Modelling Galaxies

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Prologue

- Work on surveys of the MW has led to significant advances in model Galaxies
- These advances can now be applied to external galaxies
- Made possible by the advent of techniques for evaluating action-angle coordinates in arbitrary galactic potentials (Binney 10, 12, Sanders 12, Sanders&B 14a, 14b)
- Crucially, we can now assemble dynamical models component by component (incl DM and BH components)
- For now only axisymmetric equilibrium models
- Extension to triaxial & time-dependent models lies ahead (but see S&B14b)

Outline

- Why equilibrium models
- Review of model types
- Models with specified f(J)
- Designer f(J) for spheroids & DM
- f(J) for discs
- Worked example: the MW
- Conclusions

Equilibrium models

- Essential because
  - We want distribution of mass (M/L, ρ_{DM}(x), M_{BH},..) & hope to infer these from dynamics of stars
  - Without the assumption of dynamical equilibrium, any phase-space distribution is possible
  - Information limited [in external galaxies (α, δ, v_{los})] and we hope to complement this by using constraints from dynamics
  - Best way to compensate for defects in data (seeing, spectral resolution, selection effects) is to "observe" a model
- E galaxies quickly settle to equilibrium
- Spiral structure in disc galaxies we plan to model by applying perturbation theory to an equilibrium model
- Use of p-theory brings physical understanding

Model types

- N-body
  - Final model determined by initial conditions through an obscure mapping
- Hence M2M (Syer & Tremaine 96, de Lorenzi + 08, Dehnen 09, Morganti + 13)
  - Adjust weights of particles dynamically to optimise fit to obs
- Schwarzschild modelling (Schwarzschild 79)
  - Build a library of orbits and then choose weights to optimise fit to obs
- Problems with all of above:
  - Discreteness noise (eg McMillan & B 13)
  - Hard to characterise model (non-uniqueness of description)
  - Computational cost

Models with specified DF

- Models with f(E,L) (Michie 63, King 66) fundamental to studies of GCs
  - Can quickly recover ρ(r), σ_r(r), etc from f(E,L)
- Most galaxies are non-spherical
  - Models with f(E,L_z) studied since Prendergast & Tomer 70
  - Not easy to recover ρ(R,z) from f(E,L_z)
  - Relation of observables to f(E,L_z) obscure
  - Models require $\sigma_R = \sigma_z$
- Models from Jeans eqs (Satoh 80, Binney+90) also restricted to $\sigma_R = \sigma_z$

Models with specified f(J)

- Most orbits in axisymmetric Φ are quasiperiodic $\Rightarrow$ admit three integrals of motion (Arnold 78)
  - Integrals best chosen to be actions J_i
- In spherical case
  - J_r quantifies radial oscillations
  - $J_\phi = L_z$
  - $J_\theta = L-L_z$ quantifies motion $\perp$ z=0
- These choices generalise uniquely to axisymmetric case: various notations for unique objects
  - $J_u = J_r$
  - $J_v = J_\theta = J_z$ quantifies motion $\perp$ equatorial plane
- $J = 0 \Rightarrow$ star at rest at centre
  - $J \rightarrow \infty \Rightarrow$ star becomes unbound
  - Any finite J with $J_r>0$, $J_z>0$ corresponds to a bound orbit

Models with specified f(J)

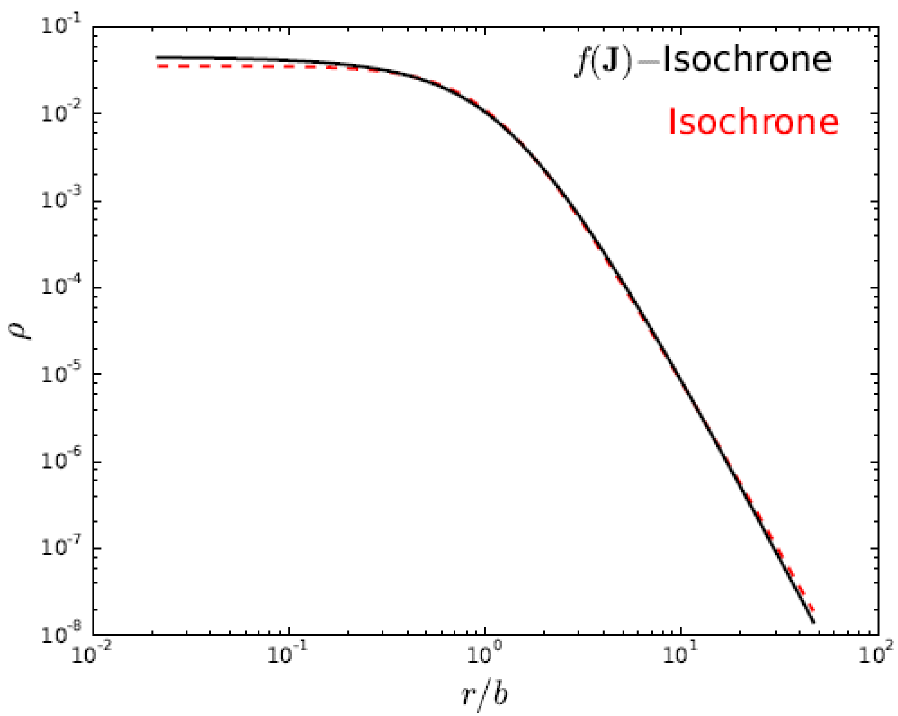
- Any non-negative f(J) with $\int d^3J\, f(J) < \infty$ defines a legitimate galaxy model
- $M = (2\pi)^3 \int d^3J\, f(J)$ is the mass
- In any Φ(R,z) we can evaluate ρ(R,z), σ_R(R,z), …
- Given 2 DFs $f_1(J)$, $f_2(J)$ with $M_i = (2\pi)^2 \int d^3J\, f_i(J)$ can build a model with 2 stellar populations cohabiting the same Φ
- So we seek $f_i(J)$ for i = 1,…,N stellar populations and $f_{DM}(J)$ for DM
- The self-consistent Φ(R,z) is easily found by rapidly convergent iteration (Binney14)

