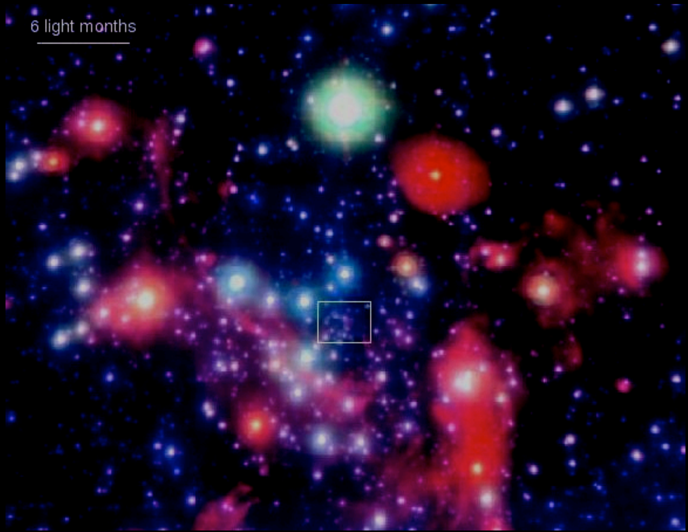
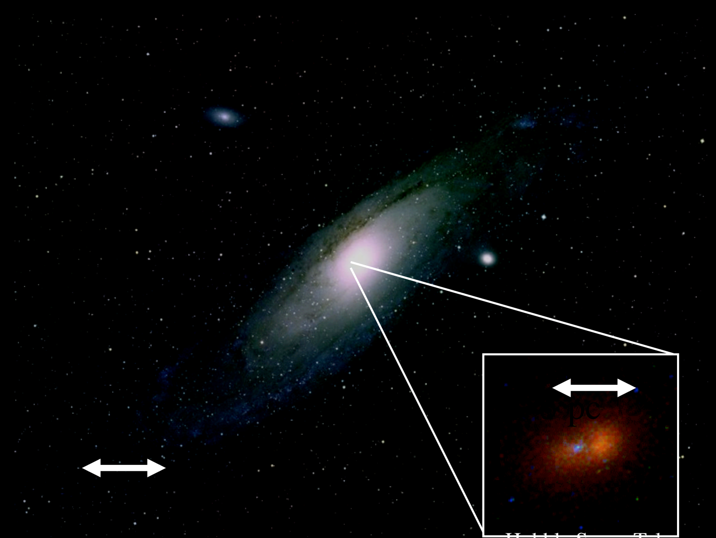
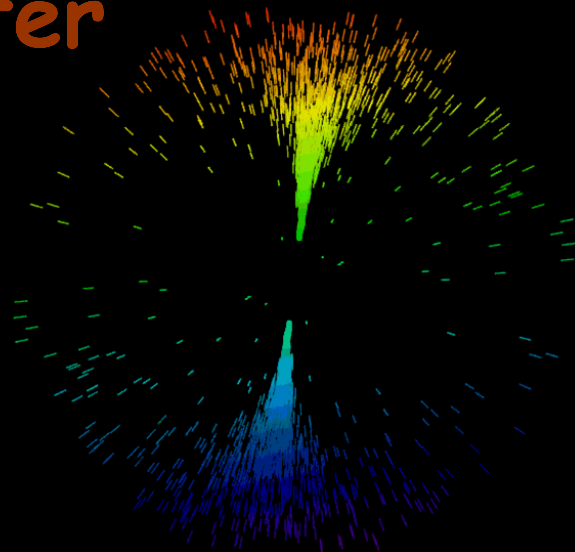


Hubble Space Telescope 3"x3"
Lauer et al. AJ 1998

# Resonant relaxation and the warp of the stellar disk in the Galactic Center



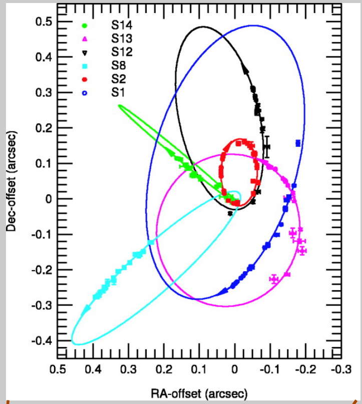
Bence Kocsis
Einstein Fellow, Harvard

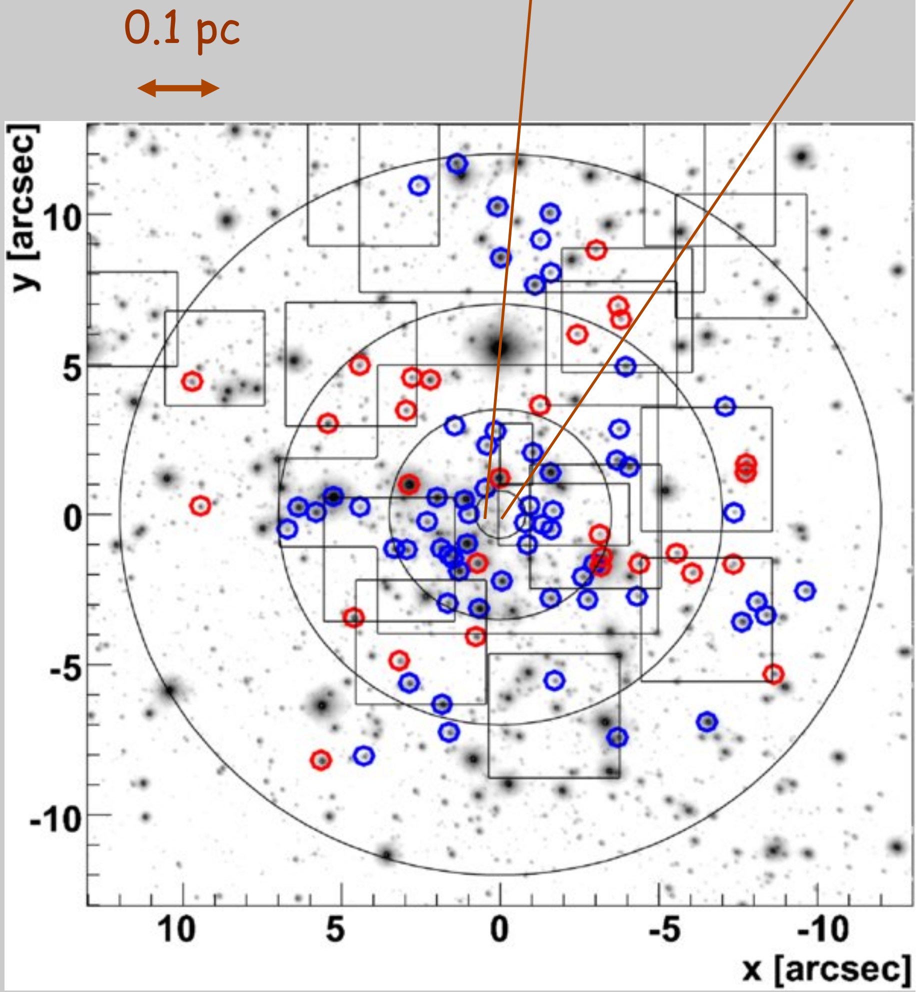
Collaboration with Scott Tremaine (IAS)

Central Massive Objects, ESO Garching, June 22, 2010

# The disk(s) in the Galactic center

- ~ 100 massive young stars found in the central parsec, age: 6 x $10^6$ yr; formation is a puzzle:
    - formation in situ from a disk?
    - disruption of an infalling cluster?
- implied star-formation rate is so high that it must be episodic
- line-of-sight velocities measured by Doppler shift and angular velocities measured by astrometry → five of six phase-space coordinates
- many of velocity vectors lie close to a plane, implying that many of the stars are in a disk (Levin & Beloborodov 2003)



