Field-of-view widening in adaptive optics system.

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1.Introduction.

Conventional adaptive optics system compensate for cumulative phase distortion due to turbulence propagation, but such compensation is only effective over an area approximately equal to the size of isoplanatic patch.[1,2] This limitation is a result of the turbulence distribution along the optical path or the vertical extent of the atmospheric-turbulence structure. Although atmospheric refractive-index inhomogeneties form a three-dimentional spatial random process, an adaptive optics system applies only a two-dimentional spatial correction, specifically the surface of the deformable mirror. That fact results to anisoplanatism of adaptive optical system. The size of isoplanatic patch depends on turbulent profile along the trace and also on controlling algorithm.

In the present paper an angular anisoplanatism is studied as a fundamental limit to adaptive-optical system performance in regime of extended object viewing. It is considered a perfect system with no other causes of the performance degradation (i.e., one with no measurement noise or fitting error).

To investigate the potential possibility of the system we neglect technical limit results from lack of information about the distortion and suppose that we can register wave-front distortion associated with the spherical wave coming from any point on the object to be viewed. With such an assumption the modified controlling algorithm that corresponds to minimum of a new phase quality criteria is introduced. We show that even in that ideal case a completed correction of the extended area is not achievable.

We also exam a potential possibility of the system where corrector may be placed to conjugate the center of one thin turbulent layer for different allocation of the layer on trace and for various width of the layer.

The results are presented in terms of the residual mean-square wave-front distortion and also confirmed by computer simulation of the image of the extended source results after the compensation.

2. The primary scheme.

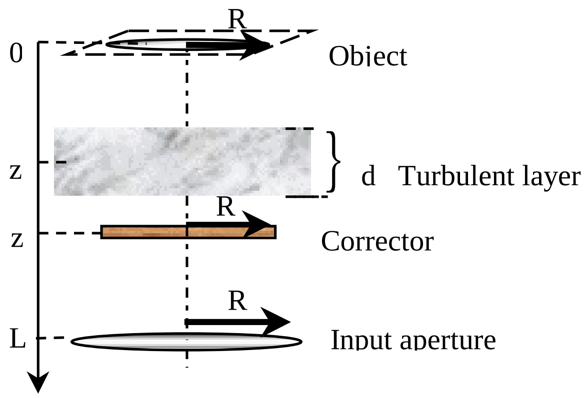

Fig.1 The primary scheme.

A schematic illustration of our concept turbulence - adaptive imaging system is shown in Fig.1.
Object in question has the dimensions that exceed the isoplanatic patch of adaptive-optics system. Distorting medium, such as atmospheric turbulent layer, degrades the received wave front. A number of references are placed in direction to the object to give information about phase distortion. Phase corrector introduces wave-front control to improve image quality of the extended source. We suggest that in this scheme wave–front corrector can be placed to conjugate any plane between the object and the pupil.

3. Mean square residual phase error.

The mean-square residual phase distortion $\sigma^2(\rho)$ is a convenient measure to describe system performance degradation caused by anisoplanatism. It represents the random wave front distortion results from the correction

$$\sigma^2(\rho) = \int \mathrm{dr}W(\mathrm{r}) \left\langle \left(\varphi(\rho, \mathrm{r}) + \mathrm{u}(\mathrm{r}) \right)^2 \right\rangle \Big/ \int \mathrm{dr}W(\mathrm{r}) \tag{1}$$

Here $\mathrm{u}(\mathrm{r})$ is used to represent phase profile of the corrector, $\varphi(\rho, \mathrm{r})$ is used to represent the random wave front distortion associated with spherical wave issuing from point ρ on the extended area to be observed and $W(\mathrm{r})$ is the aperture function. It should be noticed that we exclude the piston from the measured phase distortion because the overall piston does not take any part in image construction process. We include parameter ρ to assist in keeping point coordinate dependence.