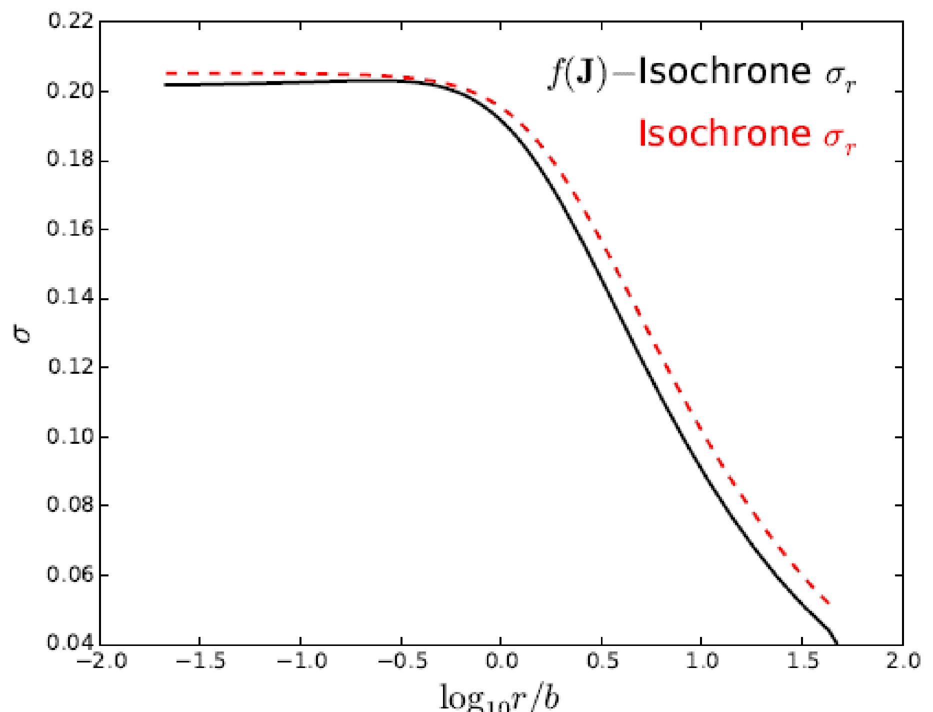
Designing f(J)

- f(J) is density of stars in 3d action space ($d^3x\, d^3v = d^3\theta\, d^3J$), so form of f(J) is readily pictured
- Start in spherical limit when Eddington gives us f(H) that generates given ρ(r) in given Φ(r)
  - In 1 special case (Henon's isochrone) we have H(J) so can immediate convert to f[H(J)]
    - This case explored by Binney 14
  - Generally don't know H(J) but good approximations not hard to find (Fermani 13, Williams+14)
  - When $\Phi(\xi x) = \xi^a(x)$ orbits can be rescaled $(x,v) \rightarrow (x',v')$ with $x' = \xi x$, $v' = \xi^{a/2}v$ by the virial thm
    - It follows that J ~ xv scales $J \rightarrow J' = \xi^{1+a/2}J$
    - Hence $H(J) = [h(J)]^{a/(1+a/2)}$ where $h(\zeta J) = \zeta h(J)$ is homogeneous of degree one
    - If self-consistent, we require $\rho \sim r^{a-2}$ and then $f(J) \sim [h(J)]^{-(4+a)/(2+a)]}$
  - Introduce core by writing $f(J) \sim [J_0 + h(J)]^{-(4+a)/(2+a)]}$

Example isochrone

- Has Kepler Φ as r $\rightarrow \infty$ and Eddington f(H)~$H^{-5/2}$
- Suggests f(J) = $A[J_0+h(J)]^{-5}$
- We choose h(J) = $J_r + (\Omega_\phi/\Omega_r)J_\phi + (\Omega_z/\Omega_r)J_z$ which would be homogeneous if frequency ratios were invariant under J $\rightarrow$ ζJ (~ energy-independent)
- Even with h(J) inhomogeneous, we have a valid DF