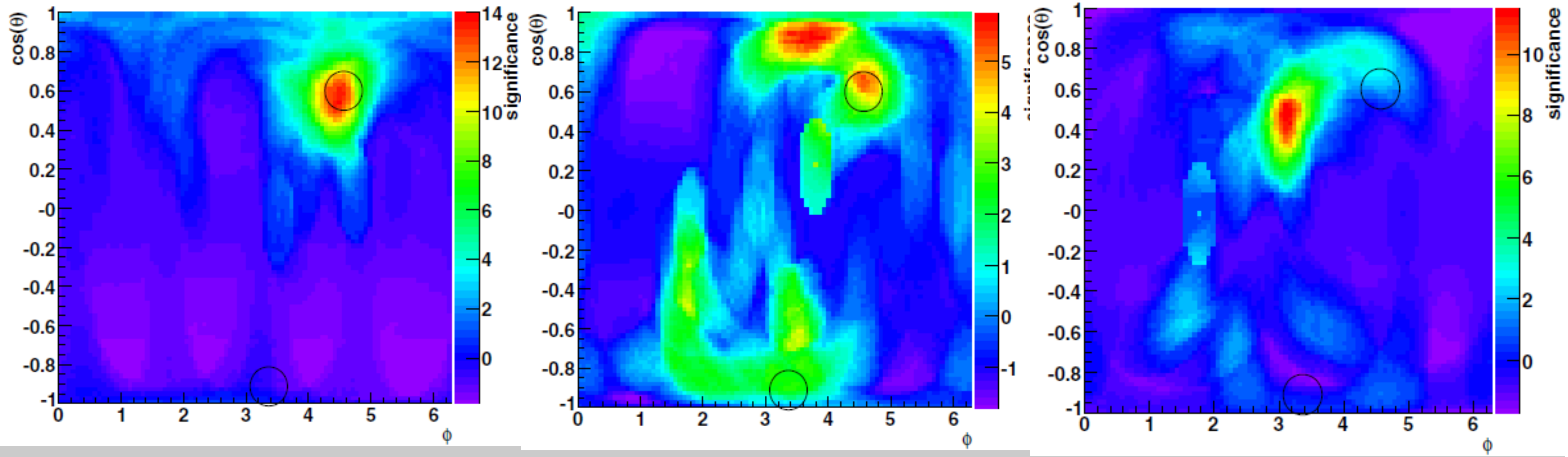




Paumard et al (2006), Lu et al. (2009), Bartko et al. (2009, 2010), …

# Observations: Warped disk

- Strong evidence for a warped disk (best-fit normals in inner and outer image differ by 60$^{\mathrm{deg}}$) – the "clockwise disk"
- Disk thickness ~10$^{\mathrm{deg}}$; rms eccentricity 0.3
- Disk is less well-formed at larger radii
- Weaker evidence for a second disk between 3" and 7" (the "counterclockwise disk")
- The two disks have the same age

Bartko et al. (2009)




Inside (1"-3.8")
0.03-0.13 pc

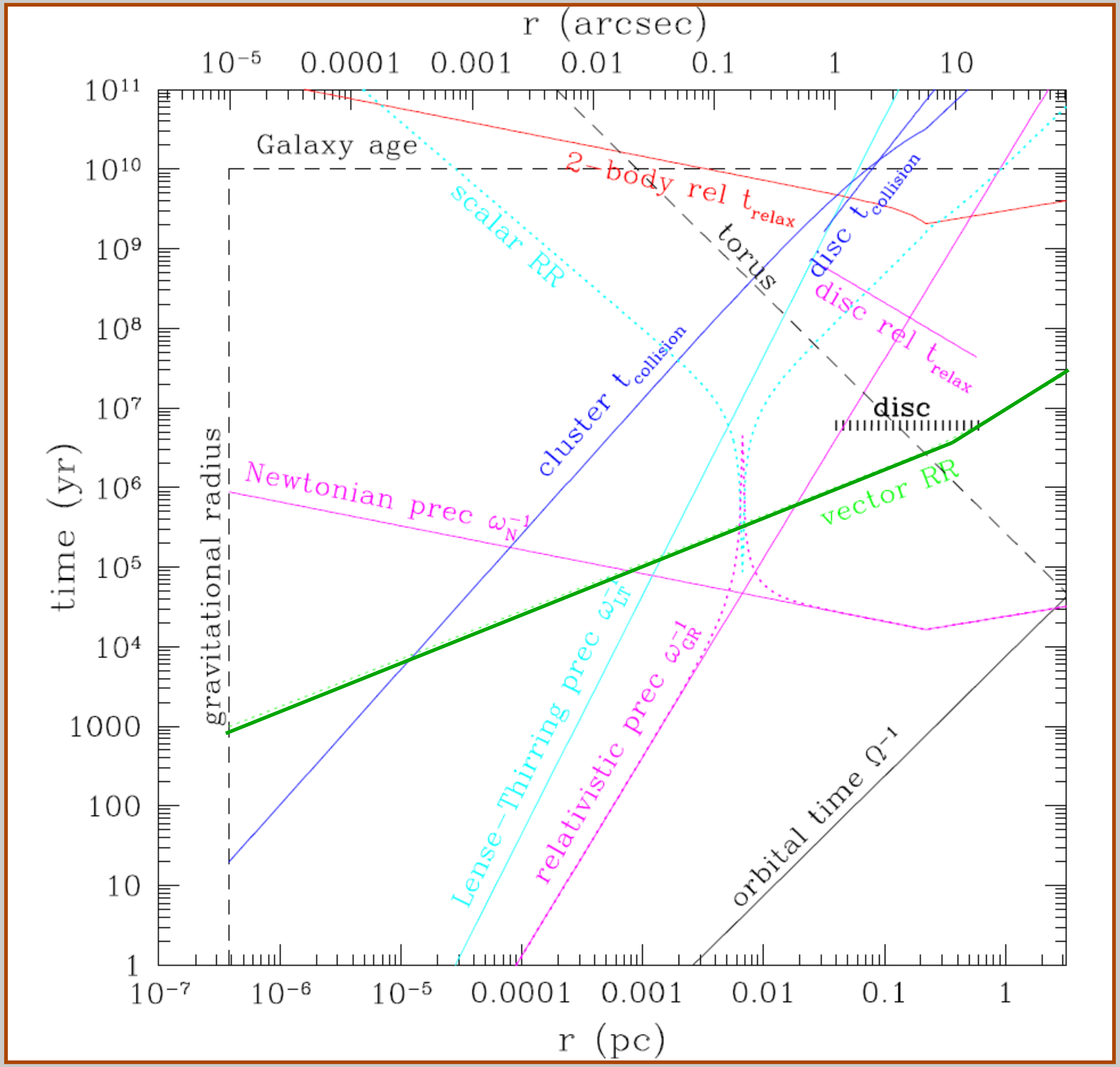
Middle (3.5"-7")
0.13-0.27 pc

Outside (7"-12")
0.27-0.47 pc

# Evolution of subparsec scale disks

- Gaseous (or stellar) disks get warped due to
  - GR frame dragging (Bardeen & Petterson 1975)
  - Radiation pressure (Petterson 1977)
  - Tori, rings (Nayakshin 2005, Subr et al. 2009)
  - resonant relaxation in NGC 4258 (Bregman & Alexander 2009)
- Isolated self-gravitating disks (also e.g. Hunter & Toomre 1969, Toomre 1983, etc.)
  - Self-gravity alone cannot explain observed inclinations (Cuadra et al. 2008)
  - Equilibrium solution can be warped by 120 deg (Ulubay-Siddiki et al. 2008)
- Warps due to infalling gas (Nayakshin & Cuadra 2005, Hobbs & Nayakshin 2009)
- Two massive inclined stellar disks warp and dissolve due to mutual torques (Lockmann & Baumgardt 2009)
- Binary companion, IMBH would warp the disk (e.g. Papaloizou et al. 1998, Yu & Tremaine 2003, Yu et al. 2007)
- Our approach:
  - Does resonant relaxation warp a thin stellar disk?
  - What is the final configuration in statistical equilibrium?

