

Polarization in Interferometry

A basic introduction

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European Radio Interferometry School (2015)

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Departures			
Destination	Airline	Flight #	Arrival Time
Atlanta	WN	751	12:55 PM
Austin	WN	3505	12:56 PM
Austin	WN	117	1:45 PM
Austin	WN	2365	1:45 PM
Baltimore	US	985	1:50 PM
Boise	AS	1100	11:15 AM
Burbank	WN	8336	11:05 PM
Burbank	WN	531	2:25 PM
Chicago-Midway	WN	791	12:55 PM
Chicago-O'Hare	WN	117	1:45 PM
Dallas-Love Field	D	1000	12:05 PM
Delta-Love Field	WN	3505	12:05 PM
El Paso-Love Field	WN	317	1:45 PM
Elmira's World	AA	1184	11:50 PM
Emerson	UNITED	3	UA 3518 11:19 AM
Emerson	WN	211	12:35 PM
Endicott	Volaris	+ Y4 931	12:50 PM
Fort Worth-Holley	WN	2366	1:55 PM
Houston	WN	452	10:50 AM
Irvine	WN	453	10:50 AM
Jackson	WN	3839	12:25 PM
Vegas	WN	2366	1:55 PM
Vegas	WN	2366	1:55 PM
Los Angeles	WN	762	11:25 AM
Long Beach	AA	2654	11:55 AM
Mogadishu	UA	3511	11:56 AM
Nogales	UNITED	3	UA 3511 11:56 AM
Nogales	WN	517	1:45 PM
pe County	WN	453	11:45 AM
pe County	WN	2336	1:25 PM

Destination	Airline	Flight	Arrival	Departure	Status
Phoenix	US	985	12:00 PM	On Time	11:55 AM
Phoenix	WN	2395	1:45 PM	On Time	12:45 PM
Baltimore/Orchard	WN	2168	12:20 PM	New	12:35PM
Salt Lake City	DL	2418	1:15 PM	On Time	12:45 PM
Seattle	WN	453	10:59 AM	Boarding	10:55 AM
San Diego	WN	2639	12:25 PM	On Time	12:15 PM
San Diego	WN	1698	1:55 PM	On Time	12:45 PM
Tokyo-Narita	NH	1075	1:05 PM	On Time	12:45 PM
Tokio	WN	762	11:25 AM	On Time	12:45 PM

Arrivals			
Baltimore	WN 1443	11:30 PM	Now 11:25AM
Barbados	WN 768	1:25 PM	On Time
Chicago Midway	WN 2338	12:55 PM	Now 12:00PM
Columbus	WN 543	1:40 PM	On Time
Detroit	WN 2738	11:10 AM	On Time
Las Vegas	WN 2108	11:45 AM	Now 12:00PM
Las Vegas	WN 2055	1:10 PM	On Time
Los Angeles	AA 3484	10:45 AM	Arrived 10:46AM
Los Angeles	WN 2714	1:20 PM	On Time
Los Angeles	WN 1163	1:40 PM	On Time
Los Angeles	WN 2333	12:35 PM	Now 12:00PM
Oklahoma City	WN 2160	11:40 AM	Now 12:00PM
Omaha	WN 751	1:25 PM	On Time
Orlando	WN 1511	12:05 PM	On Time
Orange County	WN 1308	10:50 AM	Arrived
Orange County	WN 2100	1:20 PM	On Time
Phoenix	WN 1443	11:30 AM	Now 11:35AM
Phoenix	WN 751	1:25 PM	On Time
Phoenix	WN 258	1:25 PM	On Time
Portland	AA 2185	11:20 AM	Now 11:07AM
Portland	WN 3595	11:35 AM	Now 11:08AM
Raleigh/Durham	WN 533	1:45 PM	On Time
Salt Lake City	WN 517	1:15 PM	On Time
San Antonio	WN 1163	2:10 PM	On Time
San Diego	WN 2055	1:10 PM	On Time



Goals of this lecture

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- Get familiar with some basics of polarimetry.
 - ▶ The different states of polarization.
 - ▶ The Stokes parameters.



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- Understand radioastronomical polarizers.
 - ▶ Linear dipoles and quarter waveplates.
 - ▶ Polarization and interferometry: the Measurement Equation.



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- Learn the basic calibration procedures.
 - ▶ Calibration with the Measurement Equation.
 - ▶ The effects of cross delay (phase) and leakage.