In attempt to widen field-of-view by means of updating controlling algorithm we introduce a new quality criterion of correction [3], which takes into account all points belonging a certain extended region S_0. We set residual phase error averaged over this region as such a criterion

$$\sigma^2 = \frac{1}{S_0} \int_{S_0} \sigma^2(\rho) d\rho \tag{2}$$

The goal is to minimize σ^2 and a conventional algorithm with a single reference does not meet a minimum of this criterion. In this connection there is the necessity in the additional information on the distortion, which can be received by use of a set of references [4]. We have conducted the study that presents the generalization of multi-reference principle and extended case of a set of spatial diversified guide stars to continuum. In other words, our treatment here is based on a-priori knowledge of the distortion associated with spherical wave coming from every point within the estimation area. With such an assumption minimization of σ^2 leads to the modified controlling algorithm

$$\mathrm{u}(r) = -\frac{1}{\mathrm{S}_0} \int_{\mathrm{S}_0} \varphi(\rho, r) \mathrm{d}\rho \tag{3}$$

The analytical analysis was made for the case of a circular estimation region with radius R_0 centered in point $\rho = 0$. The particular application of this research occurs for the case when only a single reference is used as an estimate of the distortion, that corresponds to zero value of estimating area $R_0 = 0$.

To determine $\sigma^2(\rho)$ as given by Eq. (1) with correction function presented by Eq (3) we must determine second order statistic for the residual phase. The covariance functions appeared after the substitution Eq. (3) into Eq. (1) can be defined through two-dimension spectrum of index-of-refraction fluctuations $\Phi_n(k, z)$:

$$\left\langle \varphi(\rho, r) \varphi(\rho', r') \right\rangle = 2\pi k_0 \int_0^{\mathrm{L}} \mathrm{dz} \iint \mathrm{dk}\, \Phi_{\tilde{n}}(\mathrm{k}, \mathrm{z}) \mathrm{e}^{i2pkr(\mathrm{z})} , \tag{4}$$

where

$$r(\mathrm{z}) = \left(1 - \frac{\mathrm{z}}{\mathrm{z_c}} \right) (r - r') + \frac{\mathrm{z}}{\mathrm{z_c}} (\rho - \rho') \tag{5}$$

κ_0 is the spatial wave vector of the light, L - distance between the object and an input aperture.

The physical assumption for the following evaluation is that Kolmogorov spectrum describes the distribution of special frequencies in the refractive-index fluctuations.

$$\Phi(k, z) = 0.00969 C_n^2(z) k^{-11/3}, \tag{6}$$

Structure parameter $C_n^2(z)$ characterizes the strength of the fluctuation as a function of the *z*.

In order to calculate integrals over the circular region S_0 we can use the expression:

$$\frac{1}{S_0}\int_{S_0} d\rho \exp\left(i2\pi k\rho(1 - z/z_c)\right) = \frac{2J_1\left(2\pi k R_0 (1 - z/z_c)\right)}{2\pi k R_0 (1 - z/z_c)} \tag{7}$$

After the calculation we obtain the following expression for the residual phase error without piston

$$\sigma^2(\rho) = \left(\frac{D}{r_0}\right)^{5/3} 0.97 \left(\int_0^L C_n^2(z)(z/L)^{5/3} dz\right)^{-1} \int_0^L C_n^2(z) Q\left(\frac{z}{L}, \rho, R_0\right) dz \tag{8}$$

where

$$Q\left(\frac{z}{L}, \rho, R_0\right) = \int_0^\infty x^{-8/3}\left[1 - \left(\frac{2J_1(x\cdot z/L)}{x\cdot z/L}\right)^2\right]\cdot \left\{1 - \frac{2J_1\left(x(1 - z/L)R_0/R\right)}{x(1 - z/L)R_0/R} J_0\left(x(1 - z/L)\rho/R\right) + \left(\frac{2J_1\left(x(1 - z/L)R_0/R\right)}{x(1 - z/L)R_0/R}\right)^2\right\} dx \tag{9}$$