# Timescales




- Disk Age:

$$t_{\rm disk} \sim 6\,{\rm Myr}$$

- Vector Resonant Relaxation:

$$t_{\substack{\rm v.res.\\ \rm relax}} \sim t_{\rm orb}(r)\frac{M_\bullet}{\sqrt{m_2 M(r)}}$$

- Pericenter precession

$$t_{\rm precess} \sim \frac{1}{t_{\rm orb}(r)\,G\rho(r)}$$

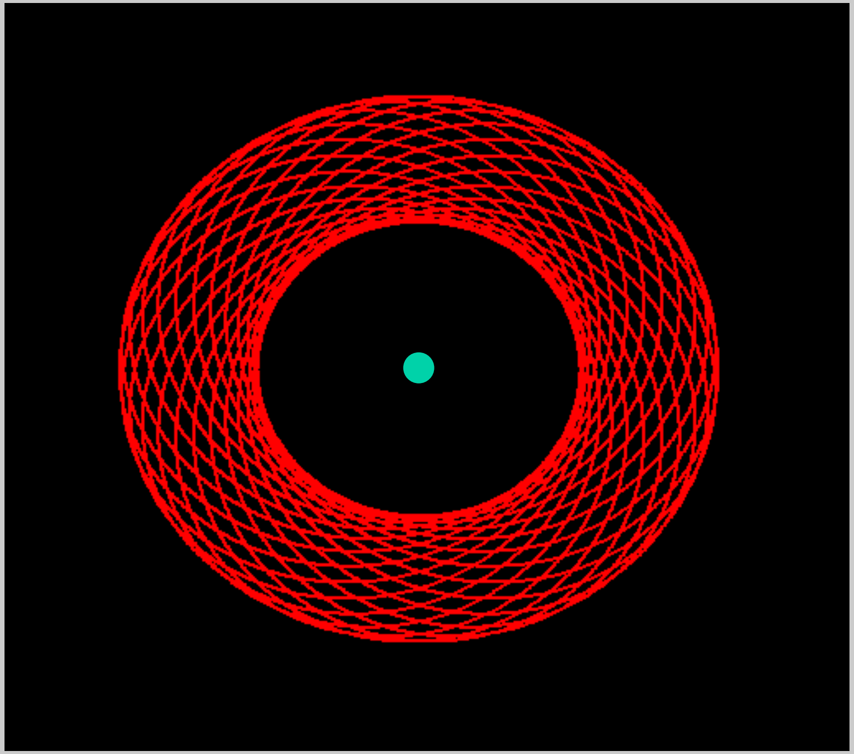
- Orbital time

$$t_{\rm orb}(r)$$

# Resonant relaxation in dense stellar systems

- on timescales longer than the orbital period each stellar orbit can be thought of as an eccentric wire
- orbits precess due to the gravity of the spherical star cluster → axisymmetric disk or annulus
- orbits are specified by semi-major axis, eccentricity, and orbit normal
- conserved for each star:
    - energy (semi-major axis)
    - angular momentum magnitude (eccentricity)
- each annulus exerts steady force on all other annuli, leading to secular evolution of the orbit normals

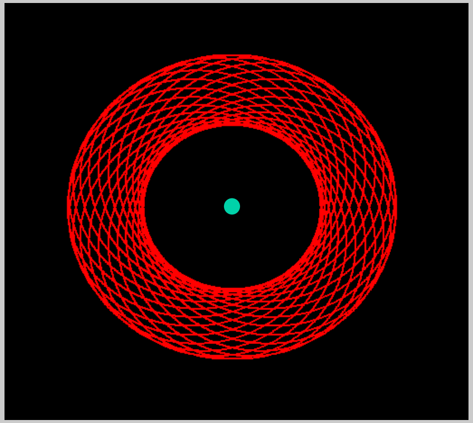
→ orbit normals change in time



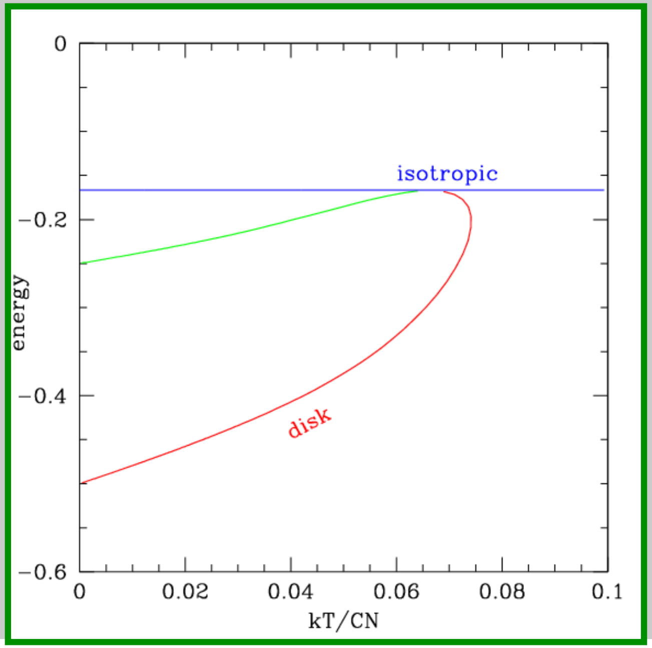
Rauch & Tremaine (1996)

# Vector resonant relaxation

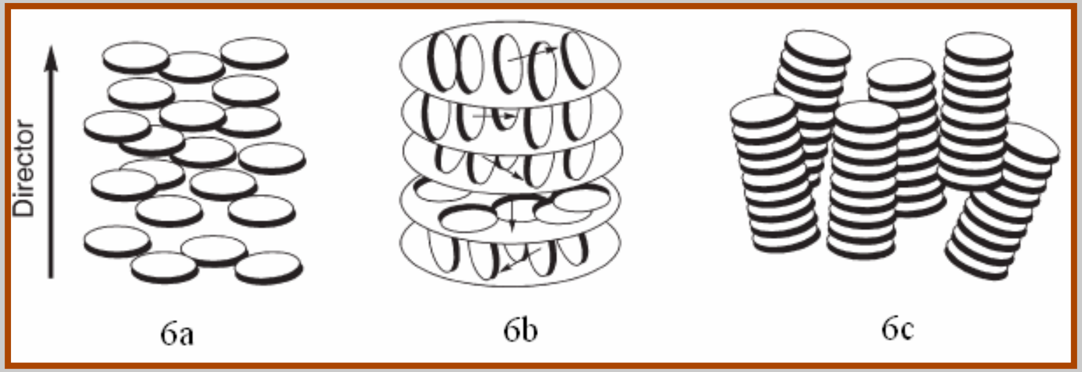
- Eccentricity + semimajor axis conserved
- Torques between axi-symmetric disks
  - Parallel configuration stable
  - Inclined orbits exert torques



1. "Thermal" equilibrium?
   - Monte Carlo Markov Chain simulation
   - Analytic solutions (Maier-Saupe model)
2. Time evolution?
   - numerical simulation (symplectic integrator)
   - analytic models (Laplace-Lagrange model)

