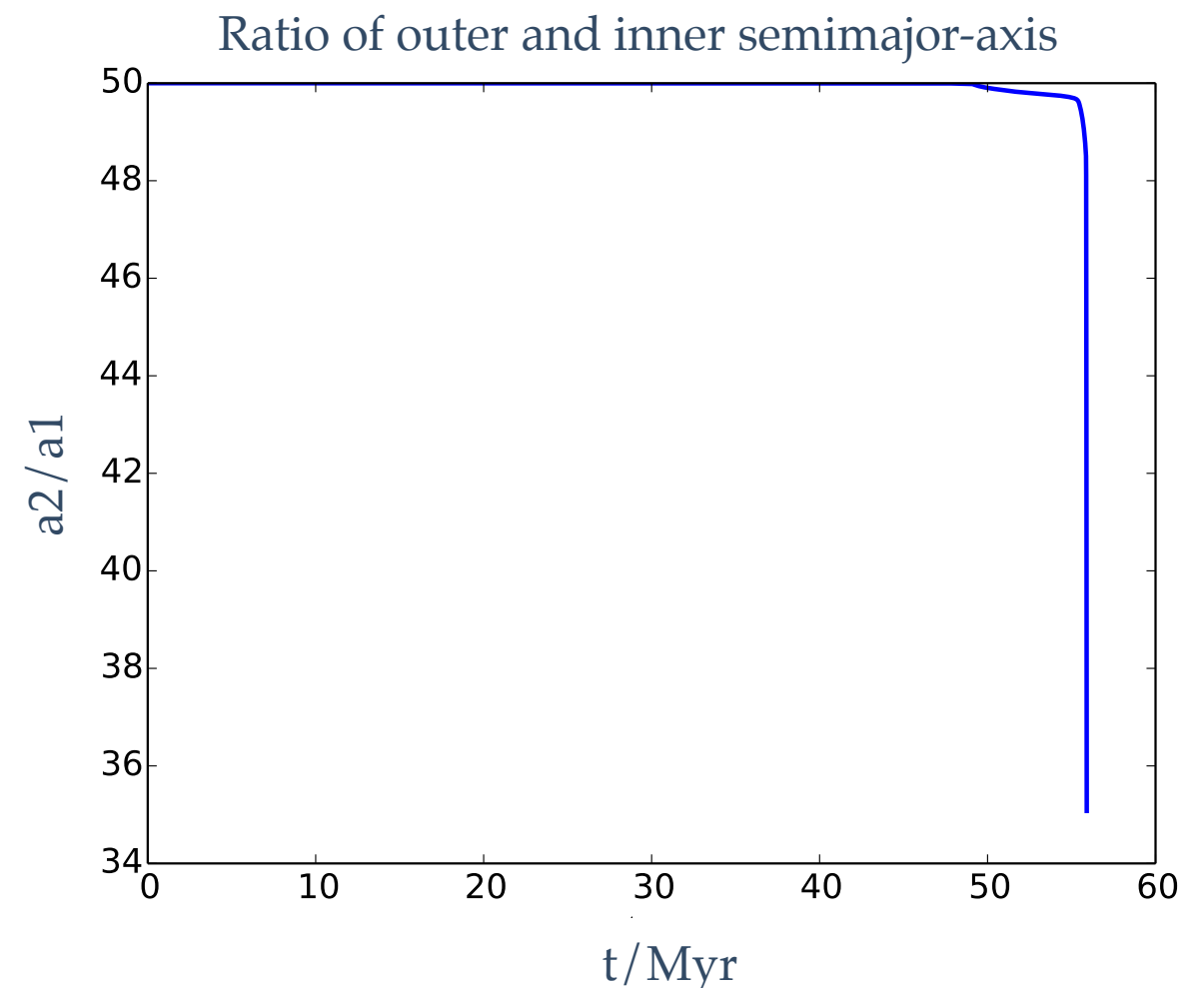