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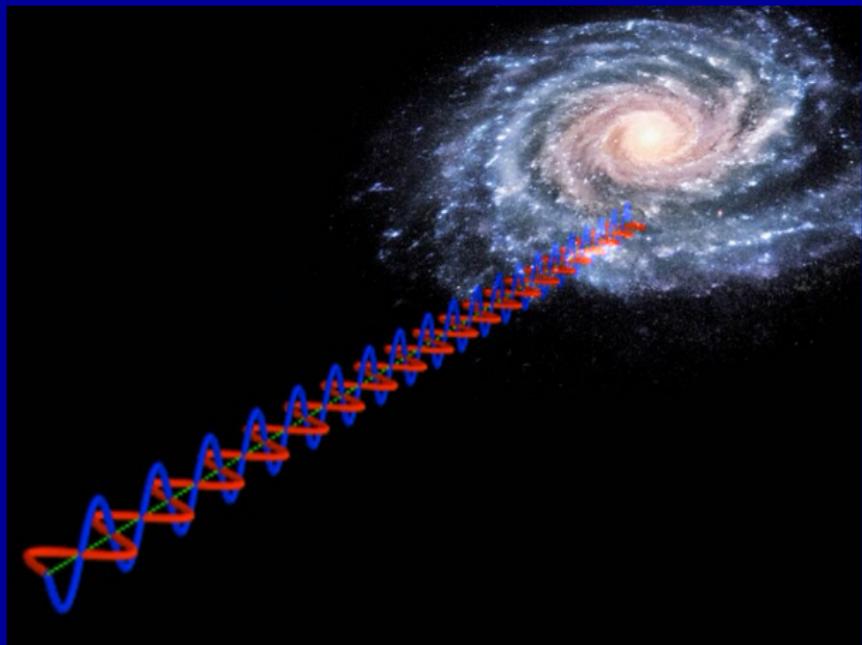
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 - ▶ Calibration with the Measurement Equation.
 - ▶ The effects of cross delay (phase) and leakage.
- Calibrate and process real observations (MERLIN C band).



Polarization of light



Electromagnetic waves



Electromagnetic waves

A random orientation of \vec{E}

\vec{E} as seen on the wave-front plane

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Polarization modes

- LINEAR
- CIRCULAR
- ELLIPTIC (i.e., LINEAR + CIRCULAR)



The Stokes parameters

The Stokes parameters

- We need **four** quantities to fully describe the polarization state:



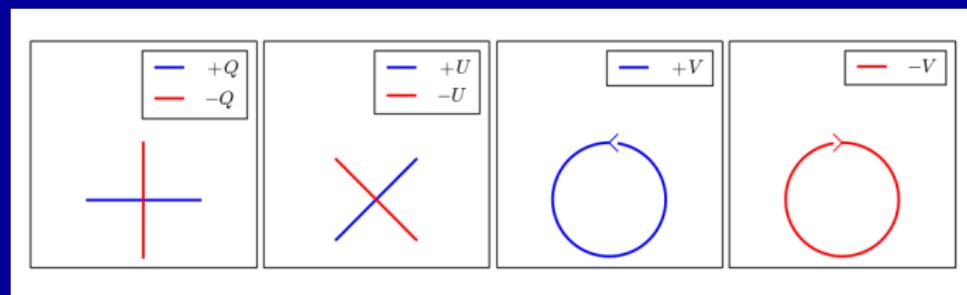
The Stokes parameters

- We need four quantities to fully describe the polarization state:
 - ▶ How much polarized vs. unpolarized light do we have?
 - ▶ What is the strength and direction of the linear polarization?
 - ▶ How much circular polarization do we have?



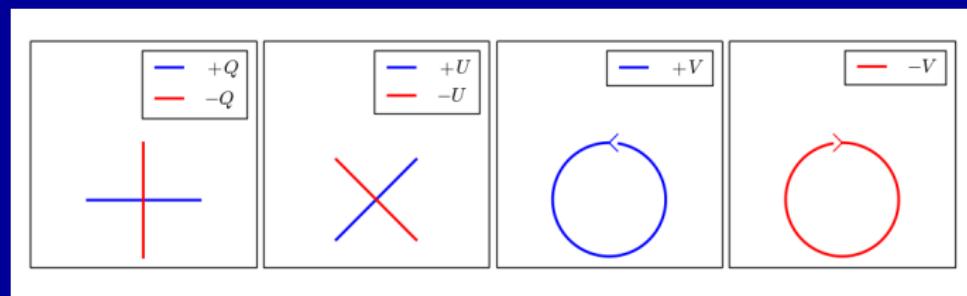
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- The Stokes parameters: I , Q , U , and V