J_i - Bessel function, R-aperture radius, R_0- estimation zone radius. For the correction using one reference placed in the center of the object R_0=0. It is convenient to present the expression for the phase variance with introducing of Fried's coherence diameter:

$$\left(\frac{1}{r_0}\right)^{5/3} = \frac{2.92}{6.88}\kappa_0^2 \int_0^L \left(1 - \frac{z}{L}\right)^{5/3} C_n^2(z) dz \cdot \tag{10}$$

The expression for $\sigma^2(\rho)$ given by Eq (8) can be calculated by means of numerical integration. In many respects, Bessel functions behave as trigonometric functions with slow damping. The best integration algorithm seems to be that of Gauss-Legendre quadrature. Integrand function in Eq.(9) decreases rapidly with large value of x, so we restricted up integration limit by $x_0 \approx 20$ without significantly affected the results. However high-accuracy calculations is required, because parameter ρ as well as radius of a the estimating area R_0 appear as arguments in Bessel function. For calculation both Eqs.(8) and (9) we divided the integration interval into m_1 equal regions with m_2 points of Gauss-Legendre quadrature on every region. The calculation has shown that sufficient accuracy (four significant digits) is achieved with m_1=20 and m_2=39 for evaluation Eq.(9) and m_1=5, m_2=9 for evaluation Eq.(8).

4. Modified controlling algorithm.

The analysis proceeds here the generalization of multi-reference principal for widening of isoplanatic patch. A study was conducted to illustrate that modification of the controlling algorithm (Eq.(3)) results to redistribution the phase error over the image so to decrease it in remote regions but only with simultaneous increase in the center. Fig.2 presents the residual phase behavior for different sizes of estimation region.

If we define an isoplanatic patch as a region within which the residual error is less than its dispersion value then some conclusions can be formulated with regard to any aberration number:

1. The widening of isoplanatic patch is possible at the expense of complete correction in center.
2. For any given R_0 isoplanatic patch is larger than R_0

$$\sigma^2(\rho) < 1.03 \quad \text{if} \left|\rho\right| \le \mathrm{R}_0$$
$$\text{and } \sigma^2(\rho) = 1.03 \quad \text{if } \mathrm{R}_0 = \infty \tag{11}$$

That means that every point inside circle R_0 has image of better quality than without correction. The degree of the image quality depends on the value of R_0. The case $R_0 = \infty$ corresponds to the absence of any correction.

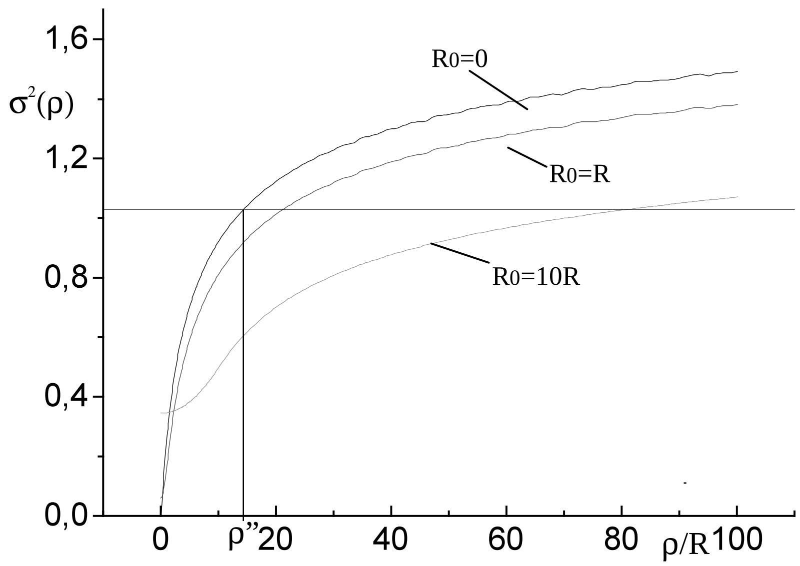

Fig.2. Residual phase error behavior on the point coordinate on the object for different sizes of estimation region R_0. Horizontal line – dispersion value that corresponds to the absence of any correction and ρ" – 'isoplanatic zone'