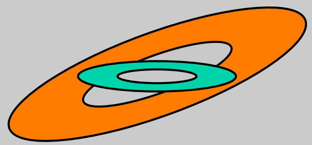



Interesting analogy: liquid crystals

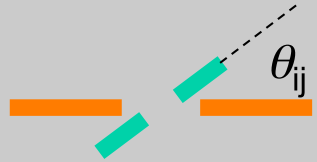



- nematic chiral nematic columnar

# Thermodynamic equilibrium for vector resonant relaxation



- mutual torques can lead to relaxation of orbit normals (angular momenta)
- self-gravitating system in terms of the orientation of orbits
- interaction energy between stars i and j




$$H = \frac{1}{2}\sum_{\substack{i,j=1 \\ i\neq j}}^{N} m_i m_j F(a_i, a_j, e_i, e_j, \cos\theta_{ij})$$

masses; semi-major axes; eccentricities; Angle between orbit normals

$$\cos\theta_{ij} = \frac{L_i \cdot L_j}{\| L_i \| \| L_j \|}$$

- Canonical ensemble: phase space probability density is

$$f(L_1, L_2, \ldots, L_N) = C \exp\left(-\frac{E}{kT}\right)$$

- What is temperature?
- More straightforward:
  - microcanonical ensemble with fixed "energy" and angular momentum

# Thermodynamic equilibrium from resonant relaxation – toy model #1

$$H = -\tfrac{1}{2} C \sum_{i,j=1}^{N} \cos^2 \theta_{ij}$$

- drastic simplification assuming equal masses, semi-major axes, eccentricities and neglecting all harmonics > l=2

- this is the **Maier-Saupe model** for the isotropic-nematic phase transition in **liquid crystals**

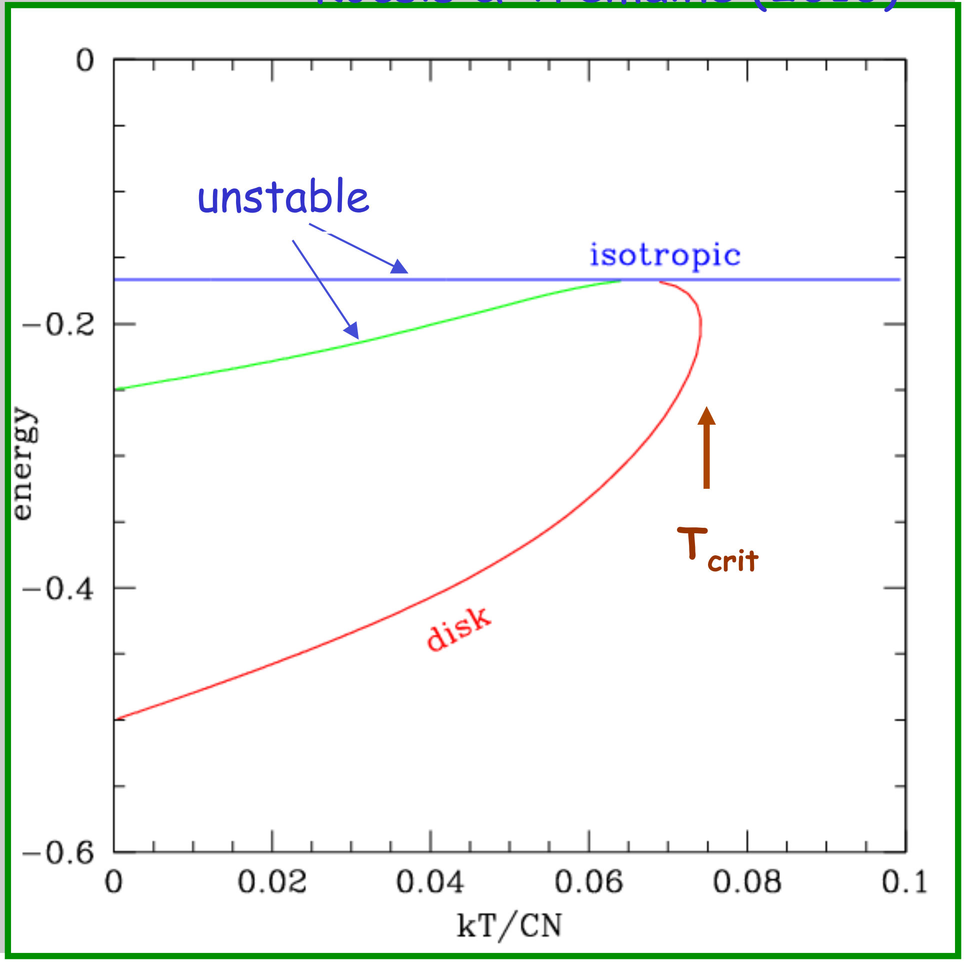
- analytic mean field solution verified by experiments

- above temperature

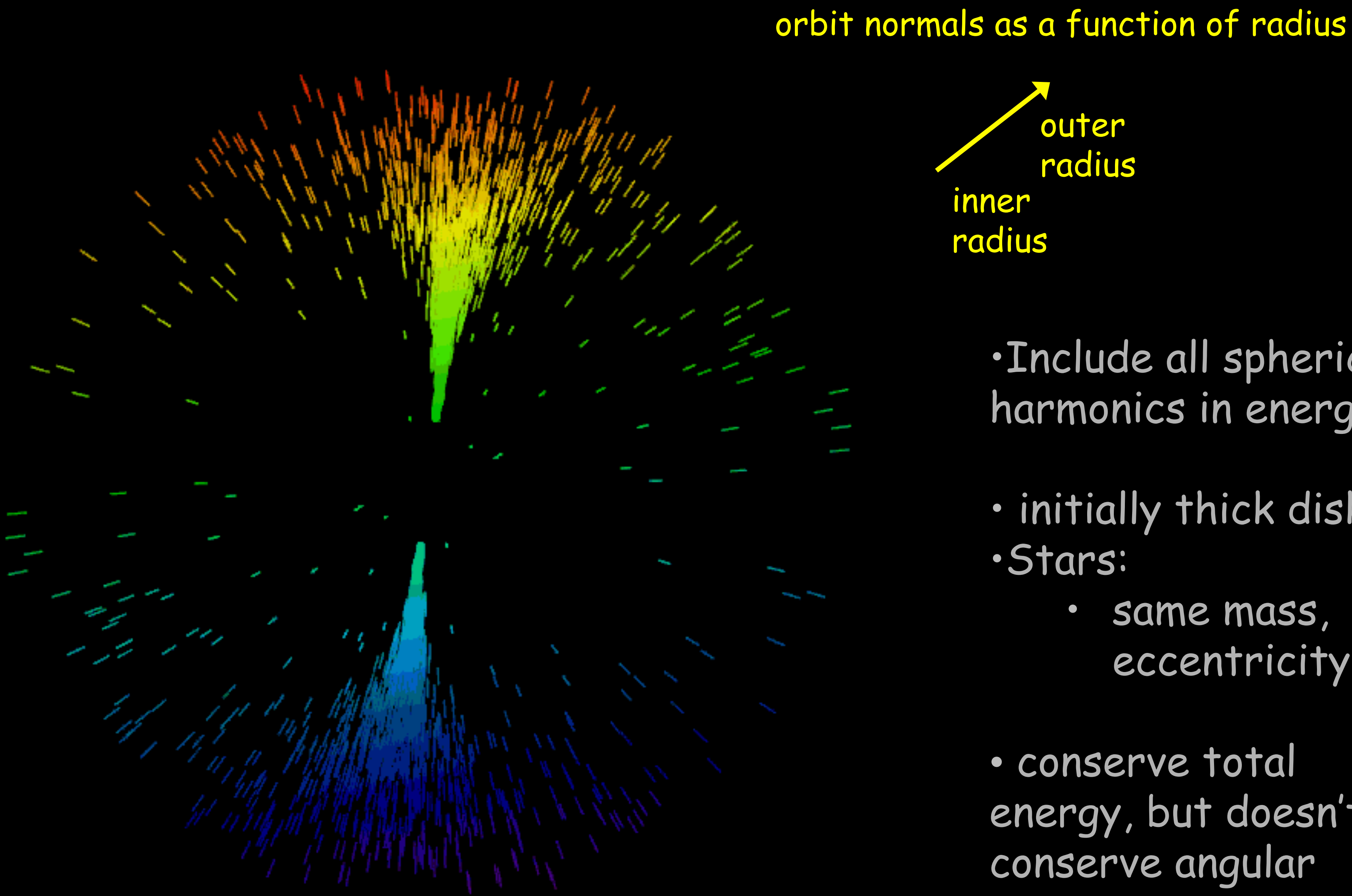
$T=T_{crit}=0.0743\ CN/k$ ← Boltzmann constant