The Stokes parameters

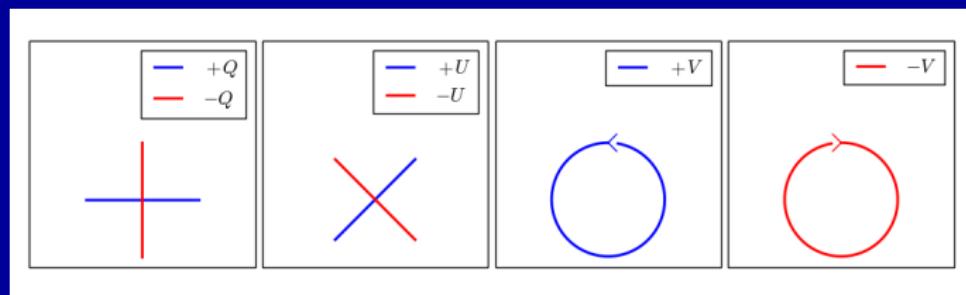
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- Linear polarization: $I_p = \frac{\sqrt{Q^2 + U^2}}{I}$, $\theta = \frac{1}{2} \arctan \left(\frac{U}{Q} \right)$

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 - ▶ How much polarized vs. unpolarized light do we have?
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- Linear polarization: $I_p = \frac{\sqrt{Q^2 + U^2}}{I}$, $\theta = \frac{1}{2} \arctan \left(\frac{U}{Q} \right)$
- Unpolarized intensity: $I_u = \sqrt{I^2 - Q^2 - U^2 - V^2}$



Polarizers in Radio Astronomy



Detecting source polarization

- The Stokes parameters describe the polarization state of light.
But how do we measure them?



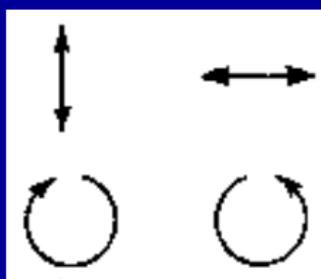
Detecting source polarization

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- Polarizing receivers (polarizers). The signal is split coherently into two orthogonal polarization states.



Detecting source polarization

- The Stokes parameters describe the polarization state of light.
But how do we measure them?
- Polarizing receivers (polarizers). The signal is split coherently into two orthogonal polarization states.
 - ▶ Linear polarizers (horizontal / vertical linear polarization).
 - ▶ Circular polarizers (left / right circular polarization).



Linear polarizers

Decomposing linear pol. with linear polarizers (no phase offset)



Linear polarizers

Decomposing **circular** pol. (left) with **linear polarizers** (90° offset)



Linear polarizers

Decomposing **circular** pol. (right) with **linear** polarizers (270° offset)



Linear polarizers

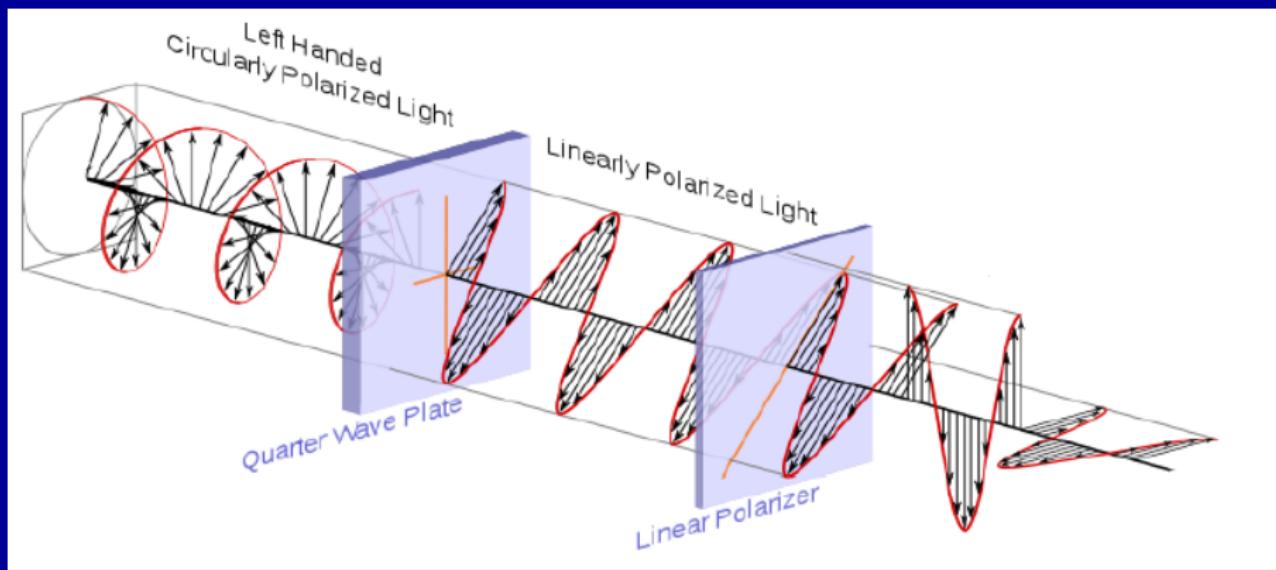
Decomposing **elliptical** pol. (right) with **linear** polarizers (generic phase offset)