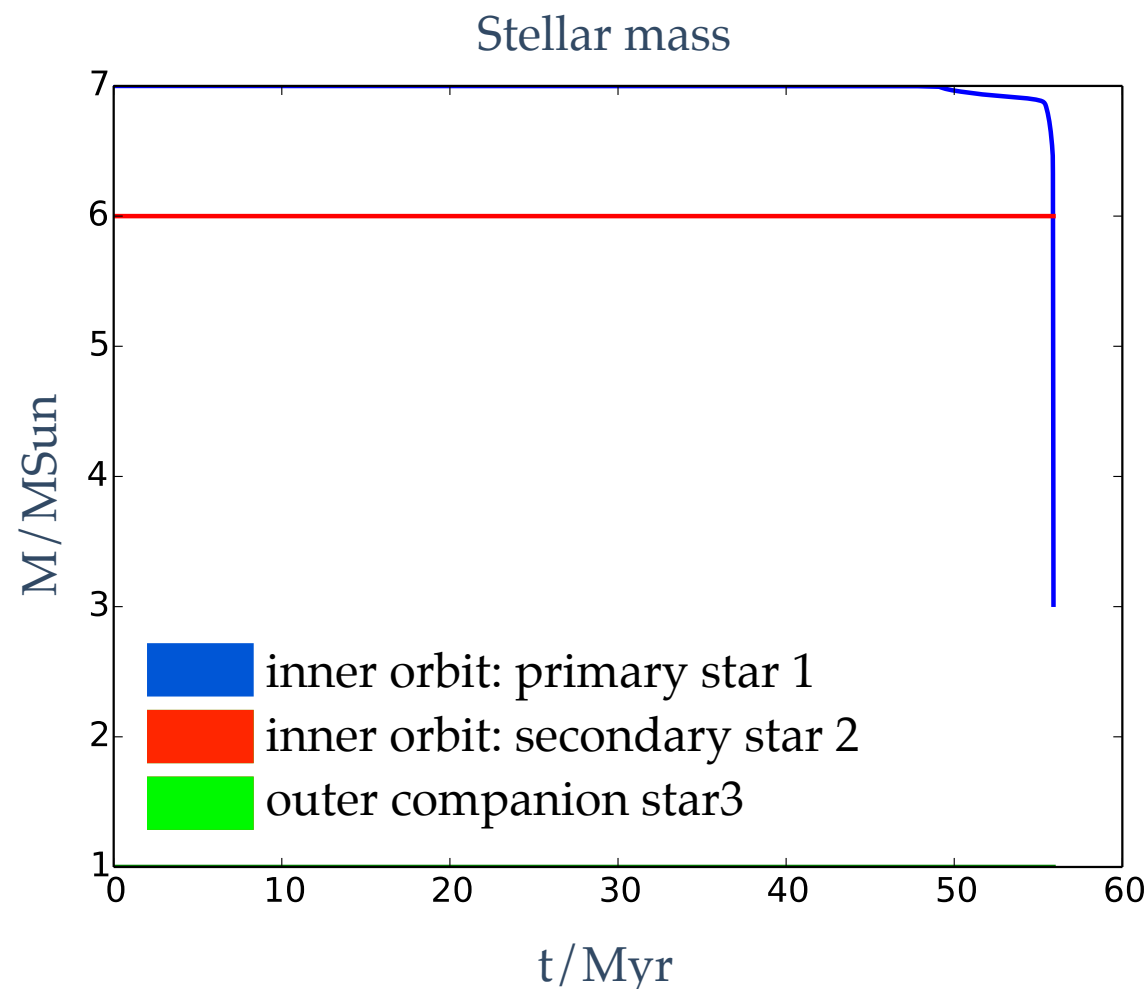