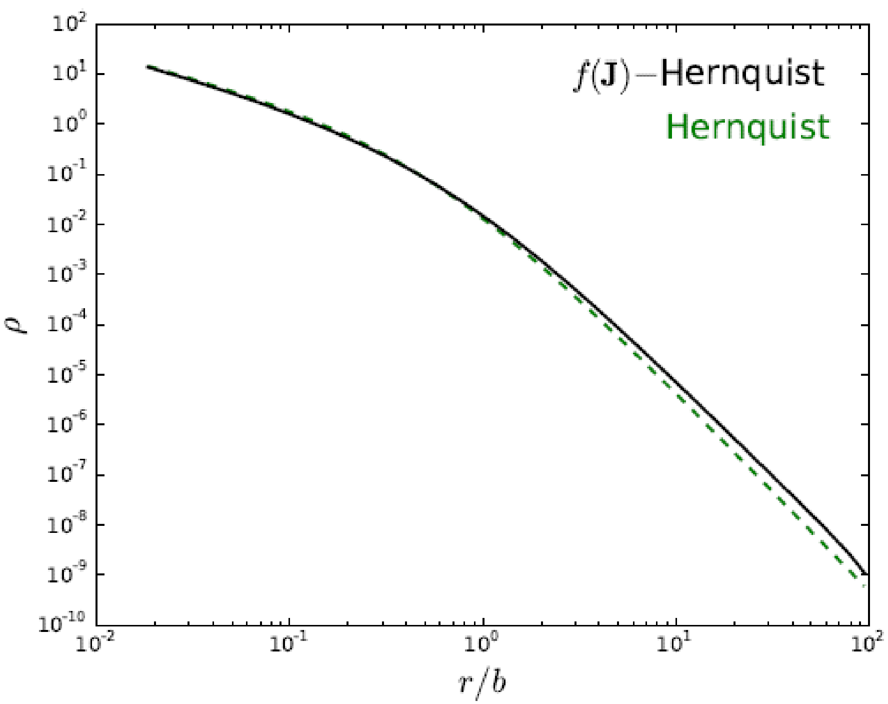


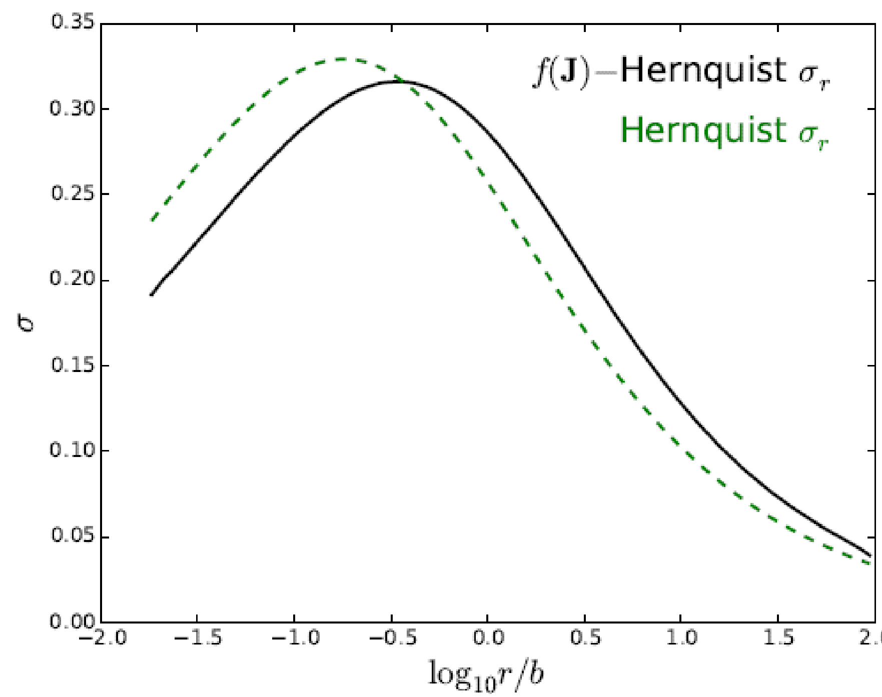


Designing f(J) – 2 power models

- Many popular models (Jaffe, Hernquist, NFW,..) are 2-power models
- Apply reasoning above to regions r $\rightarrow$ 0 (α=1, β=-1) and r$\rightarrow \infty$
- In each region Φ(r) ~ r$^{\alpha}$ ρ(r) ~ r$^{\beta}$ implies by Eddington f ~ E$^{(\beta/\alpha\,-\,3/2)}$ and therefore
- f(J) ~ [h(J)]$^{-(3\alpha\,-\,2\beta\,)/(\alpha\,+2)}$
- Example: Hernquist model

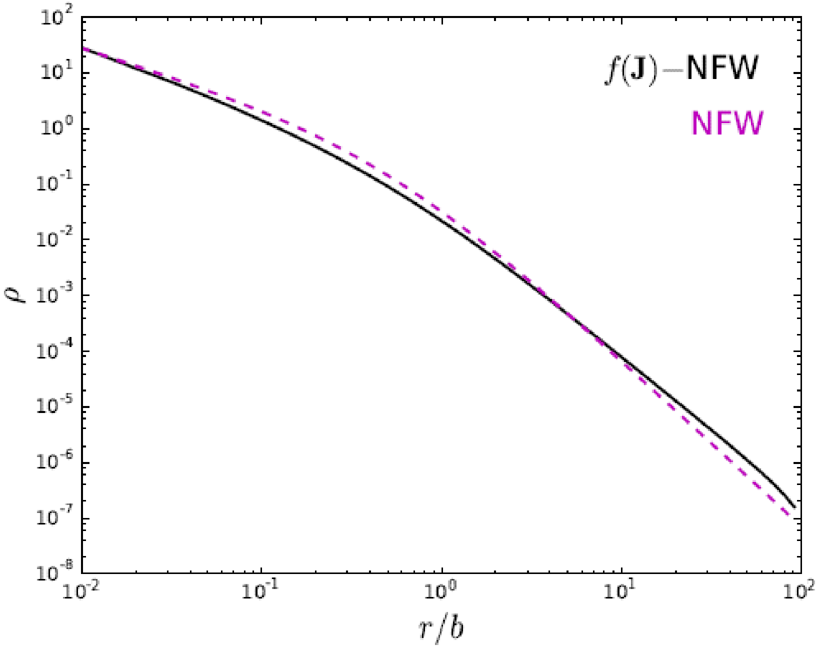
$$f(\mathbf{J}) = \frac{A}{[h(\mathbf{J})]^{5/3}\,[J_0 + h(\mathbf{J})]^{5-5/3}}$$