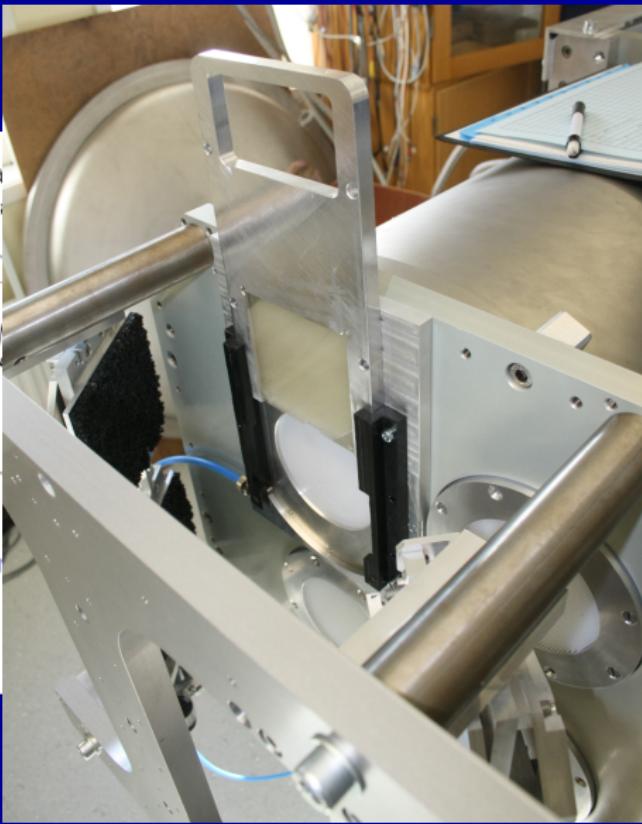
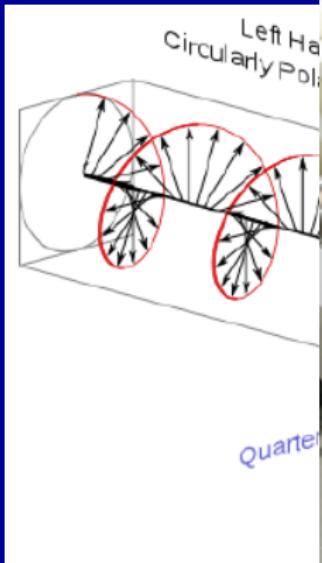
Linear polarizers

- $I = |E_x|^2 + |E_y|^2$
- $Q = |E_x|^2 - |E_y|^2$
- $U = 2 \operatorname{Re}(E_x E_y^*)$
- $V = 2 \operatorname{Im}(E_x E_y^*)$

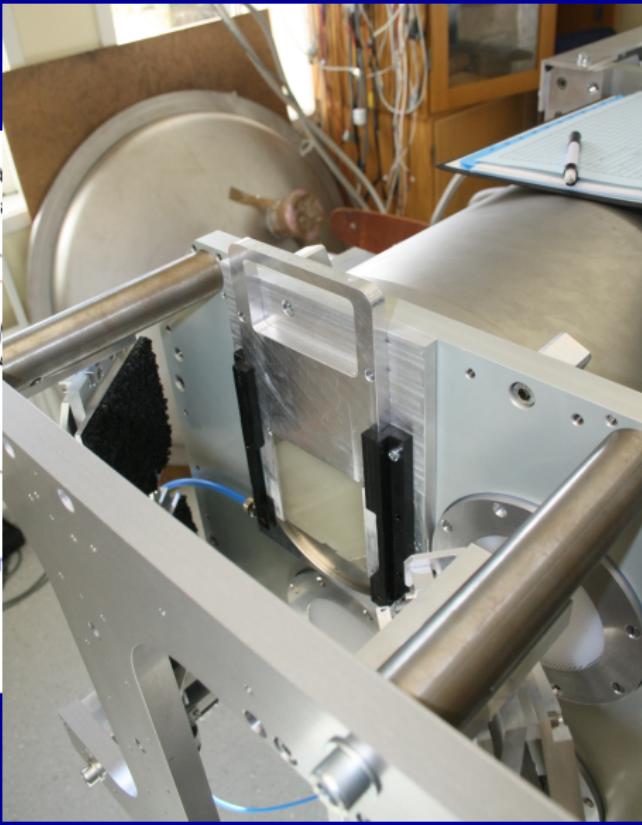
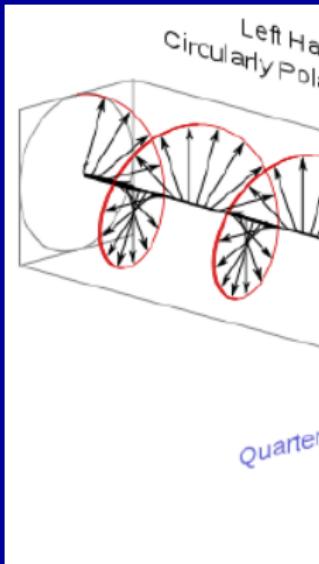
Circular polarizers