5. Stratified medium.

Other approach to a problem of widening field-of-view consists in utilization of specific stratified structure of the earthy atmosphere. As a particular case of this approach we have considered a system with a single thin layer and a phase corrector placed to conjugate the center of the layer. This investigation proceeds the multi-conjugated concept for widening of isoplanatic patch. In this scheem each mirror would conjugate a plane located at altitudes where the atmospheric turbulence strength is mainly concentrated. As a particular case of this approach we have considered a system with a single thin layer and a phase corrector is placed to conjugate the center of the layer.

Consider such a layer with thickness d allocated on between object's and pupil's plane near plane $z=z_s$ (Fig.1). In order to equalize various cases of layer's position and thickness in sense of Fried's radius we put in (8)

$$C_n^2(z) = \begin{cases} 1, z \in (z_s - d, z_s + d) \\ 0, z \notin (z_s - d, z_s + d) \end{cases} \qquad (12)$$

Suggest one reference in the center of an object $R_0=0$.

The value of isoplanatic patch depends on the layer allocation on the trace. The size of isoplanatic zone in dependence on allocation turbulent layer on the trace is presented on fig.3.

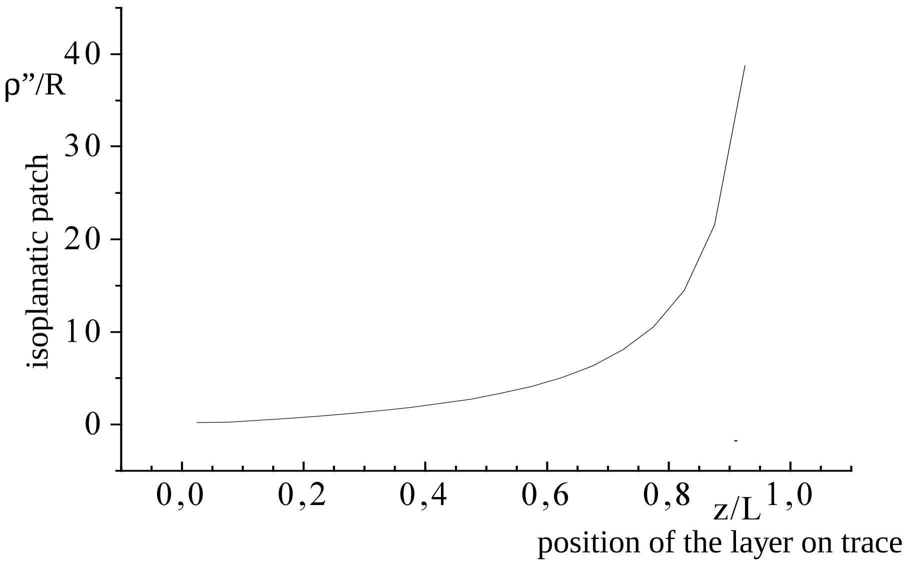

Fig.3. Size of isoplanatic patch ρ" in dependence on the position of a thin turbulent layer between object's ans pupil's planes. Width of the layer d=0.05L. z=0 corresponds to the object plane and z=L corresponds to the pupil plane. Corrector coincides with the input aperture.