the only equilibrium is isotropic. Below $T_{crit}$ there is a phase transition to a disk

Kocsis & Tremaine (2010)



# Thermal equilibrium




- Include all spherical harmonics in energy

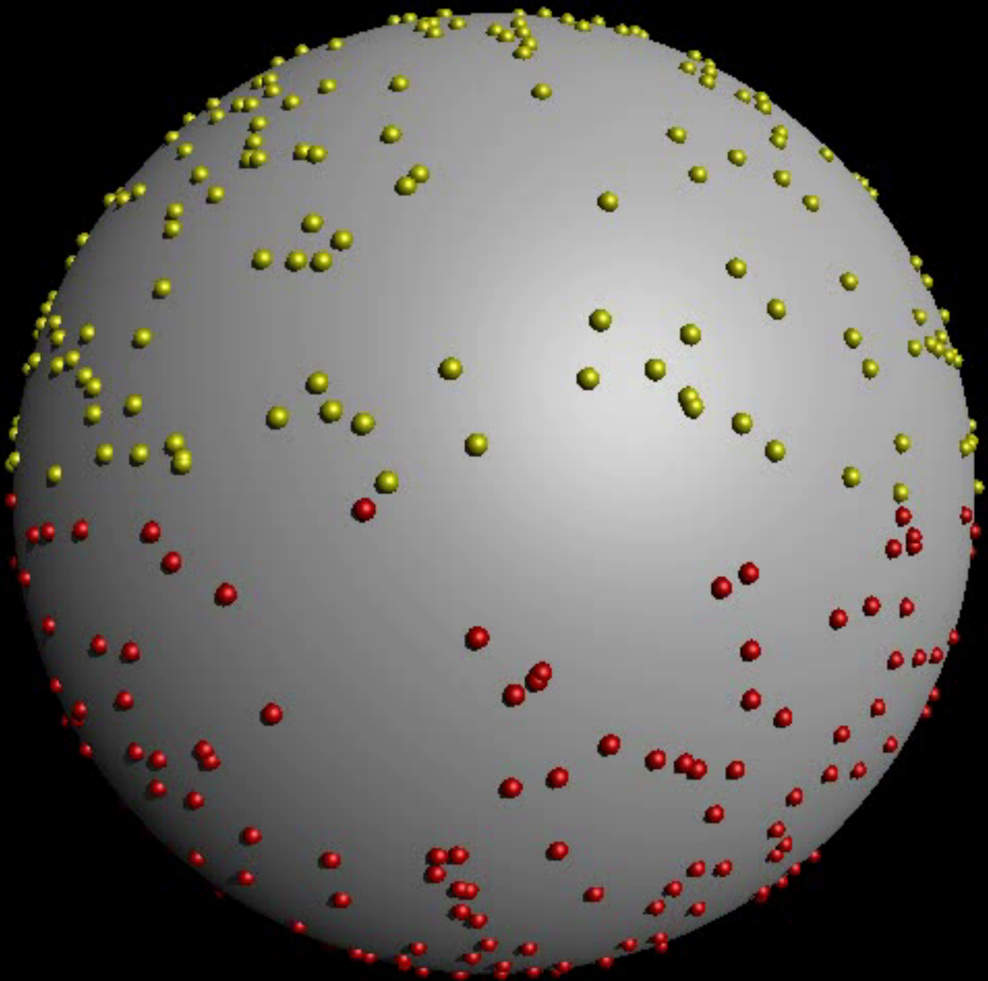
- initially thick disk
- Stars:
  - same mass, eccentricity

- conserve total energy, but doesn't conserve angular momentum

Monte Carlo Markov Chain simulation

Kocsis & Tremaine (2010)

# Time evolution



Solve Hamilton's equations of motion numerically

- same mass
- same radius
- same eccentricity

N = 800

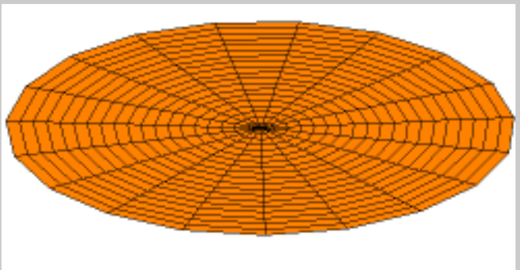
Is there chaotic mixing?

Symplectic integrator (conserves $L_{tot}$ exactly)  Kocsis & Tremaine (2010)

# Time evolution of a <u>Thin</u> Disk
## – toy model #2

- Secular forces between disk stars "exactly"
  - Small inclinations and eccentricities $e_j, I_j < |a_j - a_k|/a_j$
  - $N$ stars with masses $m_j$, semi-major axes $a_j$, inclinations $I_j$, nodes $\Omega_j$
- Torques from the cluster as a background
- This system is described by the Laplace-Lagrange Hamiltonian

$$H = -\sum_{i,j=1}^{N} (q_i A_{ij} q_j + p_i A_{ij} p_j) \quad \text{where} \quad (q_i, p_i) = m_i^{1/2} (G M a_i)^{1/4} I_i (\sin \Omega_i, \cos \Omega_i)$$



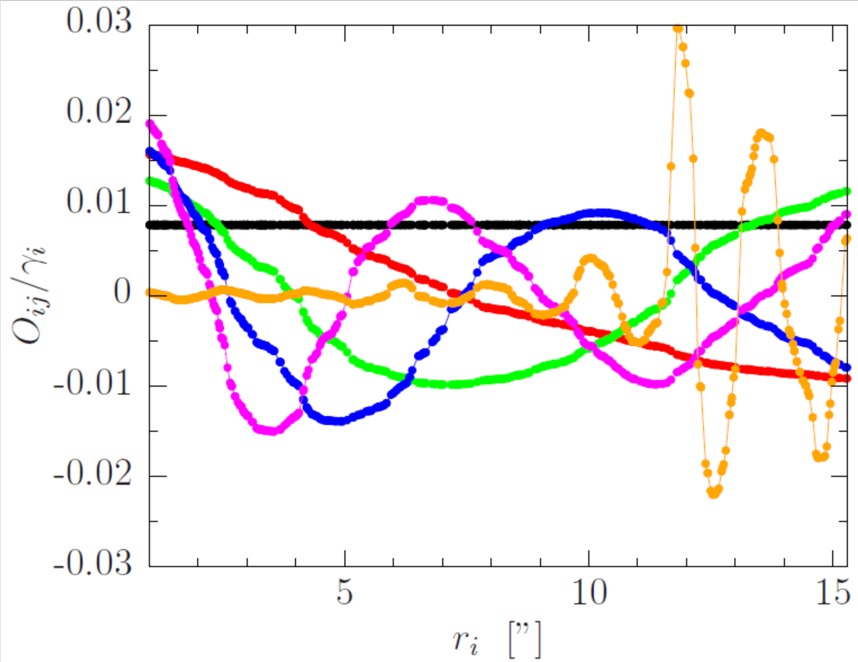
Diagonalize: → harmonic oscillator

- Normal modes (oscillate independently)
- Spherical cluster as external torque
  - stochastic (Gaussian)
- Predict power spectrum of normal modes after time t = 6 Myr