Circular polarizers



Circular polarizers



Circular polarizers

Decomposing linear pol. with circular polarizers (phase offset gives inclination)



Circular polarizers

Decomposing **elliptical** pol. with **circular** polarizers (R/L amplitude difference)



Circular polarizers

- $I = |E_I|^2 + |E_r|^2$
- $V = |E_I|^2 - |E_r|^2$
- $Q = 2 \operatorname{Re}(E_I^* E_r)$
- $U = -2 \operatorname{Im}(E_I^* E_r)$



Advanced formulation: The Measurement Equation



The Measurement Equation. Coherency matrix

- Electric field seen by antenna A : \vec{E}^A .



The Measurement Equation. Coherency matrix

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- For baseline AB , the coherency matrix is $E^{AB} = \vec{E}^A(\vec{E}^B)^H$



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- In the x-y polarization basis, the coherency matrix for baseline AB is:

$$E^{AB} = \begin{pmatrix} \left\langle E_x^A (E_x^B)^* \right\rangle & \left\langle E_x^A (E_y^B)^* \right\rangle \\ \left\langle E_y^A (E_x^B)^* \right\rangle & \left\langle E_y^A (E_y^B)^* \right\rangle \end{pmatrix}$$



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- We also define the brightness matrix, B . For x-y polarizers, it is

$$B = \begin{pmatrix} I + Q & U + j V \\ U - j V & I - Q \end{pmatrix}$$



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- The coherency matrix is related to the Fourier transform of the brightness matrix!

$$E^{AB} = \mathcal{F}[B]|_{(u,v)}$$



Coherency matrix and Visibility matrix.

- Voltage for antenna A with an x-y polarizer is: $\vec{v}^A = J^A \vec{E}^A$, where \vec{E}^A is the electric field in the x-y base and J^A is the Jones matrix that calibrates antenna A .



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- The visibility matrix (i.e., voltage cross-correlations) is:

$$V^{AB} = \vec{v}_A (\vec{v}_B)^H = \begin{pmatrix} \left\langle v_x^A (v_x^B)^* \right\rangle & \left\langle v_x^A (v_y^B)^* \right\rangle \\ \left\langle v_y^A (v_x^B)^* \right\rangle & \left\langle v_y^A (v_y^B)^* \right\rangle \end{pmatrix}$$



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- Voltage for antenna A with an x-y polarizer is: $\vec{v}^A = J^A \vec{E}^A$, where \vec{E}^A is the electric field in the x-y base and J^A is the Jones matrix that calibrates antenna A .
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- Since $\vec{v}_i = J_i \vec{E}_i$,

$$V^{AB} = J_A \vec{E}_A \left(\vec{E}_B \right)^H J_B^H = J_A \begin{pmatrix} \langle E_x^A (E_x^B)^* \rangle & \langle E_x^A (E_y^B)^* \rangle \\ \langle E_y^A (E_x^B)^* \rangle & \langle E_y^A (E_y^B)^* \rangle \end{pmatrix} J_B^H$$

The Measurement Equation. A full Stokes formalism

For a source with a generic structure, the visibility matrix for antennas A and B (with no direction-dependent calibration) will be

$$V_{obs}^{AB} = J_A \left[\int_{\alpha, \delta} B e^{-\frac{2\pi j}{z}(u\alpha + v\delta)} \frac{d\alpha d\delta}{z} \right] (J_B)^H,$$

where (α, δ) are the (normalized) sky coordinates in the source plane, and $z = \sqrt{1 - \alpha^2 - \delta^2}$.



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where (α, δ) are the (normalized) sky coordinates in the source plane, and $z = \sqrt{1 - \alpha^2 - \delta^2}$.

Let us remember the classical interferometer equation:

$$V_{obs}^{AB} = G_A G_B^* \int_{\alpha, \delta} I(\alpha, \delta) e^{-\frac{2\pi j}{\lambda}(u\alpha + v\delta)} \frac{d\alpha d\delta}{z}$$



Jones calibration matrices. Examples

- Gain, $G = \begin{pmatrix} A_x(t) e^{j\phi_x(t)} & 0 \\ 0 & A_y(t) e^{j\phi_y(t)} \end{pmatrix}$
- Delay, $K = \begin{pmatrix} e^{j\tau_x(\nu - \nu_0)} & 0 \\ 0 & e^{j\tau_y(\nu - \nu_0)} \end{pmatrix}$
- Bandpass, $B = \begin{pmatrix} A_x(\nu) e^{j\phi_x(\nu)} & 0 \\ 0 & A_y(\nu) e^{j\phi_y(\nu)} \end{pmatrix}$

The Jones matrices are multiplicative, e.g.: $J = G \times B \times K$, but care must be taken, since matrices generally do not commute.