# Dynamical instability

Triple evolution dynamical instability (e.g. Kiseleva et al '94, Iben & Tutukov '99, Perets & Kratter '11)

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# Dynamical instability

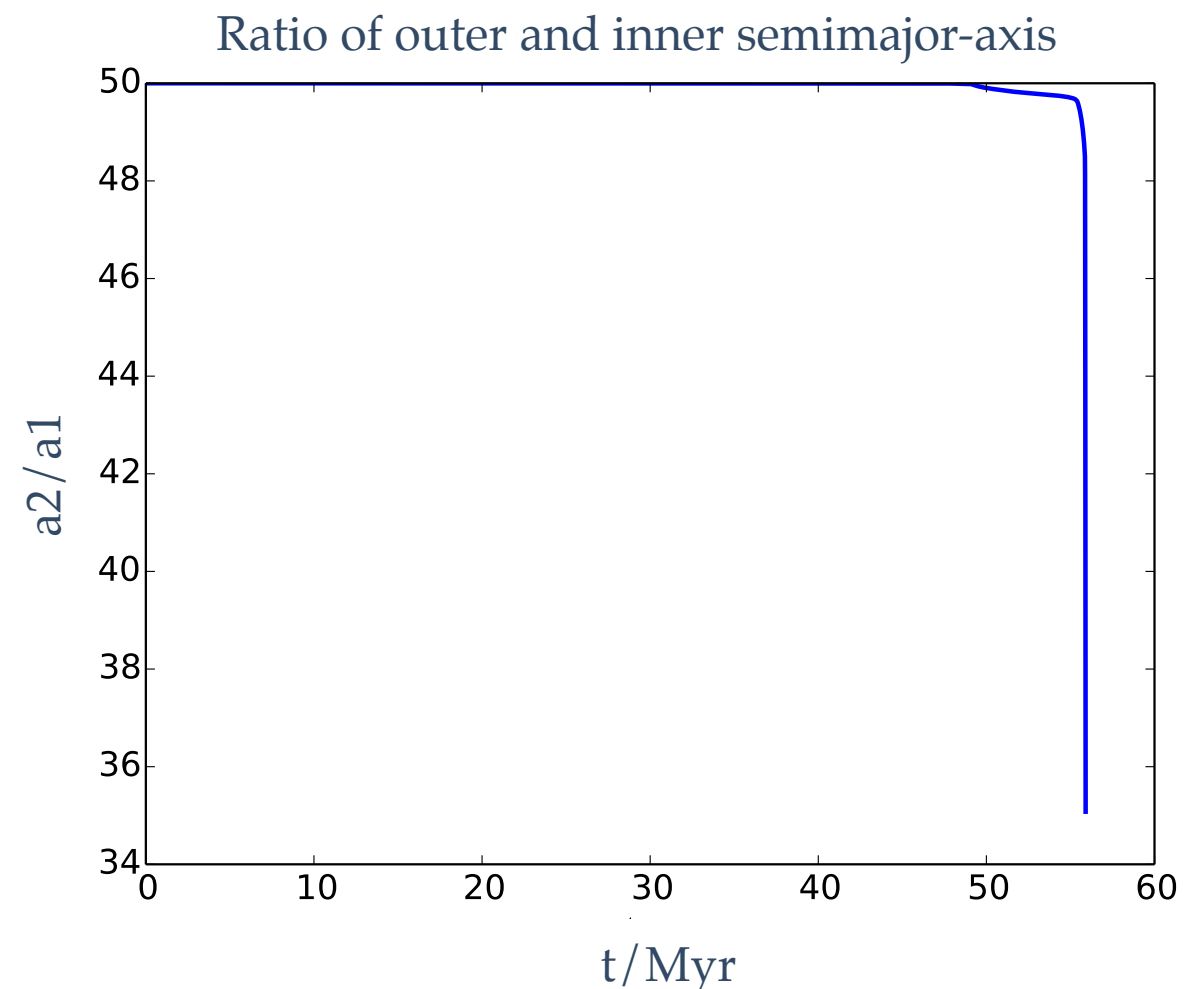
Triple evolution dynamical instability (e.g. Kiseleva et al '94, Iben & Tutukov '99, Perets & Kratter '11)

Through wind mass loss:

Example:  $M_1 = 7$ ,  $M_2 = 1$ ,  $M_3 = 6 M_{\text{Sun}}$ ,  $a_1 = 1e4$ ,  $a_2 = 5e5 R_{\text{Sun}}$ ,  $e_1 = 0.1$ ,  $e_2 = 0.8$ ,  $i = 0$ ,  $g_1 = 0.1$ ,  $g_2 = 0.5$

Effect of wind mass-loss in inner binary:

- ❖ orbits widen
- ❖ inner orbit widens more
- ❖ orbits come closer to each other
- ❖ possible dynamical instability