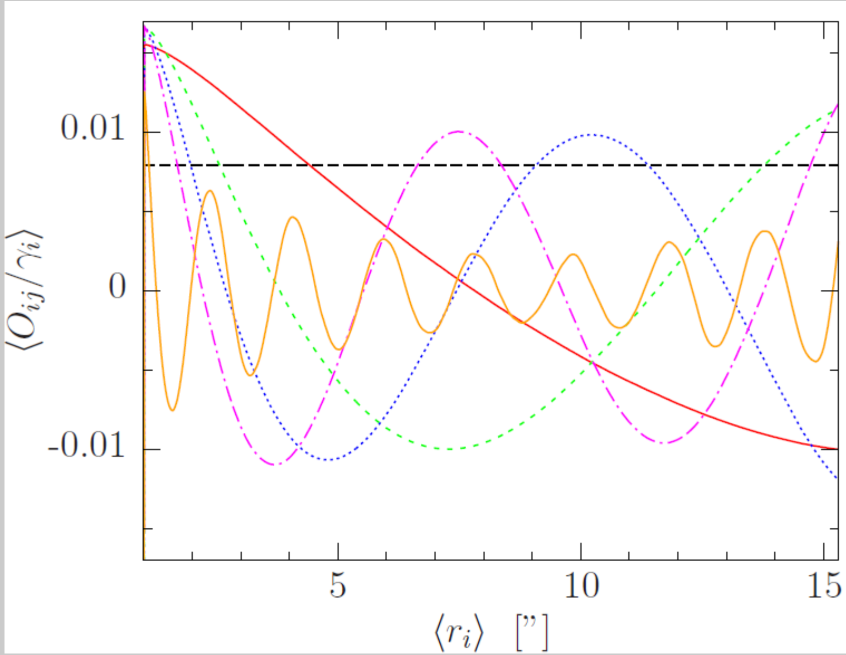
Kocsis & Tremaine, arXiv:1006.0001

# Normal modes of a Thin Disk

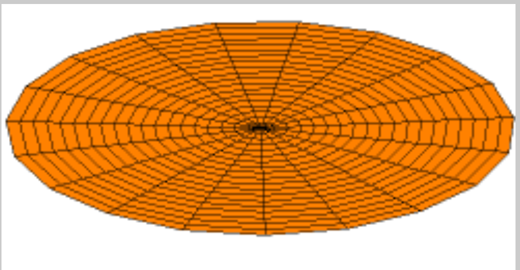
Inclination



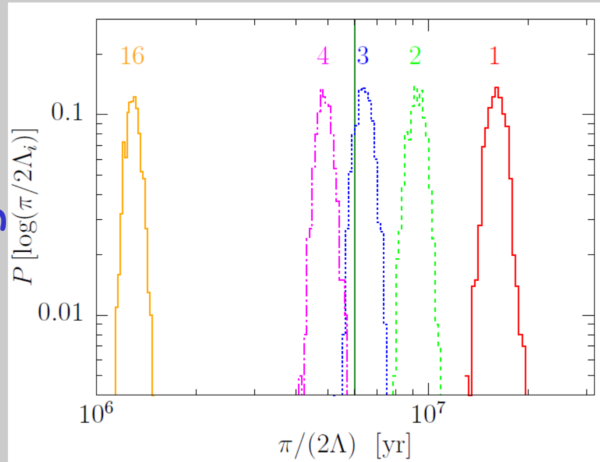

semimajor axis




semimajor axis



Histogram




½ oscillation period

**Conclude:**

- Long wavelength modes are slowest
- all but 4 modes have already saturated after 6Myr

Kocsis & Tremaine, arXiv:1006.0001

# Growth of perturbations

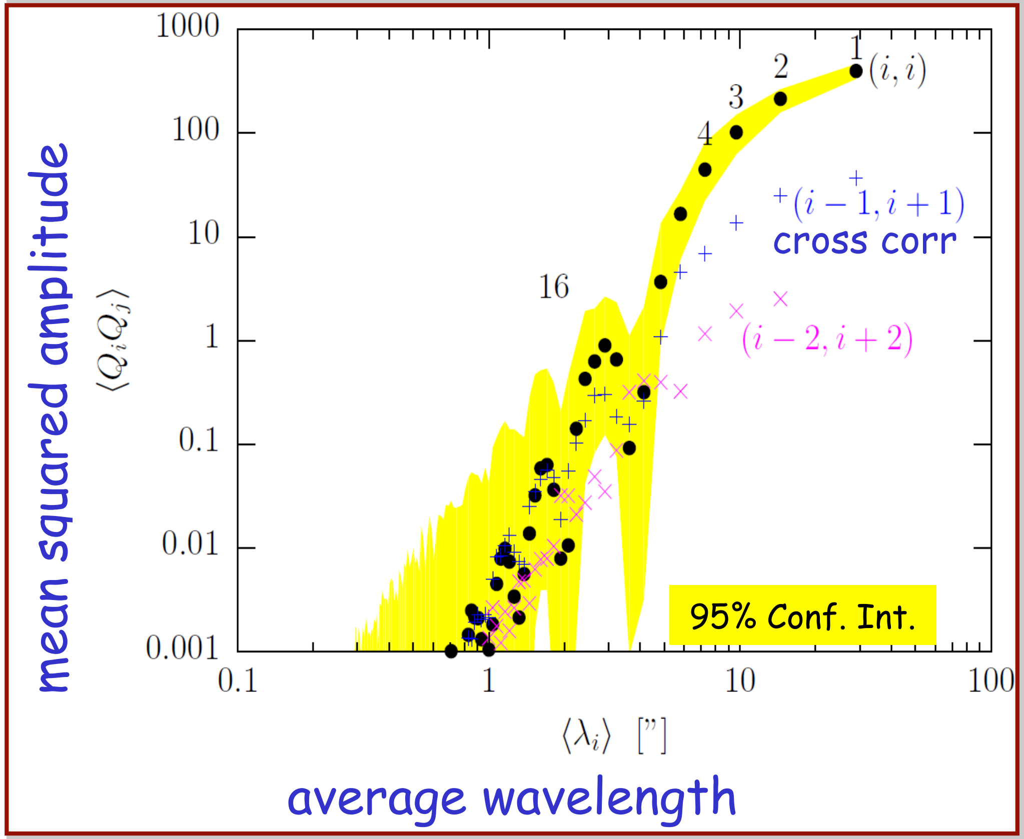
- Solve equations of motion for normal mode amplitudes
- Three timescales
  - Normal mode frequencies
  - Correlation time of coherent torques
  - Age of disk (integration time) – $t$
- Normal modes evolve differently in various limiting cases