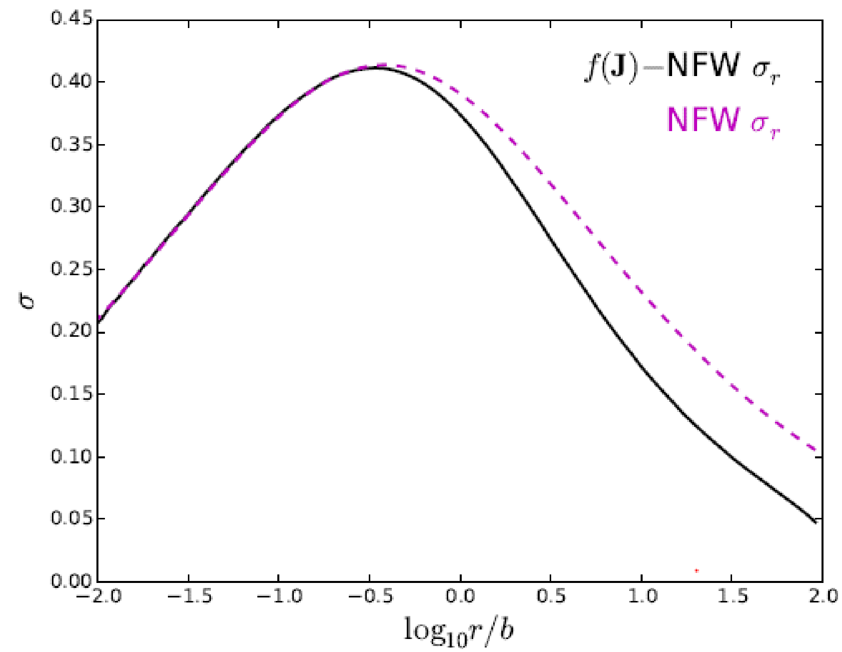




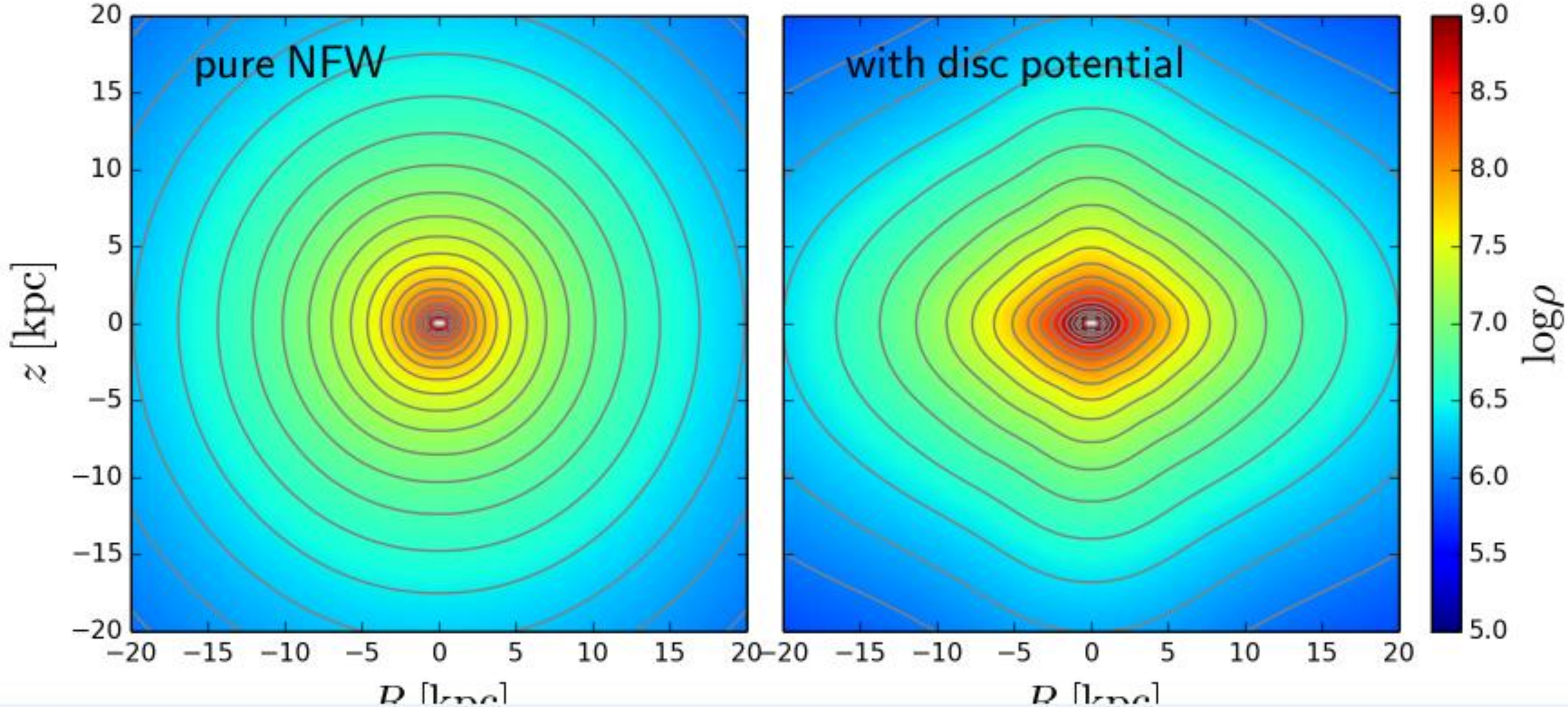
f(J) for DM (NFW)

- $$f(\mathbf{J}) = \frac{A}{[h(\mathbf{J})]^{5/3}\,[J_0 + h(\mathbf{J})]^{3-5/3}} - F_{\mathrm{M}}$$ truncation