In the case of MS destabilization the triple system is initially marginally stable (i.e. only just satisfies  $\beta > \beta_{\text{crit}}$ ; cf. Section 2.3) but due to octupole-order terms of the STD, which are important since  $\beta$  is (very) small and/or  $e_2$  is high,  $e_2$  varies periodically until it reaches a value high enough such that  $\beta \leq \beta_{\text{crit}}$ , i.e. a triple destabilization. The time when this occurs is determined by the Kozai period  $P_k$ . Similarly to the MS mergers, this occurs early in the evolution with most (90 per cent) destabilizations occurring within 10 per cent of the primary MS lifetime (cf. Fig. 7). In the other cases destabilization is triggered by mass loss in the inner orbit which, if fast and isotropic, acts to decrease  $\beta$  (i.e. the same mechanism discussed in the context of eccentric compact object mergers in Section 5.1) to a point where  $\beta \leq \beta_{\text{crit}}$ . This happens when the primary loses a significant amount of mass as it evolves from the AGB phase to a WD and similarly when this happens to the secondary. In a small number of cases both inner binary components are CO WDs when the instability occurs and since there exists a finite probability of collision in the triple evolution dynamical instability (approximately 0.1 as found by Perets & Kratter 2012) this could potentially lead to a CO WD collision. This scenario is included in Section 6.

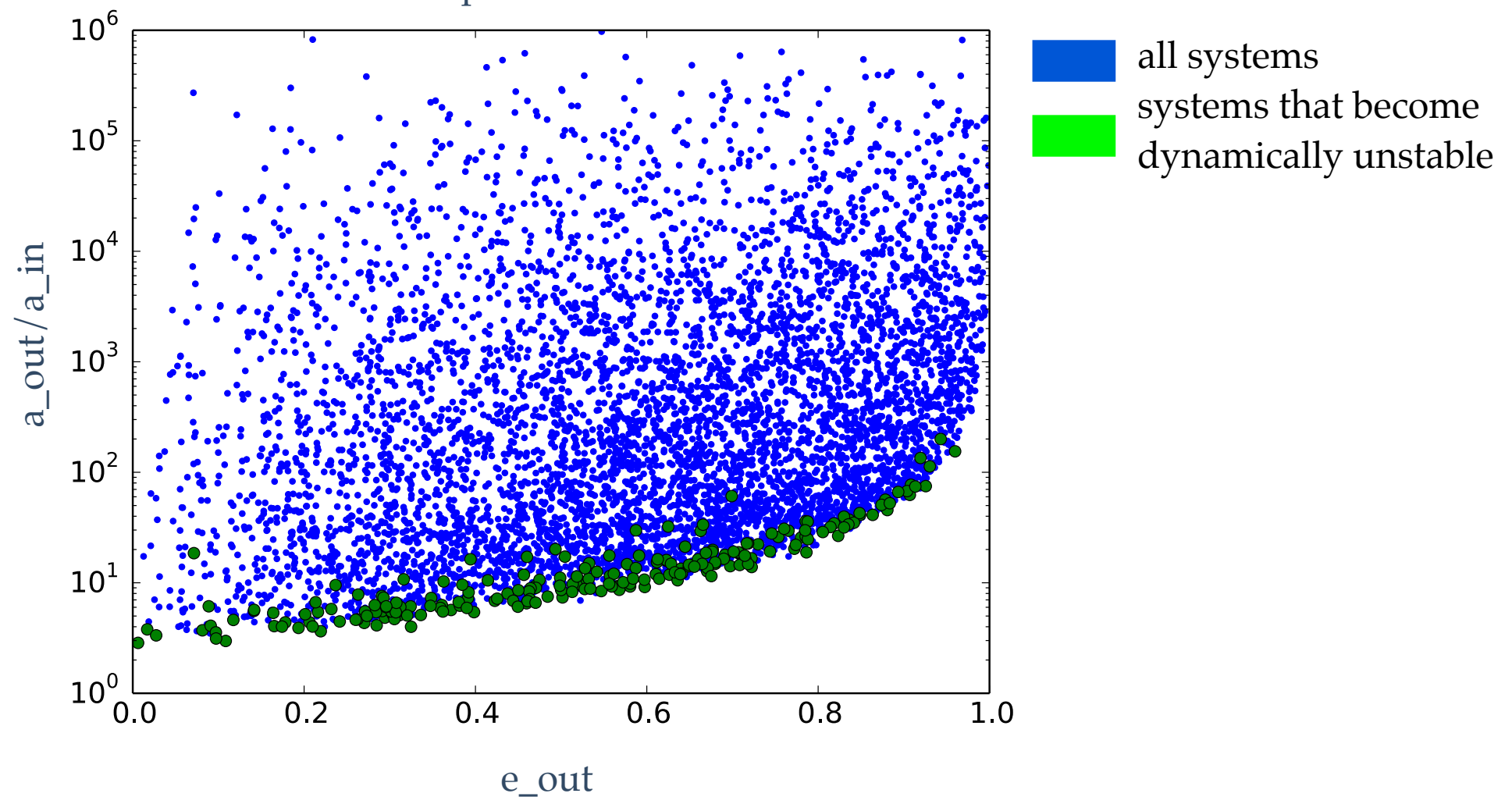
we demonstrate that the rate of stellar collisions due to the TEDI is approximately  $10^{-4} \text{ yr}^{-1}$  per Milky-Way Galaxy, which is nearly 30 times higher than the total collision rate due to random encounters in the Galactic globular