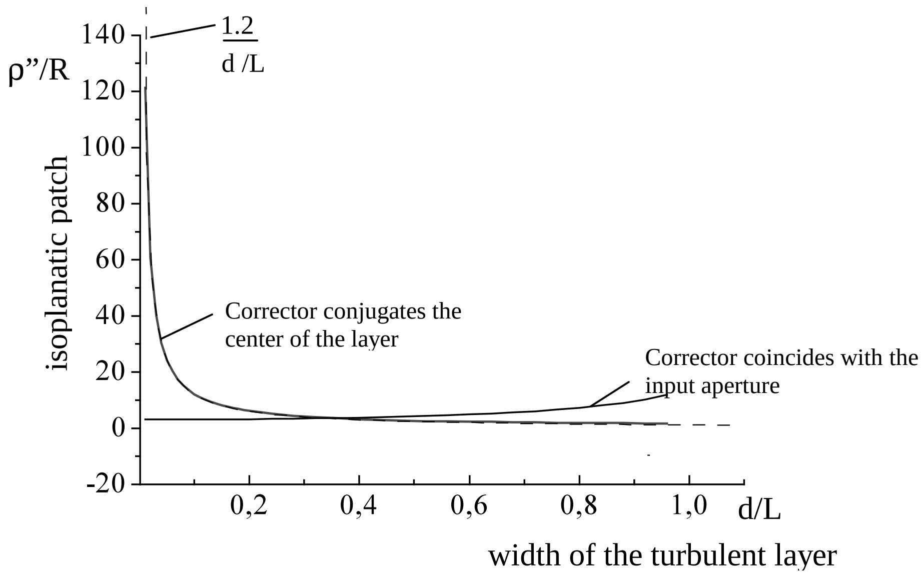

Fig.4. Size of isoplanatic patch ρ" in dependence on the width of the layer allocated in the middle of the trace.

In case of sufficiently thin layers (less than a tenth part of a trace length) size of the isoplanatic patch can be essentially increased by placing the corrector to conjugate the center of the layer. It is found that for the case overlooked above gain is defined by factor 5-6. Size of isoplanatic patch expands in 5-6 times.

In the consideration the behavior of isoplanatic patch in dependence on the lay's thickness back proportional law is set out. The size of the isoplanatic patch measured in units of aperture radius *R* is back proportional to the thickness of the layer (in units of trace length) in case when corrector conjugate the center of the layer (Fig. 4).

6. Image simulation.

From the residual phase distribution $\sigma^2(\rho)$ one can judge about the image quality. Nevertheless it does not give a sufficient visual presentation of the image itself. Only when output intensity distribution is available for analysis we can evaluate advantages and disadvantages of the adaptive optics system. On the other hand, operation of a number of adaptive optics system is based on optimization of an image quality metric dependent on image plane intensity distribution and the difficulty of optimization problem dramatically increases in case of anisoplanatism.

We have developed a direct method for distorted and corrected image simulation for realization of turbulent atmosphere. Atmospheric turbulence is modeled by set of phase screens. Using moving screens we model a wind and assuming Tailor theory to be valid we get a sample of time average image with an arbitrary expose time. Every screen in the arrange simulates current space interval on the optical path. Assuming the geometric optics propagation we evaluate phase distribution by adding phases on the screens along the ray issued from some point on the object and get to the input aperture plane.

$$\varphi_\rho(i,j) = \sum_{S=1}^{N} \varphi_s(n_s, m_s) \qquad (13)$$

Phase corrector is presented by an additional phase screen.

$$\varphi_\rho(i,j)_{cor} = \varphi_\rho(i,j) + u(i,j) \qquad (14)$$

where the correction function is defined by chosen controlling algorithm:

$$u(i,j) = f\left\{\varphi_\rho(i,j)\right\} \qquad (15)$$

The approximation precision depends on number of screens per total distance and also on the turbulence strength given as the value of C_n^2.