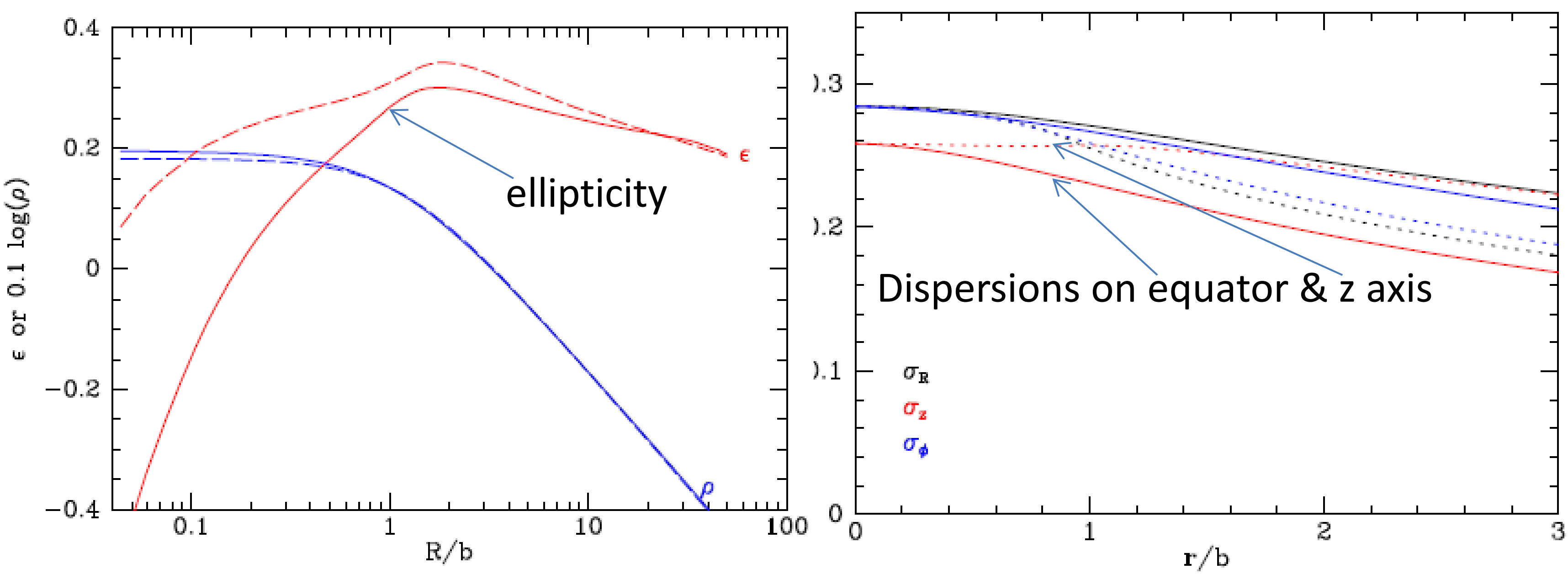


Flattening a DM halo with a disc



Velocity anisotropy

- Isotropic model has f[H(J)] so $\frac{\partial f/\partial J_r}{\partial f/\partial J_\phi} = \frac{\partial H/\partial J_r}{\partial H/\partial J_\phi} = \frac{\Omega_r}{\Omega_\phi}$
- Using $h(\mathbf{J}) = J_r + \frac{\Omega_\phi}{\Omega_r} J_\phi + \frac{\Omega_z}{\Omega_r} J_z$ ensures near isotropy because Ω_i/Ω_j ~ const
- Ω_i/Ω_j depends mostly on E, i.e. |J|
- Anisotropic mode has $f_{\boldsymbol{\alpha}}(\mathbf{J}) \equiv (\alpha_r \alpha_\phi \alpha_z) f_{\mathrm{I}}(\alpha_r J_r, \alpha_\phi J_\phi, \alpha_z J_z)$ with α_i != 1
- If $\alpha_z > \alpha_\phi$ the model becomes oblate



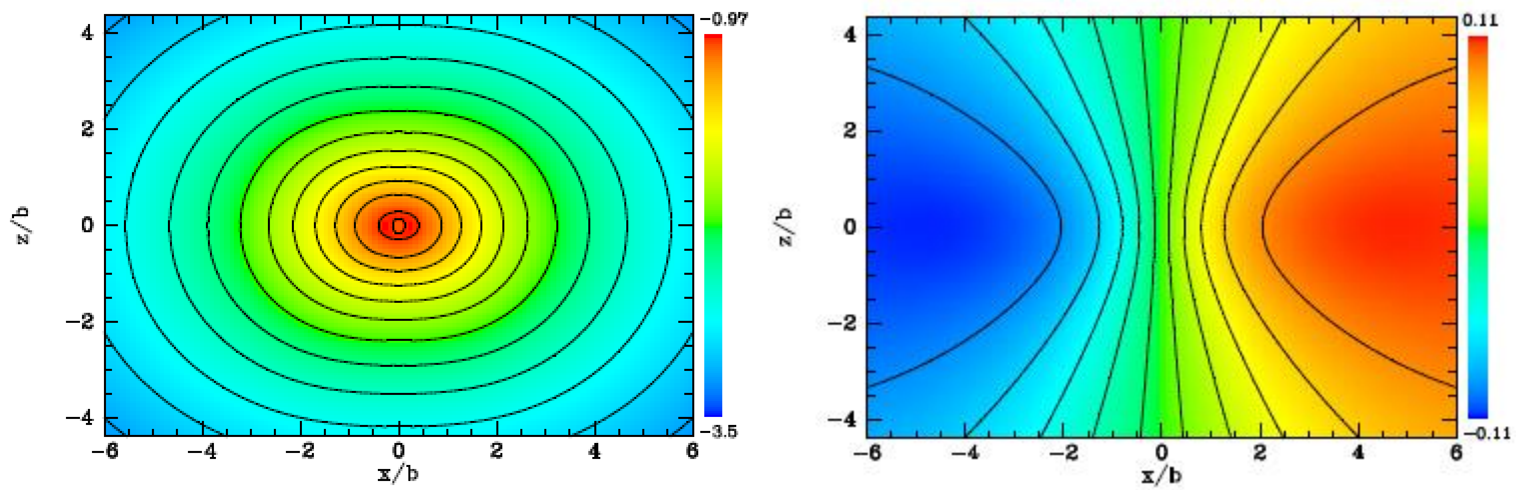
Isochrone model with α_ϕ=1, α_z = 1.5 (Binney 2014)

Rotation

- So far models non-rotating; flattened by velocity anisotropy
- Generate rotation by adding to DF part odd in J_ϕ