Polarization calibration



Polarization calibration

- Parallactic angle.
- Polarization leakage.
- Cross-Delay/phase.
- Amplitude offset.



Polarization calibration I. Parallactic angle

$$P_{xy} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \quad P_{rl} = \begin{pmatrix} e^{j\phi} & 0 \\ 0 & e^{-j\phi} \end{pmatrix}$$

- Is the rotation of the antenna mount axis w.r.t. the sky.
- Is deterministic. It's good to apply it before the phase (and delay/rate) calibration.
- It does not commute with the gains for linear polarizers.
- In VLBI, it also mixes V_{xx} and V_{yy} with V_{xy} and V_{yx} .



Polarization calibration II. Leakage

$$D_{xy} = \begin{pmatrix} 1 & D_x(\nu) \\ D_y(\nu) & 1 \end{pmatrix}$$

- Is caused by cross-talking between the polarizer channels
- Each leaked signal is modified by an amplitude and a phase.
- Introduces spurious ellipticity and linear polarization.

LIN. + LEAK

CIRC. + LEAK



Polarization calibration III. Cross-hand delay/phase

$$K_c = \begin{pmatrix} 1 & 0 \\ 0 & e^{j(\tau_c(\nu - \nu_0) + \phi_c)} \end{pmatrix}$$

- Is caused by a delay between the polarizer channels at the reference antenna.
- In linear polarizers, introduces ellipticity.
- In circular polarizers, just rotates the PA of the linear polarization.

OFFSET: 0°

OFFSET: 45°

LINEAR:

CIRCULAR:



Polarization calibration IV. Amplitude bias

$$G_a = \begin{pmatrix} 1 & 0 \\ 0 & A_c \end{pmatrix}$$

- Is caused by different T_{sys} , gain and/or bandpass between polarizer channels.
- In linear polarizers, introduces spurious linear polarization.
- In circular polarizers, introduces ellipticity.
- Not quite used *per se*, but implicit in the gain calibration.



Calibration strategy

The right order for matrix product is: $J = (G_a K_c) \times D \times P$
i.e.: $V^{cal} = P^{-1} \times D^{-1} \times (G_a K_c)^{-1} \times V^{obs}$

- STEP 1 (optional): Calibrate the cross-delay using a strong polarized source.



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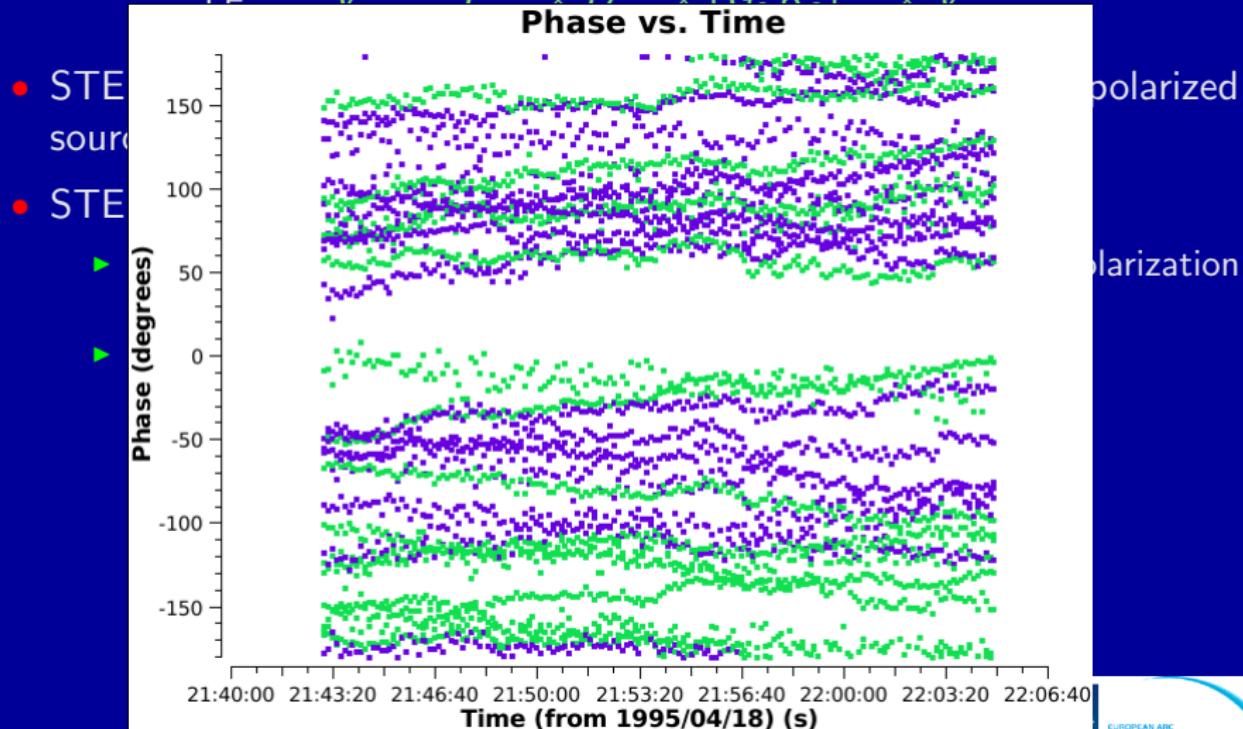
- STEP 1 (optional): Calibrate the cross-delay using a strong polarized source.
- STEP 2: Calibrate the leakage using an unpolarized source.
 - ▶ If all calibrators are polarized, solve for leakage and source polarization simultaneously.
 - ▶ Need good parallactic-angle coverage.