**Linear growth** → **Harmonic oscillation** → **Random walk**

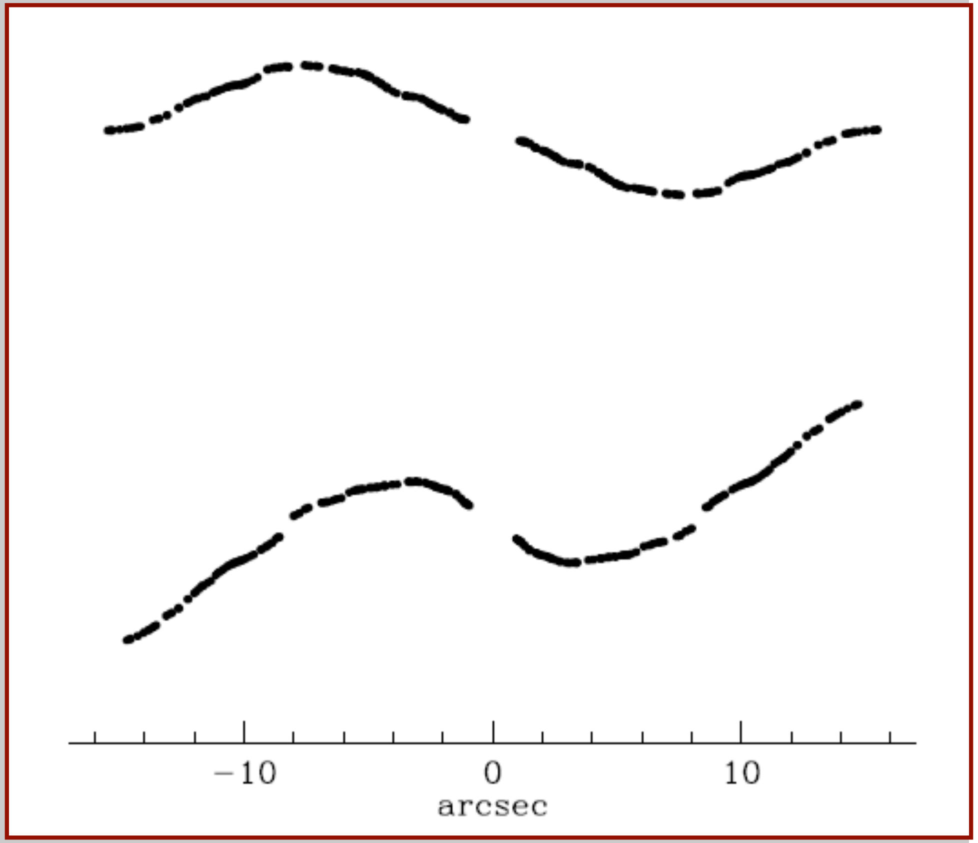
*small* $t$ | *intermediate* $t$ | *large* $t$

# Result for temporally coherent perturbations

Power Spectrum




disk cross sections (example)




**Prediction**: Long wavelength normal modes dominate

Kocsis & Tremaine, arXiv:1006.0001

# Time evolution

Two components:

• spherical cluster with light stars ($N_1$, $m_1 = 1$)

• disk with massive stars ($N_2$, $m_2 = 4$)

Total mass dominated by spherical cluster ($N_1 : N_2 = 1 : 20$
$N_1 + N_2 = 5000$)

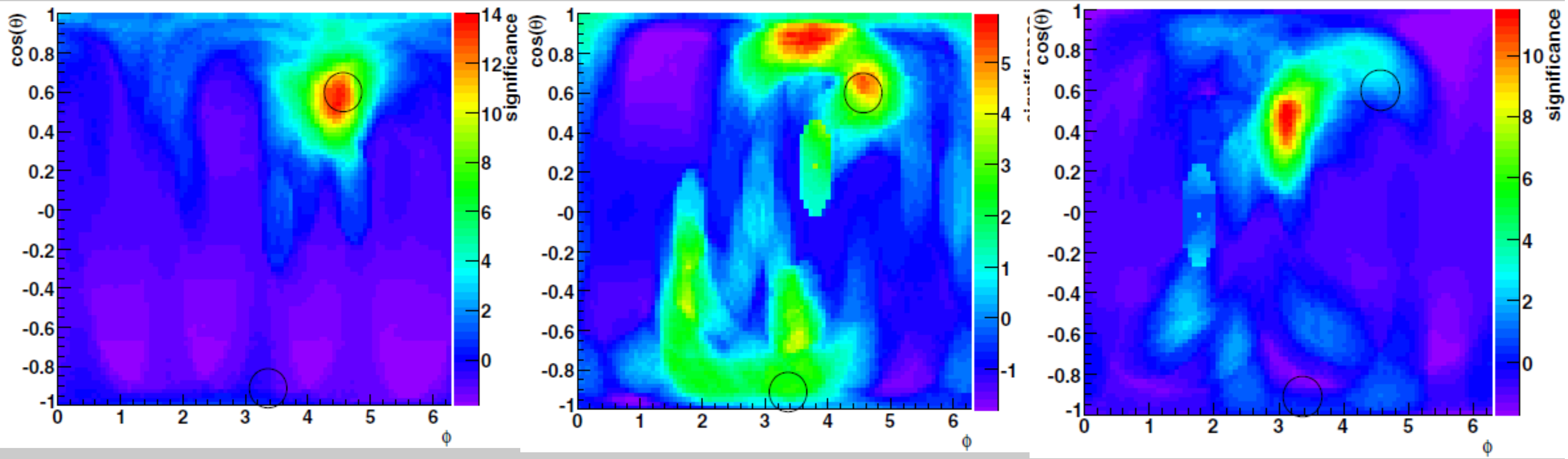
Symplectic integrator (conserves $L_{tot}$ exactly)    Kocsis & Tremaine (2010)

# Summary

- Vector resonant relaxation governs the evolution of stellar disks in galactic nuclei
  - Chaotic
  - relaxes toward thermal equilbrium
- Star cluster warps the disk
  - Long wavelength modes dominate
- Thermal equilibrium:
  - mass segration in inclinations
  - thin corotating and counterrotating disks of massive stars
  - phase transition at a given radius

# Observations: Warped disk

Bartko et al. (2009)

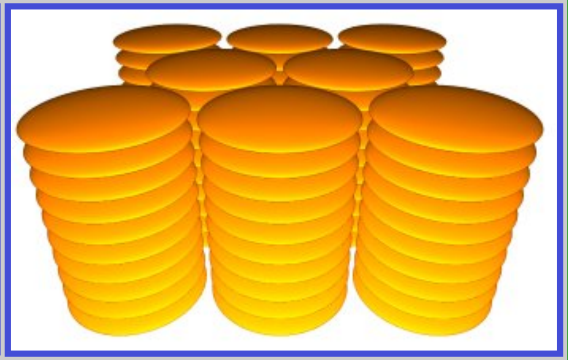



Inside (1"-3.8") | Middle (3.5"-7") | Outside (7"-12")
0.03-0.13 pc | 0.13-0.27 pc | 0.27-0.47 pc