$$f(\mathbf{J}) = (1-k)f_+(\mathbf{J}) + kf_-(\mathbf{J})$$

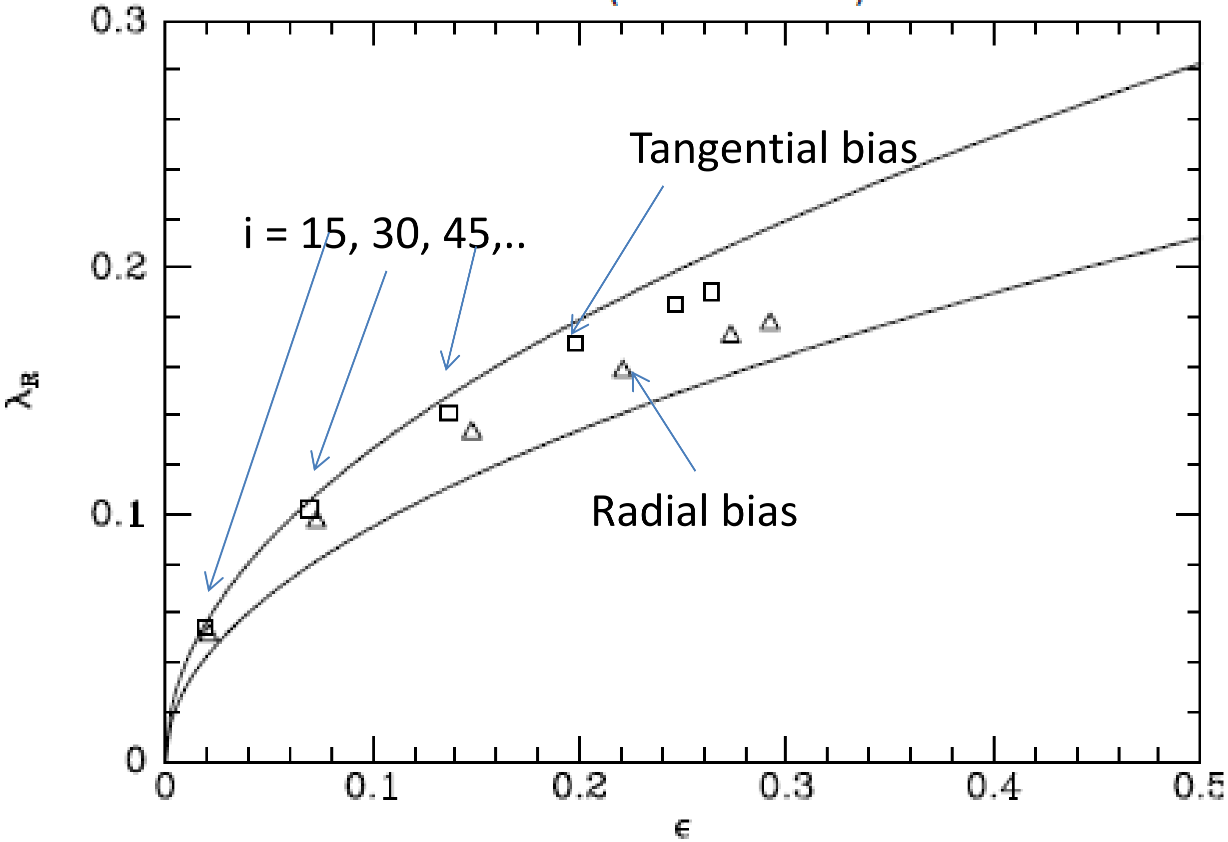
- A natural choice is $f_-(\mathbf{J}) = g(J_\phi)f_+(\mathbf{J})$
- We adopt $g(J_\phi) = \tanh\left(\frac{\chi J_\phi}{\sqrt{GMb}}\right)$



Projected density & mean velocity of maximally rotating (k=0.5) isochrone α_ϕ=0.7, α_z=1.4 (Binney 2014)

Maximal rotation (k=0.5)

- Emsellem+ 07 defined $\lambda_{\mathrm{R}} = \frac{\langle R\overline{v}\rangle}{\left\langle R\sqrt{\sigma^2+\overline{v}^2}\right\rangle}$



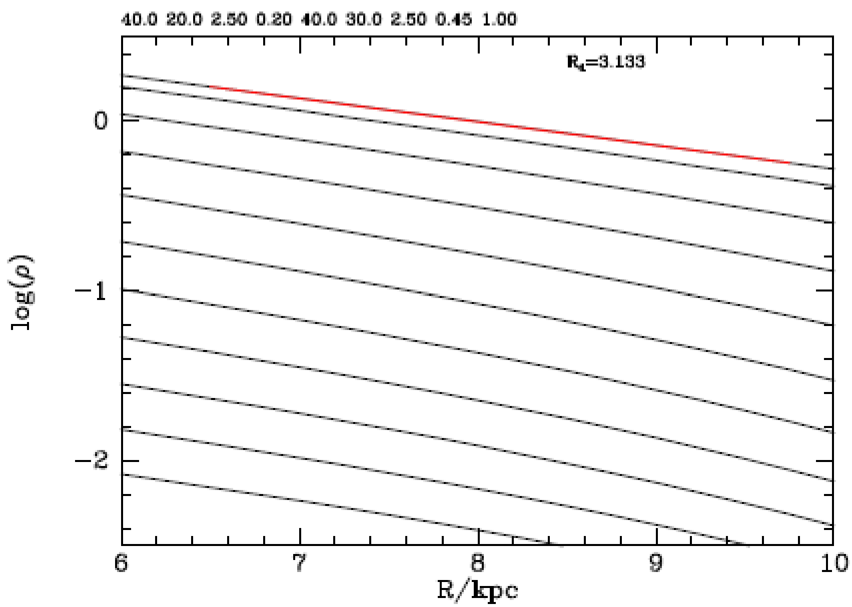
Galactic discs

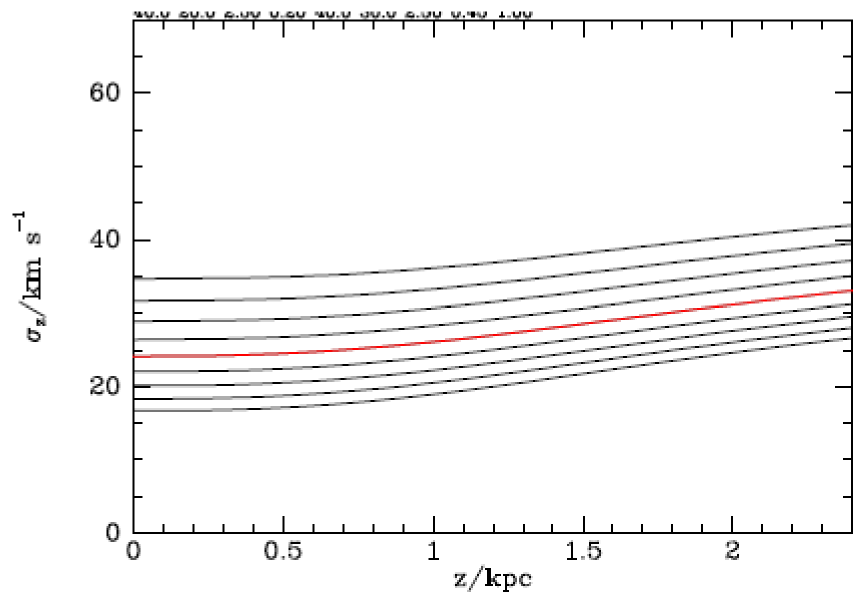
- Built up from “quasi isothermal” components (B10 B12)