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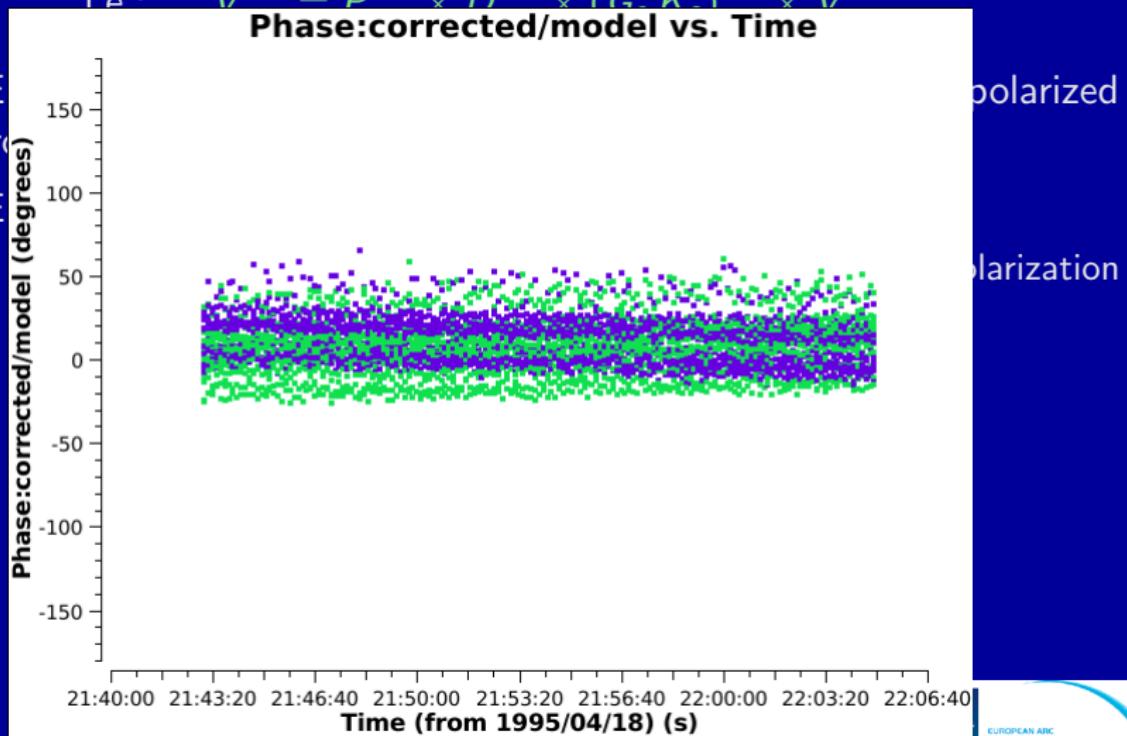
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Phase:corrected/model vs. Time

- STE source
- STE source



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- STEP 3 (optional): Refine the calibration (calibration “cross-talk”)