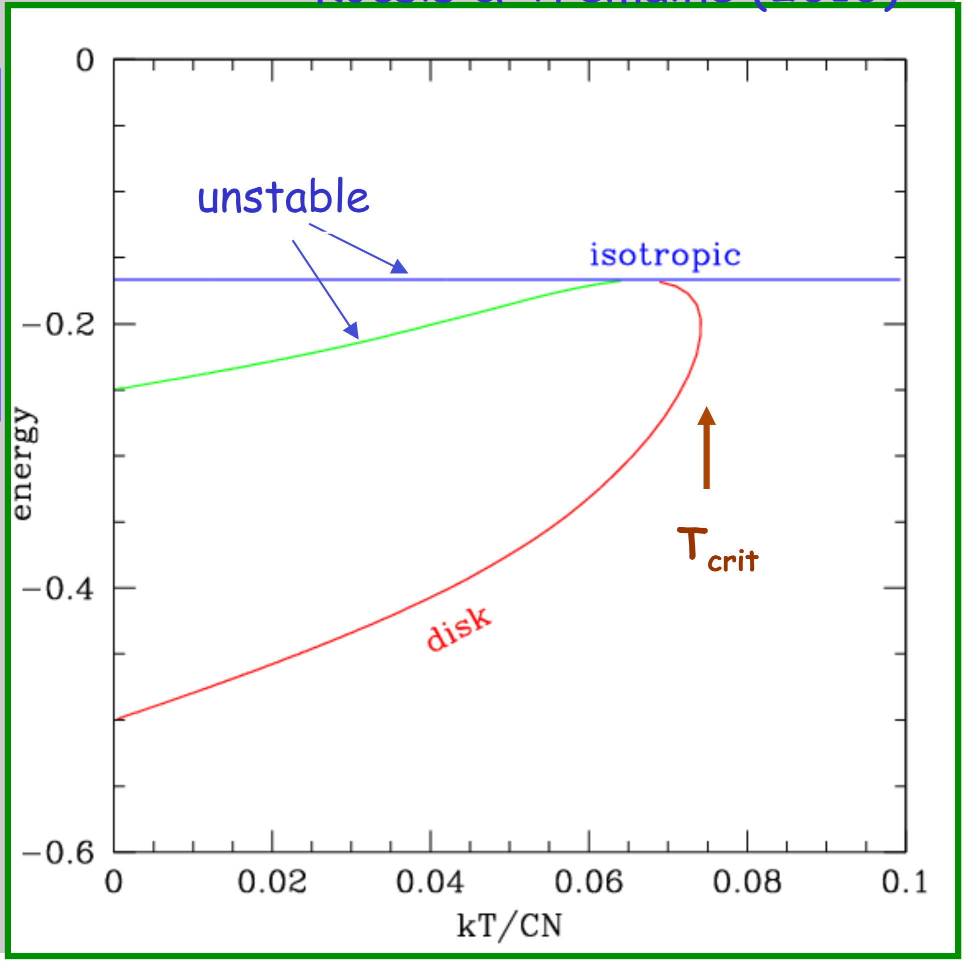
# Thermodynamic equilibrium from resonant relaxation – toy model #1

$$H = -\tfrac{1}{2} C \sum_{i,j=1}^{N} \cos^2 \theta_{ij}$$

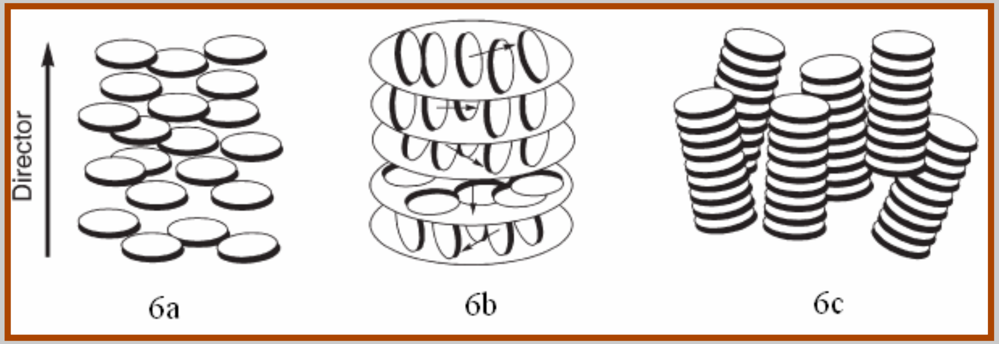
- Liquid crystals



Kocsis & Tremaine (2010)




- Many possible phases, e.g isotropic or




- nematic chiral nematic columnar

# Thermodynamic equilibrium from resonant relaxation – toy model #1b

$$H = -\tfrac{1}{2}\sum_{i,j=1}^{N}\frac{Gm_i m_j}{\max(a_i, a_j)}\cos^2\theta_{ij}$$

- Simplified interaction: only l = 2 harmonic, circular orbits
- now allow different masses and semimajor axes
- equipartition of energy :
    - effective temperature T ~ <E> / $k_B$ ~ $10^{60}$ K
    - massive objects go into a thin disk, light objects go spherical
    - objects with large semimajor axes have a smaller effective mass

        → disk becomes spherical in the outside, thin in the inside

# Laplace-Lagrange model

- Hamiltonian: $H(\boldsymbol{q},\boldsymbol{p}) = \boldsymbol{p}^{\mathrm{T}}\mathbf{A}\boldsymbol{p} + \boldsymbol{q}^{\mathrm{T}}\mathbf{A}\boldsymbol{q}$
- Diagonalize: $H(\boldsymbol{P},\boldsymbol{Q}) = \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Lambda}\boldsymbol{P} + \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{\Lambda}\boldsymbol{Q}$
- Harmonic oscillator + external stochastic torque.
  - Equation of motion:

$$\dot{Q}_i = 2\Lambda_i P_i + \sum_{j=0}^{N-1} O_{ji} f_{qj}(t), \;\; \dot{P}_i = -2\Lambda_i Q_i + \sum_{j=0}^{N-1} O_{ji} f_{pj}(t).$$

- Assume correlation function of external torque

$$\Gamma_{ij}(t,t') \equiv \Gamma_{ij}(|t-t'|) \equiv \langle f_{qi}(t) f_{qj}(t')\rangle = \langle f_{pi}(t) f_{pj}(t')\rangle$$

- Predict probability of normal mode amplitude after time t.
  - Example: coherent torque phase

$$\langle Q_i^2\rangle = \langle P_i^2\rangle = \sum_{n,m=0}^{N-1} O_{ni} O_{mi} \Gamma_{nm} \frac{\sin^2 \Lambda_i t}{\Lambda_i^2}, \quad \text{if } t \ll \tau.$$

$$\langle Q_i Q_j\rangle = \langle P_i P_j\rangle = \sum_{n,m=0}^{N-1} O_{ni} O_{mj} \Gamma_{nm} \frac{\sin \Lambda_i t}{\Lambda_i} \frac{\sin \Lambda_j t}{\Lambda_j} \cos \Delta_{ij} t$$

# Thermodynamic equilibrium from resonant relaxation

The right way to do it:

• model is specified by masses $m_i$, semi-major axes $a_i$, eccentricities $e_i$, and initial orientations of orbit normals

• interaction energy between stars $i$ and $j$ is $H_{ij}=m_i m_j f(a_i,a_j,e_i,e_j,\cos\theta_{ij})$ where $\theta_{ij}$ is the angle between the orbit normals. Evaluate $f$ numerically as an expansion in $P_l(\cos\theta_{ij})$ (can be done once and for all at the start)

$$H_{ij} = -G\frac{m_i m_j}{a_{ij}^{>}} \sum_{l=0}^{\infty} \alpha_{ij}^l \, s_{ijl} \, P_l(0)^2 \, P_l(\cos\theta_{ij})$$

Kocsis & Tremaine (2010)

where $\alpha_{ij} \equiv \dfrac{a_{ij}^{<}}{a_{ij}^{>}}$ and $s_{ijl} \equiv s_{ijl}(e_i, e_j)$

For non-overlapping orbits $s_{ijl} = \dfrac{\chi_{ij\triangleright}^{l}}{\chi_{ij\triangleleft}^{l+1}} P_{l-1}(\chi_{ij}^{\triangleright}) P_{l+1}(\chi_{ij}^{\triangleleft})$ $\chi_i \equiv \dfrac{1}{\sqrt{1-e_i^2}}$

# Thermodynamic equilibrium from resonant relaxation

The right way to do it:

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Kocsis & Tremaine (2010)

1. Evaluate interaction energy numerically and use Markov Chain Monte Carlo to find equilibrium state

2. Dynamical simulation: Numerical integration of Hamilton's equation*

* for N-annuli Hamiltonian above