$$f_{\sigma_{0r},\sigma_{0z}}(L_z,J_r,J_z)=\left.\frac{\Omega\Sigma}{\pi\sigma_r^2\kappa}\right|_{R_c}[1+\tanh(L_z/L_0)]\mathrm{e}^{-\kappa J_r/\sigma_r^2}$$
$$\times\frac{\nu}{2\pi\sigma_z^2}\mathrm{e}^{-\nu J_z/\sigma_z^2}.$$

$$\Sigma(L_z)=\Sigma_0\mathrm{e}^{-R_c/R_d}$$

$$\sigma_i(R_c)=\sigma_{0i}\,\mathrm{e}^{(R_0-R_c)/R_\sigma}$$





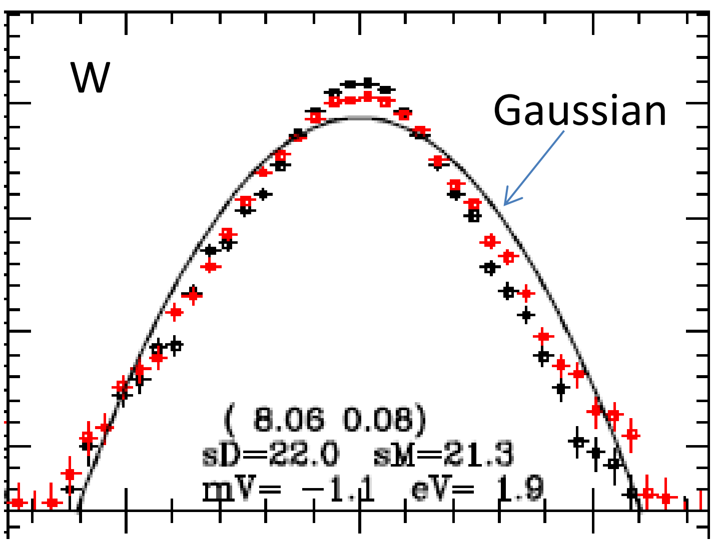
Quasi isothermal σ_{r0}=40, σ_{z0}=20 km/s (Binney 12)

Model from Geneva-Copenhagen survey (Binney 12, Binney+14)

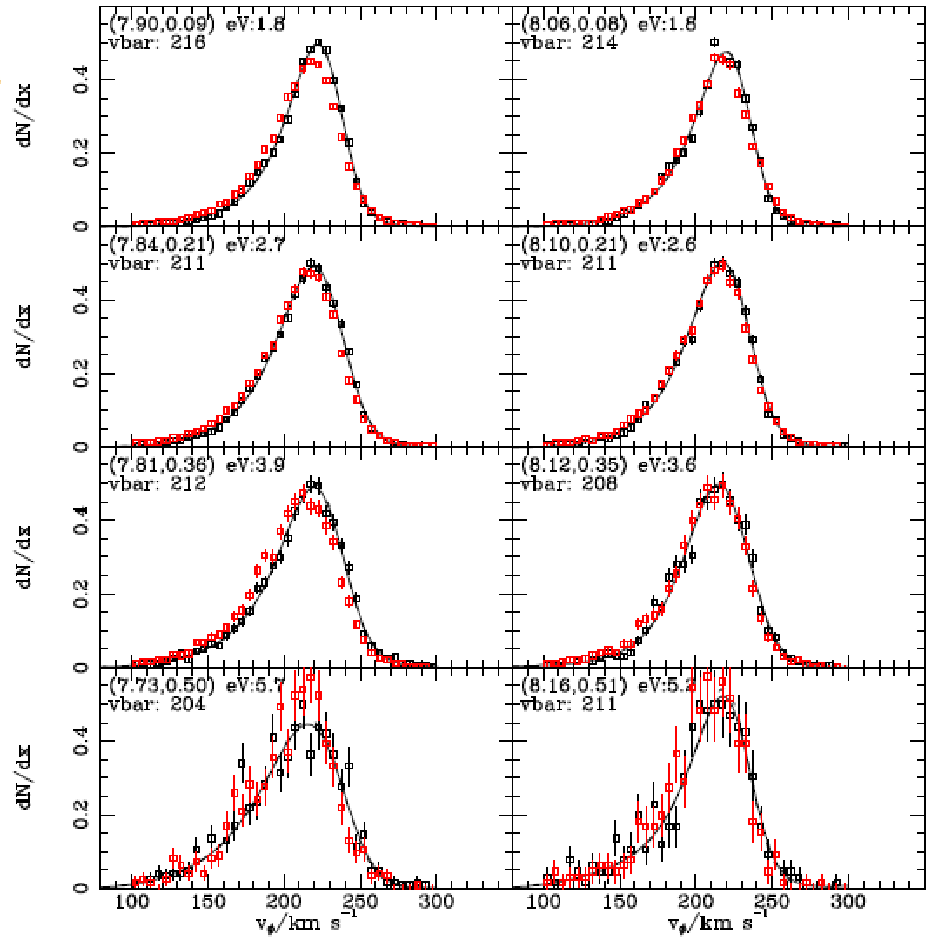
- Thick disc a single quasi-isothermal
- Each cohort of coeval stars age τ a quasi-isothermal

$$\sigma_{0i}(\tau) = \left(\frac{\tau + \tau_1}{\tau + \tau_T} \right)^{\beta_i} \sigma_{0i}$$