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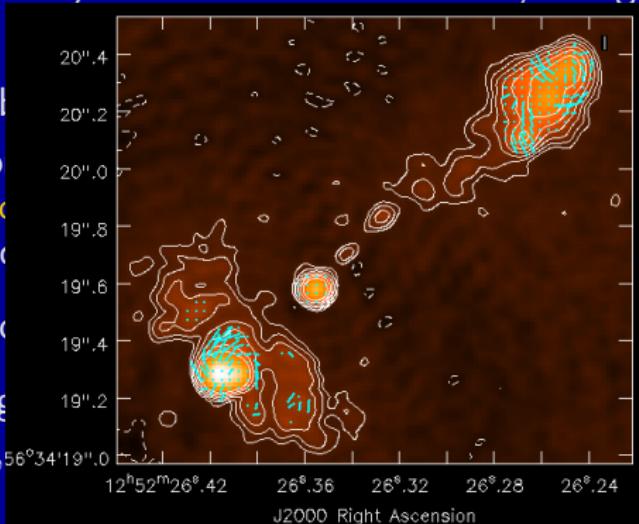
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 - ▶ Need good parallactic-angle coverage.
- STEP 3 (optional): Refine the calibration (calibration “cross-talk”)
- STEP 4: Image each Stokes parameter separately. Combine images:
 $(Q, U) \rightarrow (I_p, \theta)$.



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- STEP 1 (optional): Calibrate the cross-delay using a strong polarized source.
- STEP 2: Calibration (if needed)
 - ▶ If all calibration sources are simultaneous
 - ▶ Need good visibility coverage
- STEP 3 (optional): Check polarization on "cross-talk")
- STEP 4: Imaging (Q, U) $\rightarrow (I_p, Q, U)$



I source.
source polarization
on "cross-talk")
Combine images:

SUMMARY

- We have reviewed basic concepts of polarization.
 - ▶ Modes of polarization.
 - ▶ Stokes parameters.
- We have discussed about the different kinds of polarizers in radioastronomical receivers.
 - ▶ Linear polarizers (X-Y).
 - ▶ Circular polarizers (R-L).
- We have studied how to deal with polarization in interferometric observations.
 - ▶ The Measurement Equation.
 - ▶ The matrices for polarization calibration.
 - ▶ Calibration effects on X-Y vs. R-L polarizers.
 - ▶ Overview of calibration procedure.



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THANKS



TUTORIAL



Full-pol calibration – MERLIN C-band – 3C277.1

- Load the data into CASA. Inspect visibilities.
- Flag bad data.
- Calibrator phases and bandpass. Inspect solutions.
- Set primary flux calibrator. Inspect model.
- Calibrator amplitudes. Bootstrap flux-density calibration.



Full-pol calibration – MERLIN C-band – 3C277.1

- Calibrate phases (long time average for phase referencing).
- Calibrate leakage from phase calibrator. Understand solutions.
- Calibrate R/L phase offset using pol. calibrator.



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$$\blacktriangleright \quad V^{obs} = D_a X V_{ab}^{true} X^H D_b^H \quad ; \quad X = \begin{pmatrix} 1 & 0 \\ 0 & e^{j\alpha} \end{pmatrix} \quad ; \quad D_a = \begin{pmatrix} 1 & D_a^L \\ D_a^R & 1 \end{pmatrix}$$

$$V_{RL}^{obs} = ((D_a^R + (D_b^L)^*) I + P) e^{-j\alpha} + O(D^2)$$

$$V_{LR}^{obs} = ((D_a^L + (D_b^R)^*) I + P^*) e^{j\alpha} + O(D^2)$$



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- Calibrate phases (long time average for phase referencing).
- Calibrate leakage from phase calibrator. Understand solutions.
- Calibrate R/L phase offset using pol. calibrator.

► $V^{obs} = D_a X V_{ab}^{true} X^H D_b^H \quad ; \quad X = \begin{pmatrix} 1 & 0 \\ 0 & e^{j\alpha} \end{pmatrix} \quad ; \quad D_a = \begin{pmatrix} 1 & D_a^L \\ D_a^R & 1 \end{pmatrix}$

$$V_{RL}^{obs} = ((D_a^R + (D_b^L)^*) I + P) e^{-j\alpha} + O(D^2)$$

$$V_{LR}^{obs} = ((D_a^L + (D_b^R)^*) I + P^*) e^{j\alpha} + O(D^2)$$

- **Unpolarized** calibrator:

$$(D_a^L, D_a^R, e^{j\alpha}) \rightarrow (D_a^L e^{j\Delta} + jK, D_a^R e^{-j\Delta} + jK, e^{j(\alpha - \Delta)})$$

- **Polarized** calibrator:

$$(D_a^L, D_a^R, e^{j\alpha}) \rightarrow (D_a^L + jK, D_a^R + jK, e^{j(\alpha)})$$



Full-pol calibration – MERLIN C-band – 3C277.1

- Split the target data.
- CLEAN stokes I of target. Self-calibrate phase.
- CLEAN stokes I of target. Self-calibrate amplitude.
- CLEAN all stokes parameters of target.
- Construct the polarization image.
- Sanity check: CLEAN the polarization calibrator.