To simulate distorted image under anisoplanatic conditions we split the object into the number of the zones. The size of each zone must be less than isoplanatic patch.

$$O(x,y) \to \{O_{nm}(x,y)\}_{n=1..M;\ m=1..M} \qquad (16)$$

In image formation process every zone is distorted with its own point spread function corresponding to the central point of the zone. We assume that the image I(x,y) of the zone is related to the object O(x,y) by convolution relation with the point spread function

$$I^{nm}(x,y) \propto \iint dx_0\, dy_0 \left|h^{nm}(x-x_0, y-y_0)\right|^2 O^{nm}(x_0, y_0) \qquad (17)$$

Point spread function $h^{mn}(x, y)$ is proportional to back Fourier transform of complex pupil function that includes phase distribution:

$$h_{nm}(x,y)=\iint W(f_x f\lambda, f_y f\lambda)\exp\left(i\varphi_{nm}(f_x f\lambda, f_y f\lambda)\right)\exp\left(i2\pi(f_x f\lambda, f_y f\lambda)\right)df_x df_y \quad (18)$$

In expression (18) we take into account that point spread function for (n m) - zone is defined by phase distortion for central point of the zone ρ_{nm}. After the calculation of (17) for every n,m=1..M we build the image of the distorted zones:

$$\{I_{nm}(x,y)\}_{n=1..M;\ m=1..M} \rightarrow I(x,y) \quad (19)$$

Special software for the simulation was developed. This software allows to simulate a turbulent layer with the required parameters, to calculate and visualize phase distribution on the input plane and the corresponding point spread function and also to get distorted and corrected image of an object of arbitrary size and structure for different controlling algorithms.

The results of simulation are shown on Fig.5 for different sizes of the object. In this case the turbulence fills all space between object and the system. Structure constant $C^2_n=2*10^{-15}\ cm^{-2/3}$ doesn't varies along the trace $L=100m$. We put $\lambda=0.5\mu m$, $r_0=6\ cm$, $R=50\ cm$. Corrector coincides with the input aperture and it's radius is equal to input aperture radius. We assume that the reference is placed in the center of the object.

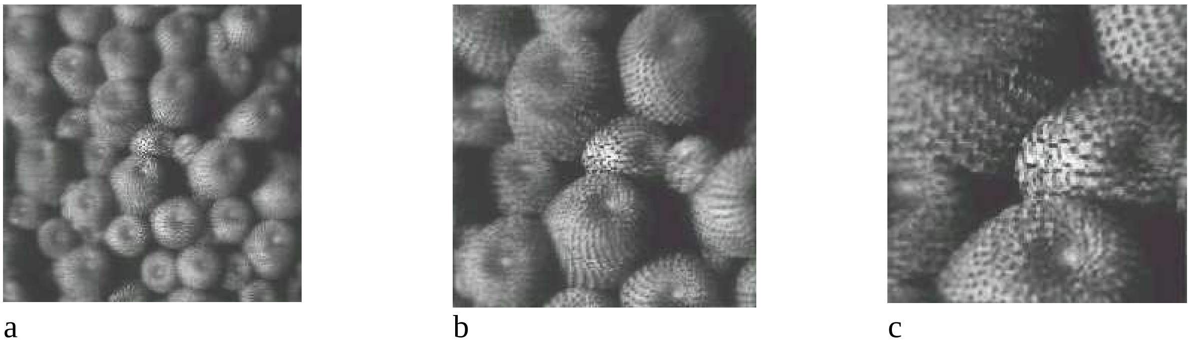

Fig.5. Corrected image for the different size of the object: a – D_0=1m, b - D_0=0.5m, c - D_0=0.25m. Single guide star is placed in the center of the object.

Simulation confirms the conclusions were made on base of the residual phase investigation. If object is small compared with the isoplanatic patch then all its parts are sufficiently corrected. For large object only a small central forms a distinctive image.

We described above the modified controlling algorithm that allows to expand the isoplanatic patch. This method is considered as theoretical case limit of multireference conception. In practice it can be applied using a number of space diverged references. The computer technique alloews to perform the simulation of corrected image using a number of guide stars. The correction function is set in the following way

$$u(r)=-\frac{1}{N}\sum_{i=1}^{N}\varphi(r,\rho_i) \quad (20)$$

The results of the simulation are shown in Fig 6.