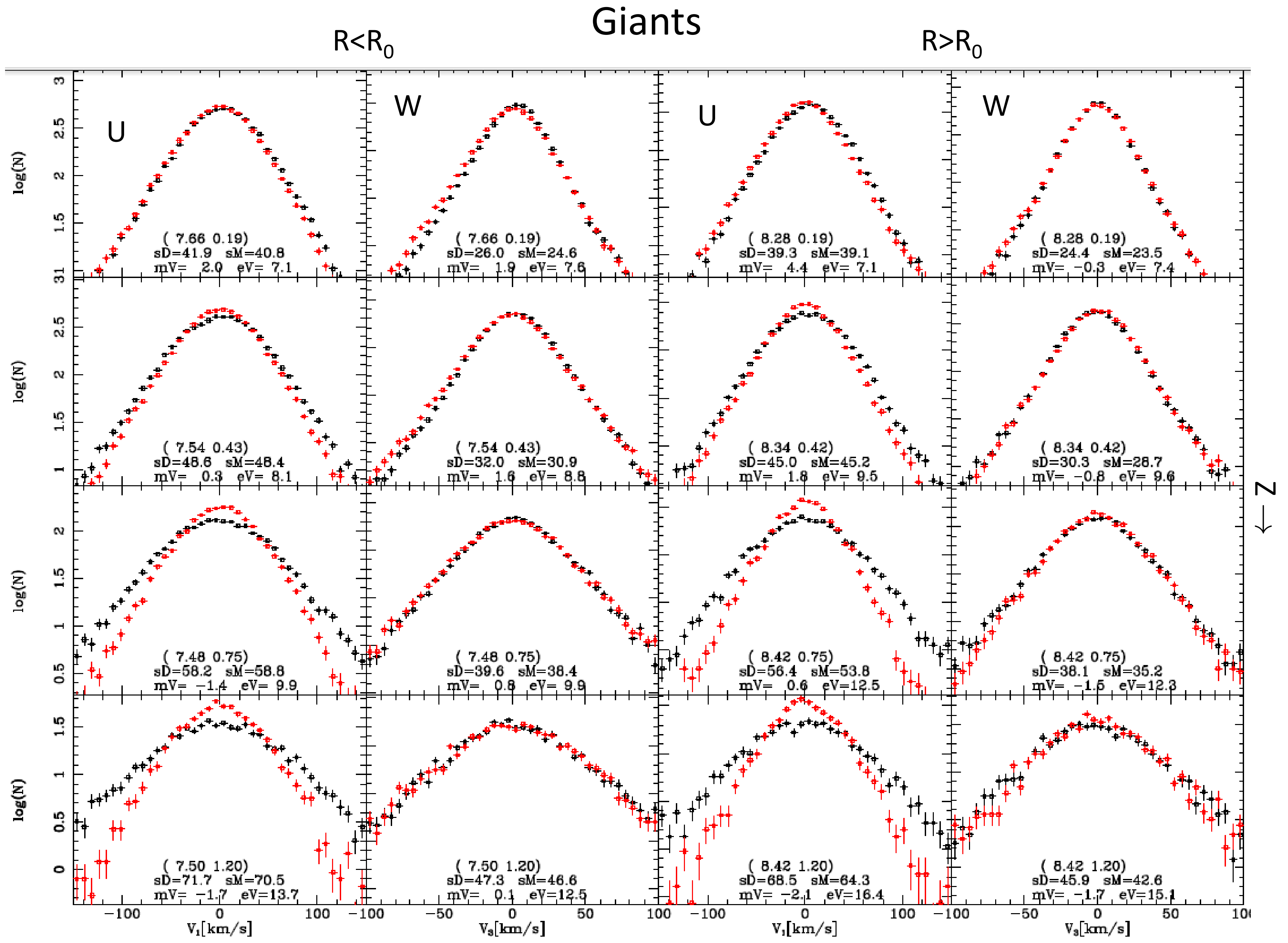
- Exponentially decaying SFR
- Fit model to GCS
- Predict velocity distributions for RAVE



Cool dwarfs: data/prediction



Dwarfs T<6000K



Giants
R<R_0
R>R_0
U
W
U
W
(7.66 0.19) sD=41.9 sM=40.8 mV= 2.0 eV= 7.1
(7.66 0.19) sD=26.0 sM=24.6 mV= 1.9 eV= 7.6
(8.28 0.19) sD=39.3 sM=39.1 mV= 4.4 eV= 7.1
(8.28 0.19) sD=24.4 sM=23.5 mV= −0.3 eV= 7.4
(7.54 0.43) sD=48.6 sM=48.4 mV= 0.3 eV= 8.1
(7.54 0.43) sD=32.0 sM=30.9 mV= 1.6 eV= 8.8
(8.34 0.42) sD=45.0 sM=45.2 mV= 1.8 eV= 9.5
(8.34 0.42) sD=30.3 sM=28.7 mV= −0.8 eV= 9.6
(7.48 0.75) sD=58.2 sM=58.8 mV= −1.4 eV= 9.9
(7.48 0.75) sD=39.6 sM=38.4 mV= 0.8 eV= 9.9
(8.42 0.75) sD=56.4 sM=53.8 mV= 0.6 eV=12.5
(8.42 0.75) sD=38.1 sM=35.2 mV= −1.5 eV=12.3
(7.50 1.20) sD=71.7 sM=70.5 mV= −1.7 eV=13.7
(7.50 1.20) sD=47.3 sM=46.6 mV= 0.1 eV=12.5
(8.42 1.20) sD=68.5 sM=64.3 mV= −2.1 eV=16.4
(8.42 1.20) sD=45.9 sM=42.6 mV= −1.7 eV=15.1
log(N)
V_1[km/s]
V_3[km/s]
Z

Secular evolution

- Equilibrium models just the start
- Given f(J) one can solve for evolution of population as stars diffuse through action space
- Diffusion coefficients $\langle J_i^2 \rangle$ computed from 1st-order perturbation theory based on fluctuations in Φ
- If you have (J,θ), you can also use method of "perturbation particles"
- N-body in which particles used only to represent fluctuations

Conclusions

- Equilibrium dynamical models are key tools
- Advent of techniques for computing J(x,v) in general potentials gives new possibilities for galaxy modelling
- Start from f(J) for each population
- Population defined by age, metallicity, place of birth,…
- f describes star density in 3d action space
- Transparent relation between f(J) and structure of component
- Simple functional forms for f(J) yield models similar to familiar spheroids and exponential discs
- Can inspect observables for component in any Φ
- Can readily find Φ self-consistently generated by a sum of components (incl DM component)