Moreover, we find that the dominant type of stellar collisions is qualitatively different; most collisions involve asymptotic giant branch stars, rather than main sequence, or slightly evolved stars, which dominate collisions in globular clusters.

# Chemical instability

## Open in triples? Correlated binaries I Correlated binaries II (Tokovinin) Anton Ling & Aarseth '01

Initial parameters



# Dynamical instability

How often does this happen in triples?

- ❖ 3.6% for model uncorrelated binaries I
- ❖ 2.2% for model uncorrelated binaries II (Tokovinin)
- ❖ 2.4% for model Eggleton

Stability criterion of Mardling & Aarseth '01

In good agreement with:

- ❖ Hamers et al '13
- ❖ Perets & Kratter '12 (3.5%, 5.3%) based on hybrid method without secular Kozai dynamics
  - => close encounters, collisions, stellar exchanges, eccentric binaries
  - => high collision rate, involving AGB stars

Moreover, we find that the dominant type of stellar collisions is qualitatively different; most collisions involve asymptotic giant branch stars, rather than main sequence, or slightly evolved stars, which dominate collisions in globular clusters.

we demonstrate that the rate of stellar collisions due to the TEDI is approximately  $10^{-4}$  yr $^{-1}$  per Milky-Way Galaxy, which is nearly 30 times higher than the total collision rate due to random encounters in the Galactic globular

- ❖ Lastly, at each BINARY\_C time-step the triple system is checked for dynamical stability by means of the stability criterion formulated by Mardling & Aarseth (2001), including the ad hoc inclination factor  $f = 1 - (0.3/\pi) i_{\text{tot}}$  (with  $i_{\text{tot}}$  expressed in radians). Whenever  $\beta \leq \beta_{\text{crit}}$ , where  $\beta_{\text{crit}}$  is given by this stability criterion, the STD

# Future plans...

- ❖ More testing of the importance of secular evolution on the trip population. How different do the inner binaries evolve compared to isolated binaries?
  - ❖ in particular for close inner binaries
- ❖ What is the effect of a different initial eccentricity distribution?
- ❖ Fabrycky & Tremaine '07 showed that KCTF is important for the formation of close MS binaries. How important is its role in binaries with more evolved stars?

❖ One possible consequence of the subsequent orbital cycles with tidal context of (solar mass) m 1979; Eggleton & 2007; Kisseleva-Fabrycky & Tremaine for producing close which is consistent likely (96 per cent). It remains to be seen in higher mass binaries because their cores are much less effective than their counterparts, which, on the other hand, as stars develop convective and asymptotic giant become much more effective at significant stages of the RGB/AGB phase.

# Let's start with binaries

- Population synthesis
- A new code for simulating the evolution of triples, including:
  - stellar evolution
  - regular & non-regular dynamics
- Preliminary results
  - Common evolutionary pathways, dynamical instabilities through TEDI, mass transfer in eccentric orbits etc...
- Consensus on the principles of binary evolution
  - Many questions remain, e.g. regarding mass transfer stability, unstable mass transfer (common-envelope phase), accretion