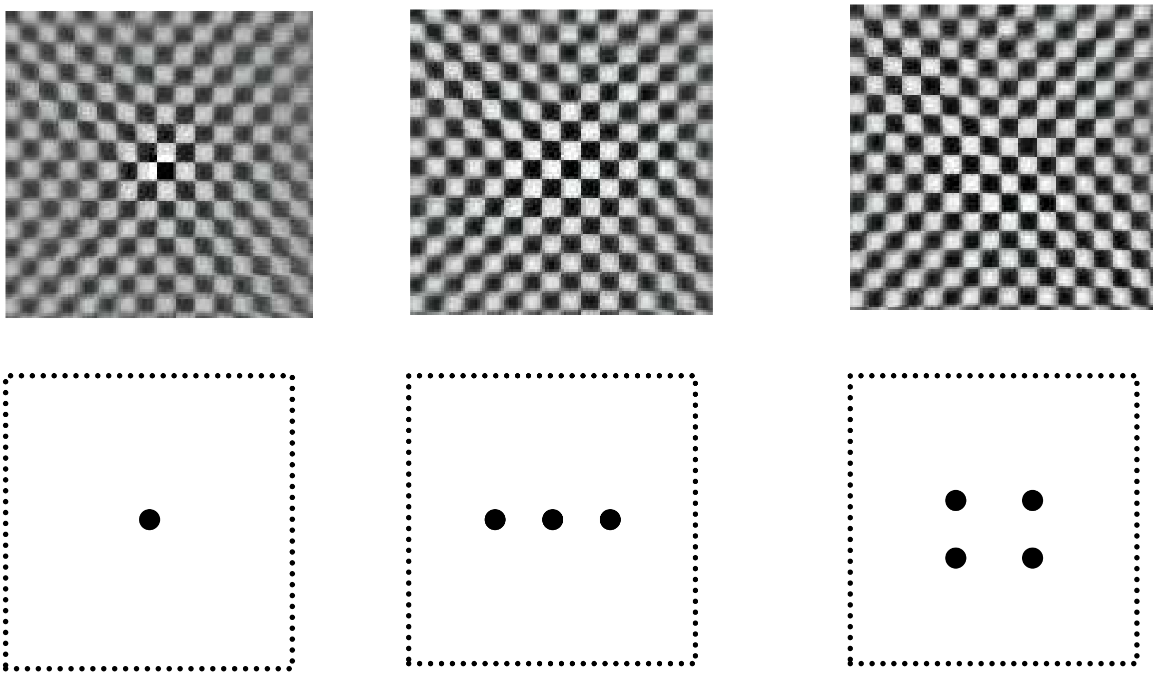

Fig.6. Image with the correction using the number of guide stars: a – N=1, b - N=3, c - N=4. Upper row – finale intensity distribution, below .- position of guide stars

7. Conclusion

The main goal of our research is to examine the opportunities of different approaches for widening field of view, including modification of controlling algorithm and the conjugation method. Our analysis was conducted for the ideal, simplest system, nevertheless it demonstrates the basic principals, advantages and the potential limits of considered concepts.

Modified correction method allows to widen of isoplanatic patch with simultaneous increase of residual phase error in center of the object when the modified is applied. In practice this method can be utilized as adaptive correction with a number of guide stars and we have shown that it is possible to increase field-of-view even in case of a small number of references.

When the opportunity of the scheme in which corrector is coincided with thin turbulent layer was investigated the back proportional between isoplanatic size and layer thickness was revealed. The investigation of the case of thin turbulent has proved the widening of isoplanatic patch with conjugated system.

Computer technique has been developed for the task of simulation corrected image in anisoplanatic condition for realization of turbulent medium (short exposure image). The results of the simulation confirm the conclusions were made on base of the analytical research.

8.References

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